AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

INTERSECTIONS

(1) Plot the graphs of the three straight lines y = x, y = 1.1x - 0.2 and y = 0.9x – 0.1
What do the graphs tell you about the solutions to the two pairs of simultaneous equations:

(1a) y = x
y = 1.1x - 0.2
and
(1b) y = x
y = 0.9x – 0.1

(2) What do you notice about these two pairs of simultaneous equations:

(2a) x + 0.99999y = 2.99999
0.99999x + y = 2.99998
and
(2b) x + 1.00001y = 2.99999
0.99999x + y = 2.99998.

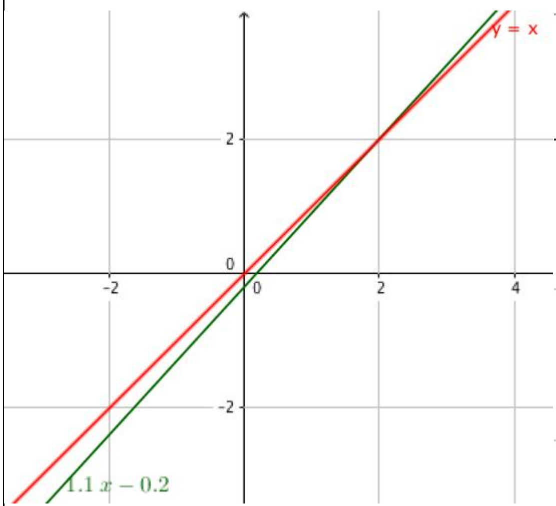
Using a calculator, check that the solutions to (2a) are x = 2 and y = 1 and the solutions to (2b) are x = -199 998 and y = 199 999.

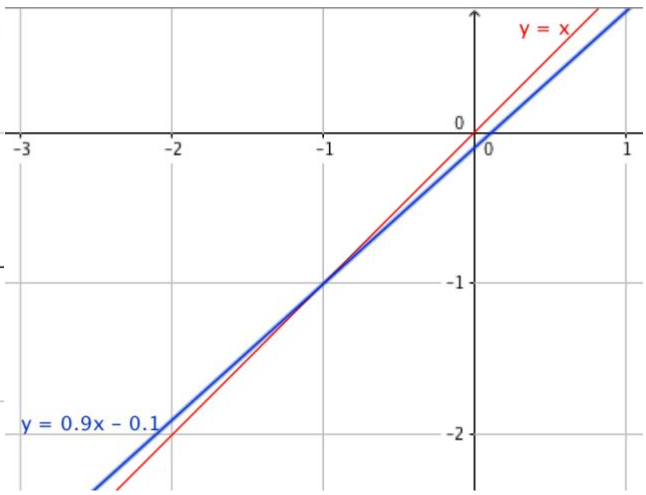
Consider the geometrical properties of the lines and explain why although the pairs of equations are nearly identical, the solutions are very different.

HELP

Study these two graphs and the pairs of simultaneous equations.
What do you notice about these two graphs?
What do the graphs have to do with the equations?
What do the graphs tell you about the solutions of the pairs of simultaneous equations?

y = x
y = 1.1x – 0.2 and y = x
y = 0.9x – 0.1





NEXT

Make up your own similar example like the one given in the HELP section. What do the graphs tell you about the solutions of the two pairs of simultaneous equations?

