AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

SYMMETRY is the theme

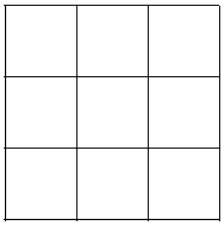
for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 4 to 18+

Just choose whatever seems suitable for your group of learners

The CHALLENGE activity was designed for Primary and Secondary

SYMMETRY CHALLENGE



How many symmetric patterns can you make by shading whole squares in a 3 by 3 grid?

HELP

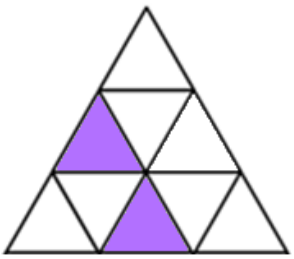
You could start with a 2 by 2 grid to help you to feel confident that you understand what to do.

NEXT

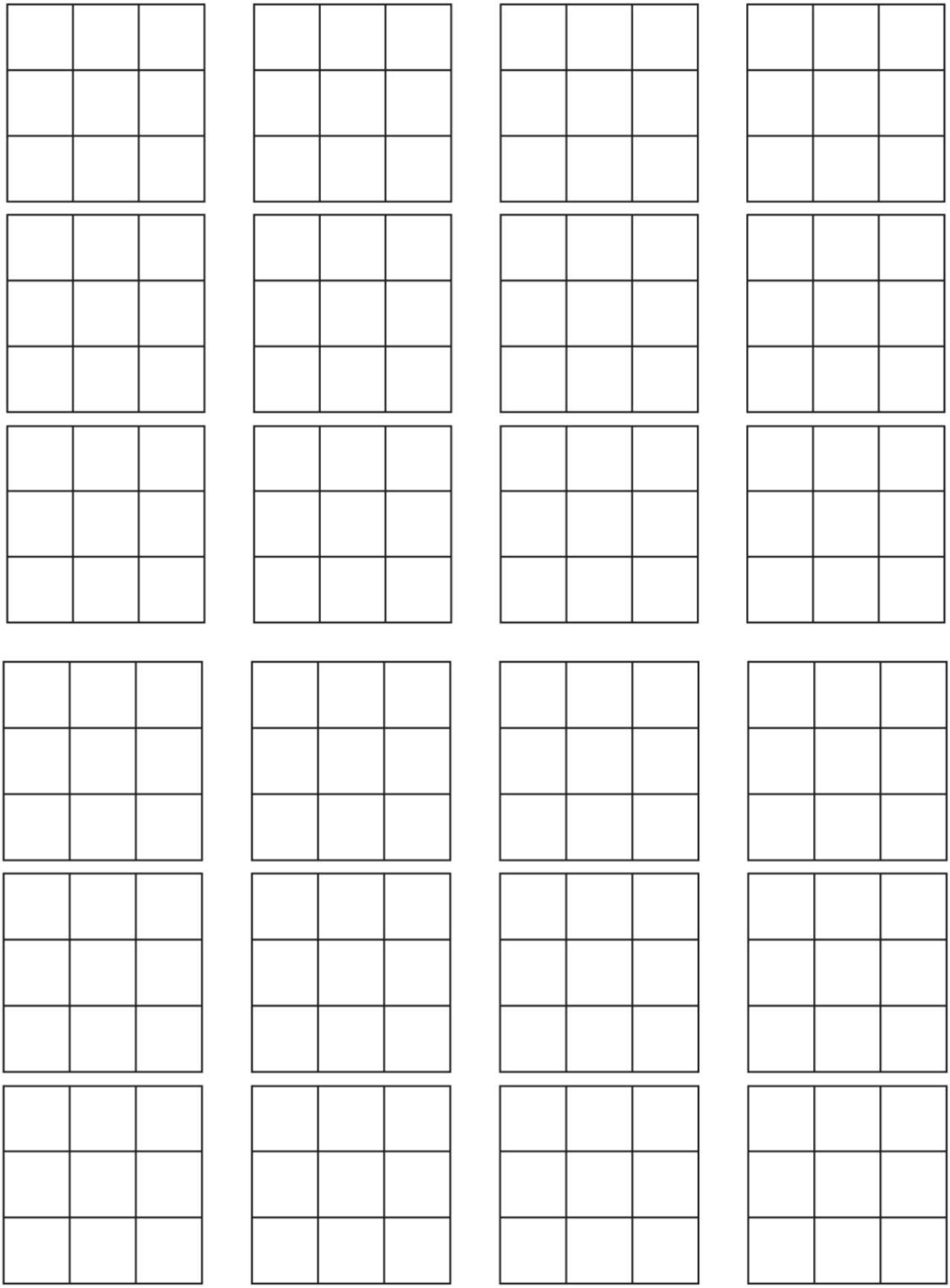
A simple extension: do any of the patterns have rotational symmetry?

