AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE

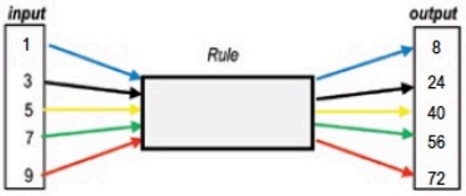
THE FUNCTION GAME

with related learning activities for all ages from 4 to 18

on the theme of INPUT AND OUTPUT

Choose what seems suitable for the age or attainment level of your learners

FUNCTION GAME – GUESS MY RULE



What is the rule (or function) that gives the outputs corresponding to the inputs shown in the diagram mapping 1 to 8 and 3 to 24 etc?

This is like translating from one language to another – input English and out comes isiXhosa!

ACTIVITY FOR ALL AGES

The Function Game is played in silence. Players have to guess the rule in the teacher's mind.

She asks players to give her numbers and writes them in a column of inputs.

For the rule she is thinking of, she writes an output beside each input. The teacher says **"The outputs give clues for finding my rule" and asks them to tell her what she is doing.**

The rule can be simple or complicated to suit the players,

Other players can take the teacher's role.

A player who thinks he or she has guessed what the teacher is doing puts up a hand and the teacher suggests a number and asks the learner to do the same with it and give the answer. If the learner gets the right output the teacher congratulates him or her and writes it on the list.

In class, the game continues until about half of the learners have guessed the rule. Then learners who have guessed it explain what the teacher was doing for the benefit of those who did not guess it and the game is repeated with another rule.

For example the rule might be "multiply the number by 3 then add 5".

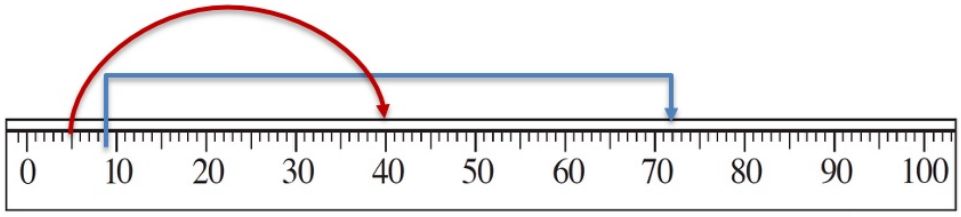
That gives $\mathbf{2 \rightarrow 11,\ 3 \rightarrow 14,\ 10 \rightarrow 35}$ etc.

Algebraically the rule is called a **function** and written as $\boldsymbol{x \rightarrow 3x + 5}$

FUNCTION GAME FOR 2 PLAYERS

The 2 players, take it in turns to think of the rule and the other player tries to guess the rule with the fewest clues. Have 5 turns each, the player who has made fewest guesses in all wins, and the player who needed more guesses is the loser.

You can show what happens on a number line. What rule does this show?



HELP

If 1 → 2 and 2 →4, and 3→6 and 4→8 can you guess the rule?

What about 1 → 3 and 2 →5, and 3→7 and 4→9 can you guess this rule?

Try again to find the rule for the picture in the top box.

NEXT

VARIATION TO INVERSE FUNCTIONS

When we think '+7 days and -7 days' for this time next week or this time last week and '+12 months and -12 months' for forwards and backwards in years and '+24 hours and -24 hours' for tomorrow and yesterday – forward and back 24 hours on the clock, we are thinking of functions and their inverses without knowing exactly what the process is called.

COMBINATIONS OF FUNCTIONS

Think about this example: for the function 'multiply by 3 then add 5' (written $x \rightarrow 3x+5$), if the output is 41 how do you find the input?

You first undo the 'add 5' by subtracting 5, then undo the 'multiply the number by 3' by dividing by 3 which gives the input 12.

The inverse function is $x \rightarrow (x - 5)/3$.

