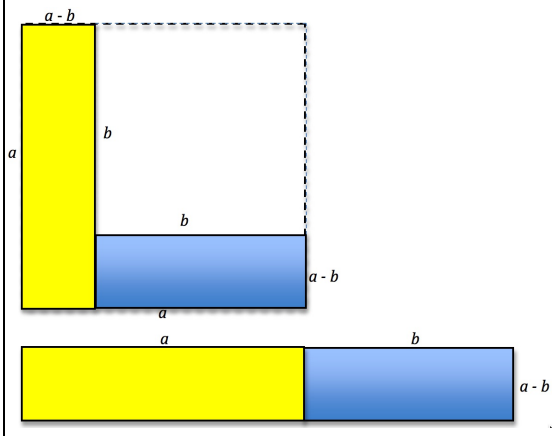
DIFFERENCES OF SQUARES AND AREA



ACTIVITY FOR ALL AGES

What do you see in this image?

Cut out your own square from scrap paper (any size).

Cut away a smaller square to form an L shape.

Cut your L shape into two rectangles as shown.

Arrange your two rectangles to make a single long thin rectangle as shown.

What can you deduce from your experiment?

What is the area of the bigger square edged by dotted line segments of length a in terms of a?

What is left of that area if you take away a square with edges of length b?

What is the area of the rectangle formed by the yellow and blue rectangles placed end to end?

What can you deduce from this?

HELP

You could cut a copy of the diagram into pieces and rearrange the pieces in different ways.

This is all about area and connecting areas of rectangles with multiplication of numbers and multiplication of algebraic expressions.

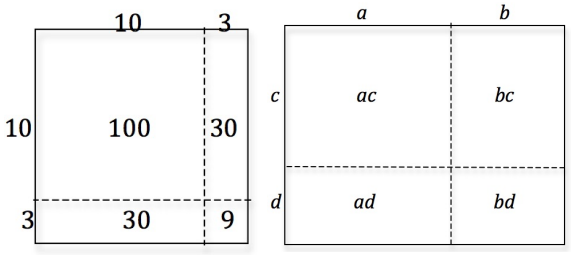
Here are two other examples.

Perhaps they will help you with the difference of two squares.

Can you explain why one shows

$13 \times 13 = 169$ and the other shows

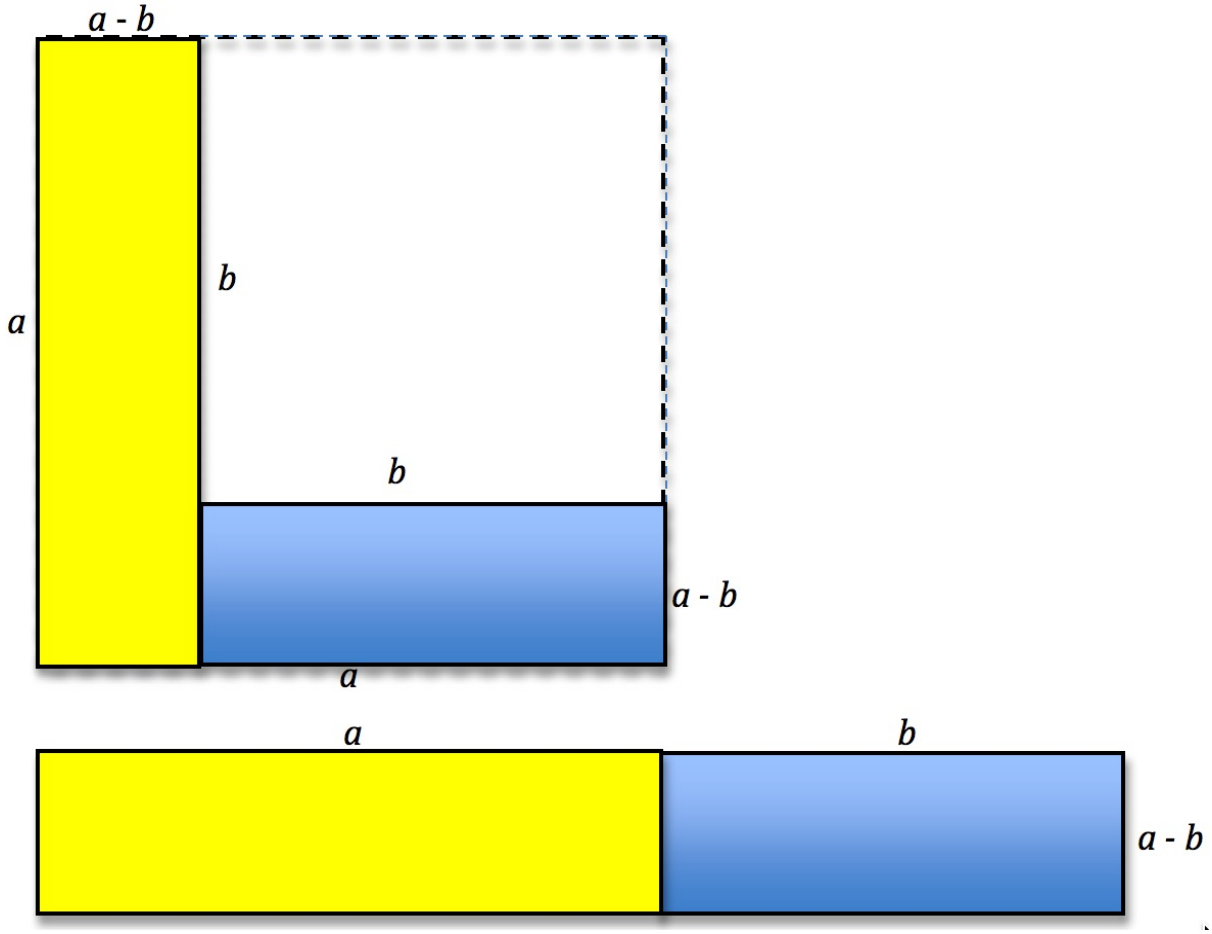
$(a + b)(c + d) = ac + ad + bc + bd$?