Use the chart on page 3 to fill in all possible distinct symmetric patterns.
There are 64 solutions.

The challenge can be extended to larger square lattices, e.g. 4 by 4, and to investigating whether there are any differences between even and odd lengths of side.

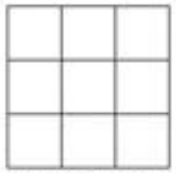


Shading symmetric patterns in an isometric grid is another possibility.

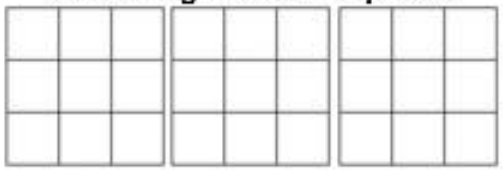


SYMMETRIC PATTERNS IN A 3 BY 3 GRID

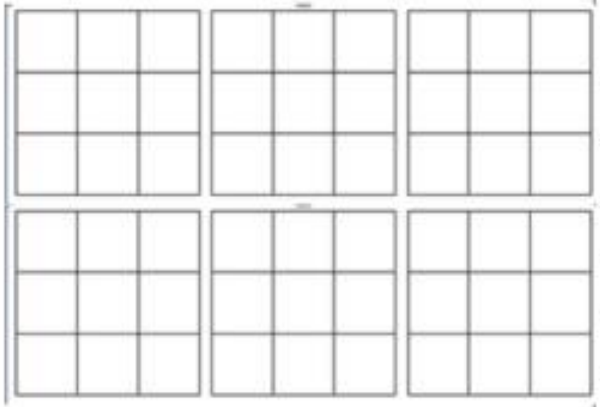
Zero or nine shaded squares



One or eight shaded squares

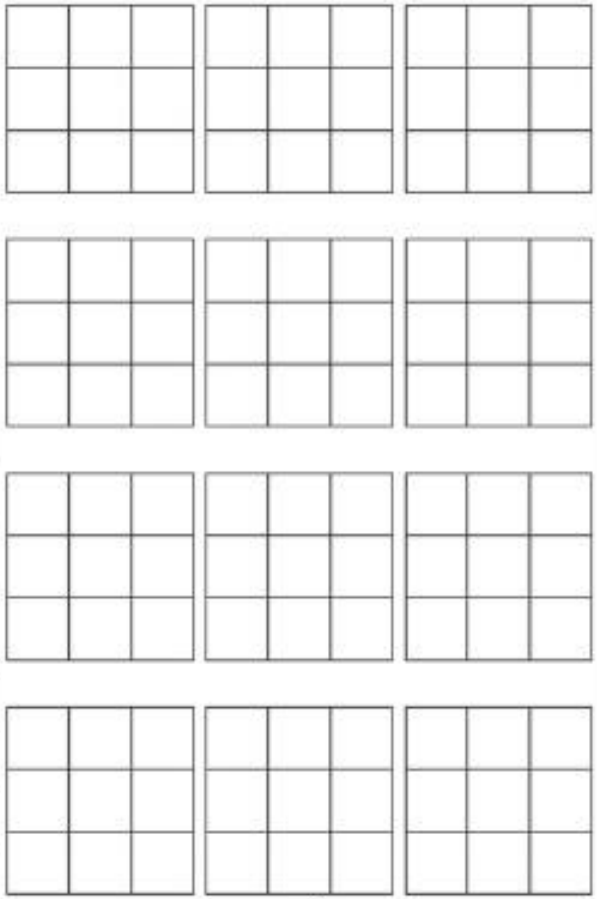


Two shaded squares



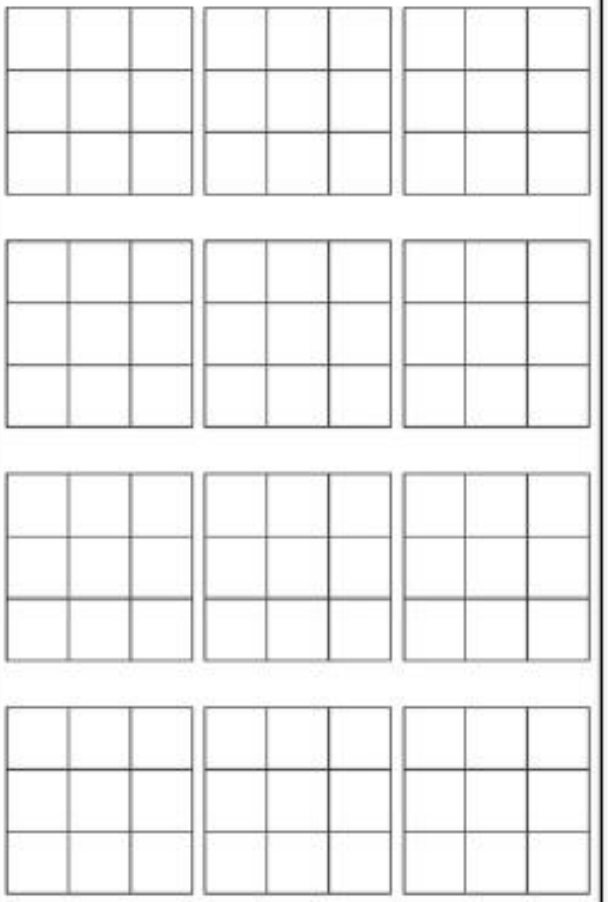
or 7

Three shaded squares



or 6

Four shaded squares



or 5

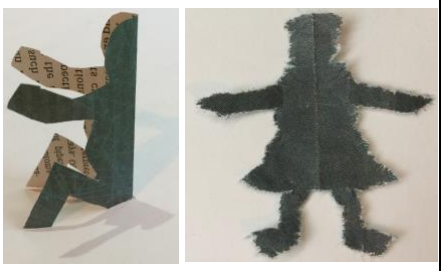
Early Years and Lower Primary

Get the learners to look at objects, and look at pictures, and talk about them.