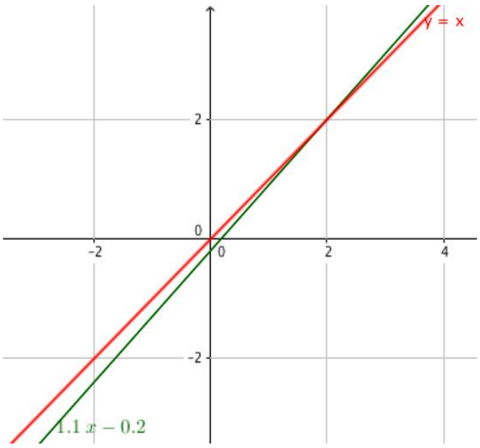
NOTES FOR TEACHERS

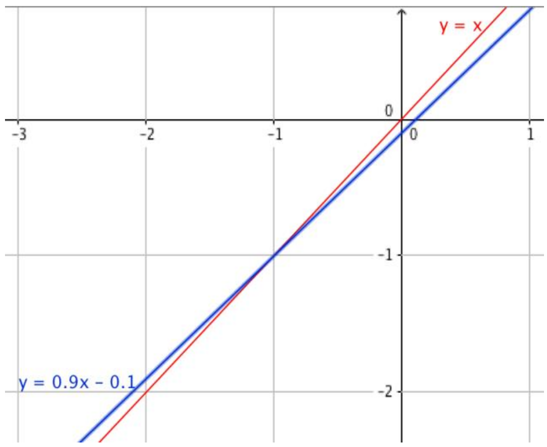
SOLUTIONS

The pair of equations

y = x
y = 1.1x - 0.2
have solutions x = 2, y = 2

and the pair of equations
y = x
y = 0.9x – 0.1
have solutions x = -1, y = -1





Although the pairs of equations have coefficients that are close in value, when comparing the two sets of graphs, in one case the second line is steeper than the line y = x and so the graphs intersect in the first quadrant. In the other case the second line is less steep than the line y = x and so the graphs intersect in the third quadrant where the coordinates of the point of intersection which give the solution are both negative.

This effect is much more extreme in the following example where the differences in gradients much smaller.

(2a) x + 0.99999y = 2.99999
0.99999x + y = 2.99998
and
(2b) x + 1.00001y = 2.99999
0.99999x + y = 2.99998.

The solution for (2a) are x=2, y=1 and
the solutions for (2b) are x=−199998, y=199999.

All four lines cut the y-axis very near to (0, 3).

For the first two lines one gradient is -1.00001 and the other gradient is -0.99999.
For the second pair of lines the gradients are -0.9999900001 and -0.99999, so the lines are nearly but not quite parallel.

Because the pairs of lines in each case are nearly parallel the slight change in each line's gradient away from minus one causes the intersection to occur in very different places.

Why do this activity?

It is helpful for students to make the connection between algebraic methods, like solving equations, and geometry or graphs. This is a good question for a revision lesson for learners who should know how to solve pairs of linear simultaneous equations and who also know how to plot the graphs of straight lines.

Learning objectives

In doing this activity students will have an opportunity to:

- practise solving simultaneous equations;
- deepen understanding of algebra by noticing connections to graphs.

Suggestions for teaching

You may choose to give part (1) to the learners and discuss the solutions with the class before assigning part (2). You can decide whether or not to let the learners use Geogebra (or other graphing software) if they have access to it.

When the learners have solved the equations in part (1) and drawn the graphs have a class discussion about what they have noticed. In the discussion ask key questions that will prompt the learners to notice that the pairs of lines in each case are nearly parallel. If they don't remark on it, ask further questions to draw their attention to the slight change in the gradients away from minus one which causes the intersections to occur in different places.

Then ask the learners to do part (2) but not to plot the graphs. As usual, the HELP box on page 1 can be given to learners who have difficulties to help them get started.

Key questions

Study the pairs of graphs and the corresponding pairs of simultaneous equations.

(1) What do you notice about the pairs of graphs?
(2) What do the graphs have to do with the equations?
(3) What do you notice about the gradients of the lines corresponding to the equations?
(4) Why are the solutions of the two pairs of simultaneous equations so different when the equations are so nearly the same?

Follow up

Some further questions on simultaneous linear equations:

Matchless https://aiminghigh.aimssec.ac.za/matchless/

Symmetricality https://aiminghigh.aimssec.ac.za/symmetricality/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6