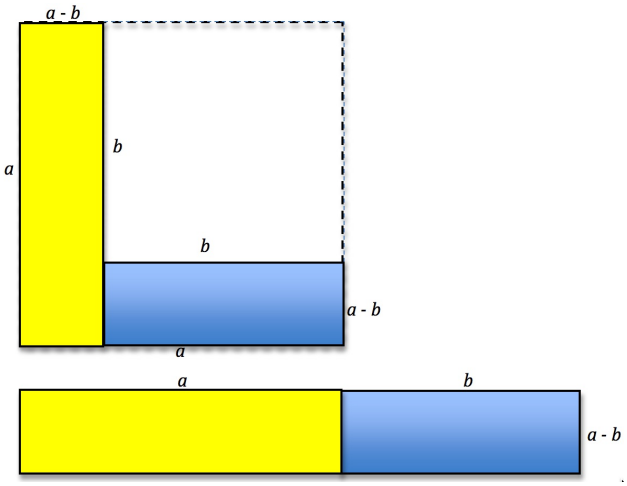
NEXT

Can you write all the numbers from 1 to 10 as the difference of two squares?

For example $3 = 2^2 - 1^2$.



NOTES FOR TEACHERS



SOLUTION

The bigger square edged by dotted line segments of length a has area a^2?

The smaller square with edges of length b has area b^2?

When you take away the square with edges of length b from the bigger square the area left, made up of the yellow and blue rectangles, is $a^2 - b^2$.

The area of the rectangle formed by the yellow and blue rectangles placed end to end is $(a + b)(a - b)$.

This illustrates the formula $a^2 - b^2 = (a + b)(a - b)$.

Multiplying the two expressions in brackets gives:

$(a + b)(a - b) = a^2 - ab + ba - b^2$ and we know $ba = ab$ so this is

$= a^2 - b^2$

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 for D".

Factorise the expression:

$x^2 - 49$

A $(x - 7)(x + 7)$

B Can't factorise

C $(x - 7)^2$

D $(x - 7)(x - 7)$

1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer so that they improve their mathematical thinking and communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. **After the learners have answered you can ask the learners who chose A to explain why they chose it. Then ask everyone to multiply (x – 7)(x + 7). Then ask them again to put up fingers to choose A, B, C or D.**
6. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A. is the correct answer.

Common Misconceptions

B. Learners may think this because this expression has only 2 terms and they have met examples of factorising trinomials.

Learners who choose **C** or **D** are making the mistake of thinking that to square what is in brackets you just square the separate terms.

https://diagnosticquestions.com

Why do this activity?

This activity gives learners the chance to think about the diagram and to see in it the proof for the formula for the difference of two squares based on what they already know about areas of squares and rectangles.

Thus they can move on **from what they know to new ideas without the teacher telling them the formula.** Brain research tells us that the best way to learn is to think flexibly about different representations of the same mathematical relationship, to see it in a visual image, and to make connections between the representations.

The activity can be used as a springboard to the use of algebra and to generalisations, conjectures and proofs.

Intended learning outcomes

In doing this activity students will have an opportunity to:

- benefit from practice in number work and spotting patterns;
- seeing the connection between multiplication, areas of rectangles and how the factorisation formula is derived, and understanding how this is illustrated in the diagrams;
- high flyers may make conjectures and be able to prove some of them.

Generic competences

In doing this activity students will have an opportunity to:

- think critically and mathematically;
- reason flexibly, be creative and innovative, and apply knowledge and skills;
- develop the skill of interpreting and creating visual images to represent concepts and situations.

Suggestions for Teaching

Start with the Diagnostic Quiz and explain that the word '**difference**' is used to describe the answer when you **subtract** one number from another.

You could introduce the activity by showing this diagram and giving learners an opportunity to work, individually or in pairs, to decide how to explain how the diagram illustrates the formula for the difference of two squares. Later in the lesson ask several learners to explain it to the class. You might want to cut out the pieces of the diagram and demonstrate how they can be rearranged. Finish the lesson with a summary of what has been learned.

Key questions

- Can you find the areas of both the squares?
- Can you find the areas of both the rectangles?
- Suppose you subtract the area of the small square from the area of the bigger square, can you write down something about the area left?

Follow up

Partitioning https://aiminghigh.aimssec.ac.za/years-7-9-partitioning/

Use area to find x https://aiminghigh.aimssec.ac.za/use-area-to-find-x/

Differences of Squares Investigation

https://aiminghigh.aimssec.ac.za/differences-of-squares-investigation/

Algebrarea https://aiminghigh.aimssec.ac.za/algebrarea-product-of-two-brackets-and-area/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel**

https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13