Play with making symmetrical shapes.

Fold a piece of scrap paper.

Tear or cut a shape and then open it out. You will have a symmetric shape with the fold line making a line of symmetry (mirror line).



Introduce the word reflection.

Look at your hands. How are they the same.

How are they different.

They are symmetric to each other.

Is one hand on its own symmetric?

Look in a mirror. Ask yourself: "What else in my body is symmetric?"

Gradually help learners to notice and describe patterns and help them to develop their vocabulary. In year 3 introduce the words rotation and centre and angle of rotation, and translation.

What do you notice about these pictures?

Is Toni touching her right ear or her left ear?

This is a selfie taken by a girl in a lift? What do you notice about it?

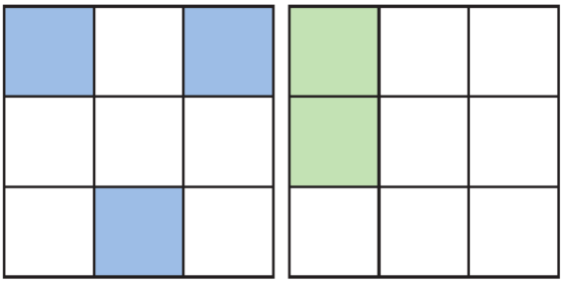
Describe these patterns.

Introduce the words rotation and rotational symmetry.

For real life examples, look at flowers.

Upper Primary and Lower Secondary

Give each learner a sheet of 3 by 3 grids (page 2).



You could start by looking together at these two shaded grids and asking the learners to say what they notice about their symmetry.

Are they symmetric or not?

Is there one line of symmetry or more than one?

What happens to the pattern if you rotate the grid? Try it.

The first one has a mirror line down the middle, it is symmetric. The second one has no symmetry.

Start with finding all the **symmetric patterns** by shading one square in the grids. Talk about it. There are only 3 distinct possibilities, a corner square, the middle square or a square in the middle of an edge. All other patterns are rotations of one of these solutions.

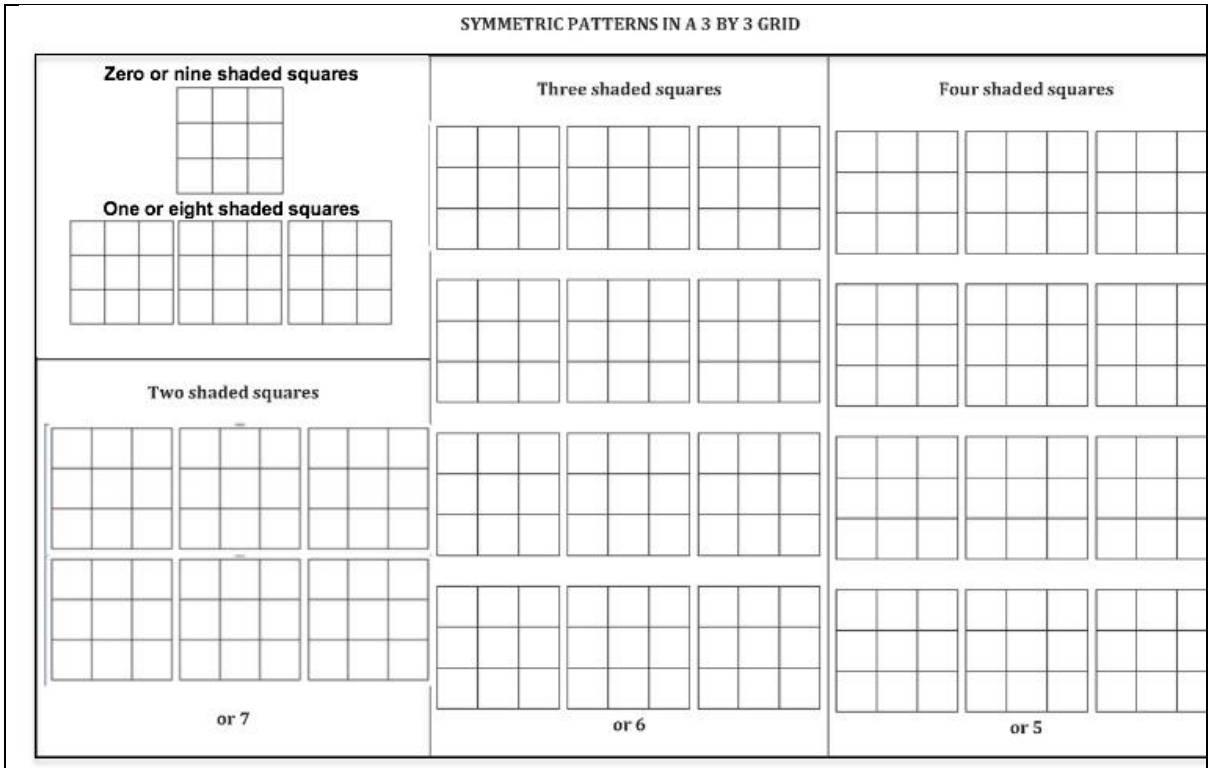
Move on to shading two squares and then three squares.
Ask:

- Are all the patterns you have shaded symmetrical?
- Where are the lines of symmetry (mirror lines)?
- Which patterns are the same and which are different?
- Are there any more symmetric patterns with the same number of squares shaded?

Ask questions and talk about some of the issues:

- What counts as the same?
- Have we found all the possibilities? How do we know we have found them all?
- How can we find other solutions?
- What if we shaded 8 of the nine squares? Or 7 …?

Then give copies of the poster template to each learner (page 3) and suggest that, you could together make one large poster. At the end of the session, if you have not found all 64 solutions then you can continue to think about the challenge and try to find the missing solutions in the next few days or weeks. You must be sure that you don't record any solution twice by recording the solution and also one of its images. Whenever someone finds a new solution their achievement should be celebrated.



Key questions

- Try shading 1 square only. What symmetries are possible?
- Have you found all the possibilities? How do you know that?
- If you try shading 2,3,4…squares, what symmetries are possible now?
- How will you record your findings?
- The 3 by 3 grid has four lines of symmetry and rotational symmetry of order 4. How might this help?

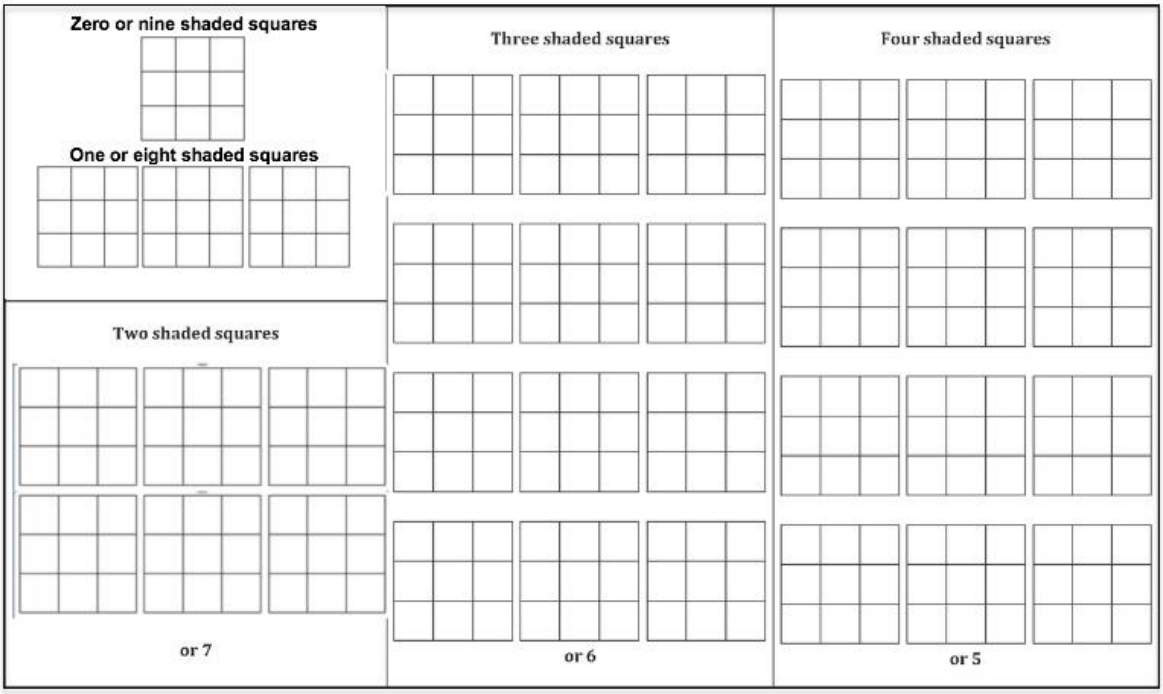
Years 9 to 12

Start this activity as for Lower Secondary but the challenge for Upper Secondary is to work systematically to find **all possible symmetric patterns** by shading whole squares. Talk about the poster until everyone understands that it provides a chart that can be used to ensure that all possible distinct solutions are found.

However, you must be sure that you don't record any solution twice by recording the solution and also one of its images by rotation.

Discuss the reason why it is a very important skill in the workplace, and in life, to be able to make a good plan for working systematically to make sure that you take into account all possible cases. Can you suggest some examples?

SYMMETRIC PATTERNS IN A 3 BY 3 GRID



If your group has not found all 64 solutions then, in odd moments, you and the children can think about more possible solutions and fill them in on the poster as you discover them.

Years 12 and 13

`Write your own version of the explanation below as to why there are 64 solutions.

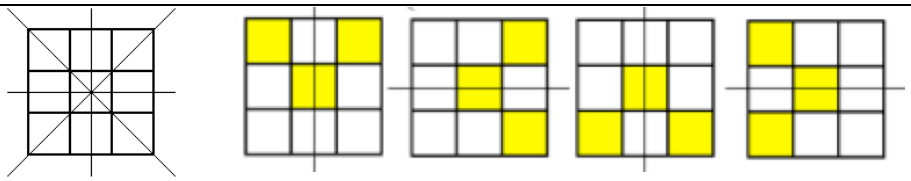
How many different symmetric patterns can you make by shading some squares in a 3 by 3 grid?

There are more than 60 possibilities. What can you say about the symmetries?

Take the top half above the diagonal; there are three regular and three half squares, a total of six options.

Each option is either "on" or "off", so two possibilities. With each option there are double and then each additional square doubles the total number of possibilities. that's why there are $64 = 2^6$ solutions.

SOLUTION



There are 4 lines of symmetry in the grid. Patterns are counted as **the same if one is an image of the other by rotation**. For example, the 4 diagrams above are counted as the same; just include ONE of these patterns, not all four. The patterns belong to 4 equivalence classes under rotation. Learners have met equivalence classes with fractions. Just as we regard $\frac{1}{2}$ as representing **all** other fractions that have the value 0.5, here we only count one pattern from each of the equivalence classes.

This gives 64 different patterns (see below). Some patterns are repeated 4 times by images under rotation, some are repeated twice, and other patterns are not repeated.

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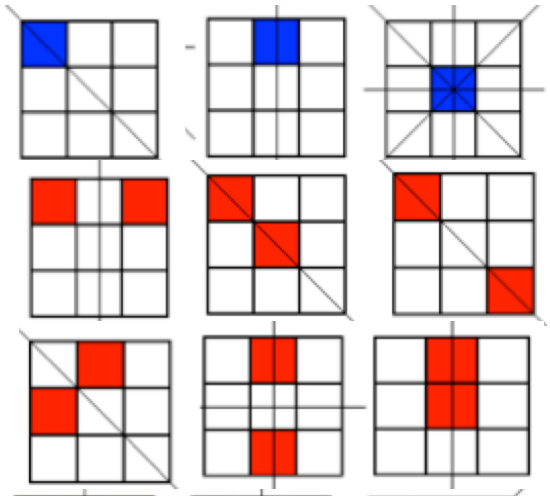
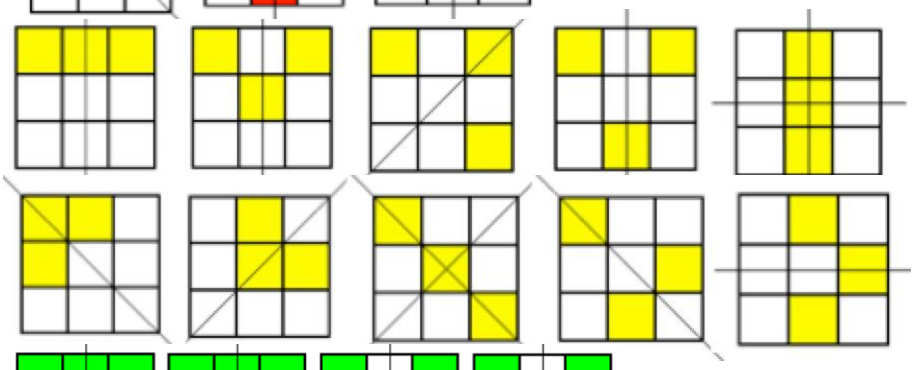
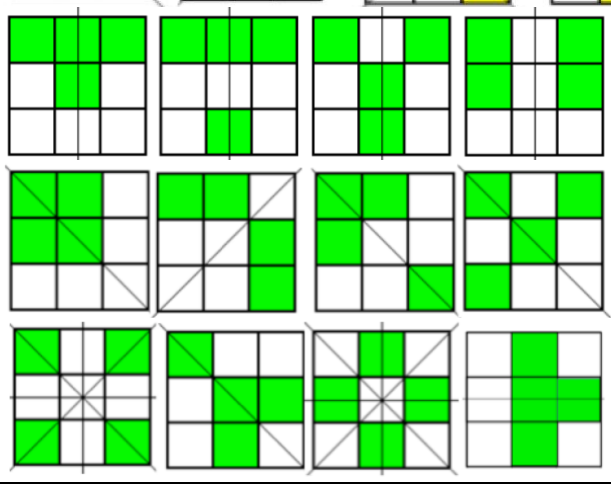


Shading just one square gives 3 symmetric patterns. The same diagrams show that, by shading the other 8 squares, there are also 3 symmetric patterns.

Shading 2 or 7 squares gives 6 symmetric patterns in each case.



Shading 3 or 6 squares gives10 symmetric patterns in each case.



Shading 4 or 5 squares gives 12 symmetric patterns in each case.

Shading 0 or 9 squares gives 2 symmetric patterns.

This makes a total of 64 patterns.

Why do this activity?

The activity requires learners to recognise and visualise the transformations of a 2D shape. They are encouraged to work systematically in a spatial environment. The problem is accessible to most learners even if they need support in organising and presenting their ideas and ensuring the completeness of their solutions.

This Symmetry Challenge activity can be done in odd moments, over a period of weeks. By recording the new patterns as they find them, and filling in solutions on a poster, children work systematically as a team to find all possible solutions.

Learning objectives

In doing this activity students will have an opportunity to:

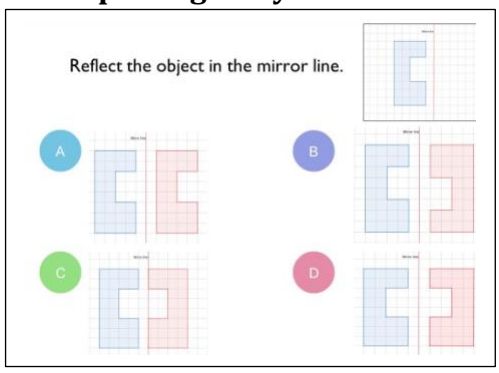
- develop a familiarity with reflection and rotation symmetry;
- learn the language of transformations.

Generic competences

In doing this activity students will have an opportunity to:

- develop visualization skills and spatial awareness;
- gain experience of working systematically to find all possible solutions to a problem;
- gain experience of sharing work in a team so as to cover all cases and achieve a set goal

Diagnostic Assessment This can be used before or after the lesson.
Show this question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer so as to clarify their own thinking by putting it into words, and to develop their communication skills.
3. If you have a group, try to make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers

The correct answer is: D
https://diagnosticquestions.com
Possible misconceptions:
A. This is a translation
B. This is a rotation
C. The two shapes are not equidistant from the mirror line (line of symmetry).

Follow up

Paper Dolls https://aiminghigh.aimssec.ac.za/years-5-8-paper-dolls/
Tangram Pattern https://aiminghigh.aimssec.ac.za/years-4-7-tangram-pattern/
Mirror Mirror https://aiminghigh.aimssec.ac.za/years-8-12-mirror-mirror/
Reflecting Squarely https://aiminghigh.aimssec.ac.za/grades-8-10-reflecting-squarely/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13