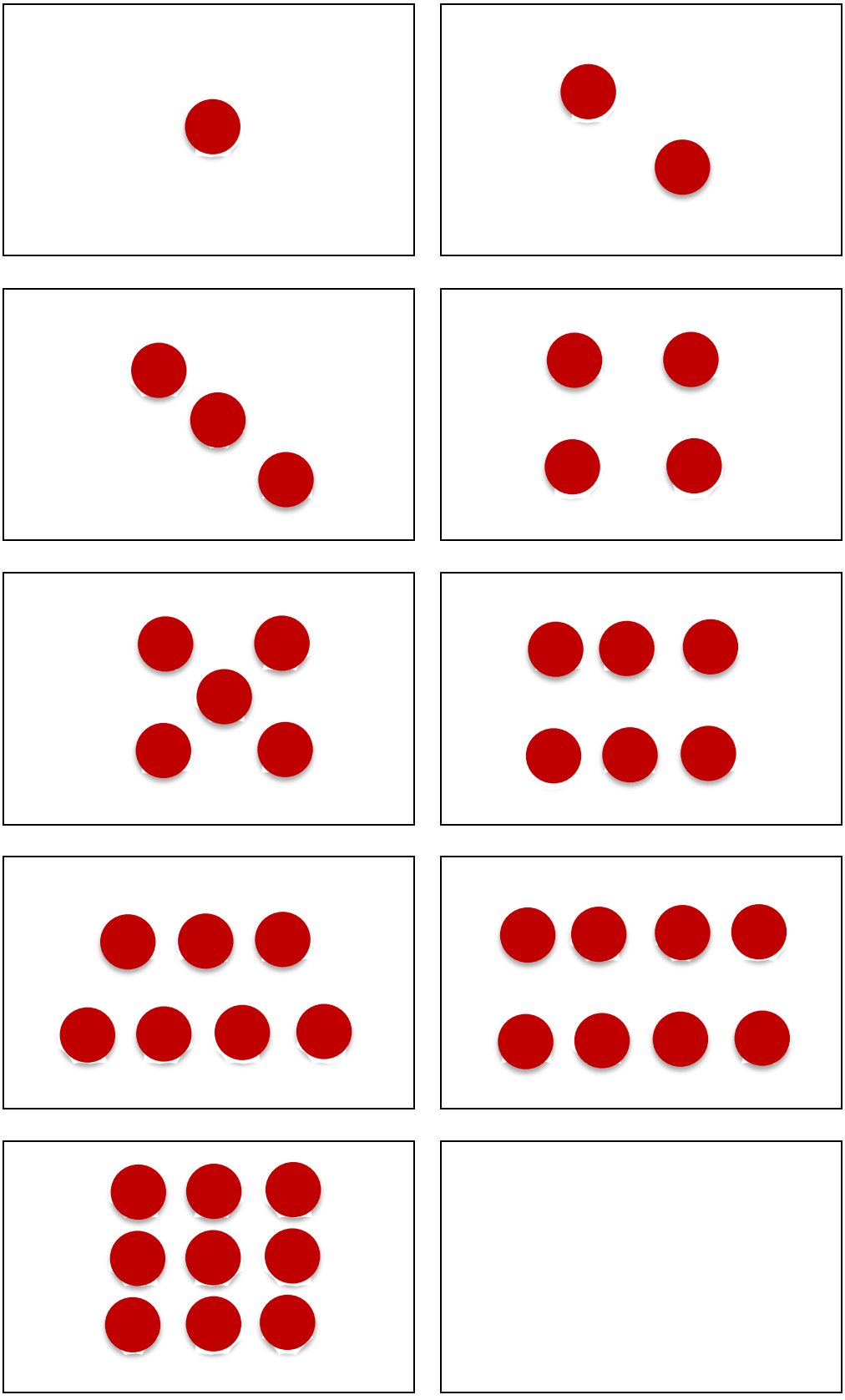
Investigate what happens when other functions are combined with their inverses and answer the question:

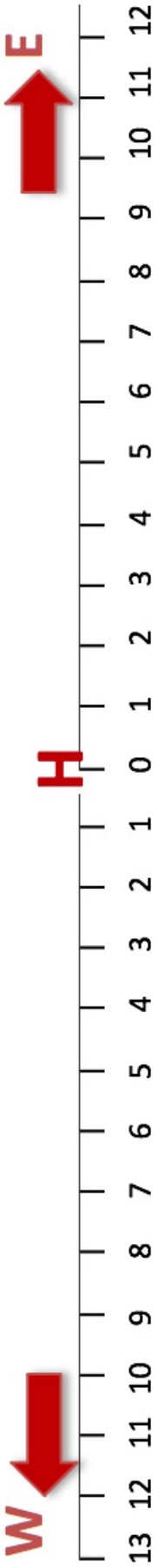
What happens when you combine a function with its inverse function?

THE INVERSE FUNCTIONS GAME FOR 2 PLAYERS.

Player 1 gives the outputs.
Player 2 (thinking of both a function and its inverse function) writes Player 1's number in the **ouput** column and the corresponding number in the **input** column.
This continues until Player 1 guesses the rule (also called a function) and also the inverse function and gets his total number of guesses as his score.
Then the two players exchange roles.
The players have 5 turns each, the player who has made fewest guesses in all wins.
The player who needed more guesses is the loser.

1	2
3	4
5	6
7	8
9	0





EAST WEST GAME

Resources: An East West Number Line as shown, 2 counters and 2 dice or spinners.

In this game, you go some steps East or West in the direction shown by the arrow.

Players start at H. Player W wants to get to 12West. Player E wants to get to 12East. They take turns to throw 2 dice. One score (say on the red die) gives the number of steps. The other score (say on a green die) gives the direction: 1, 3 or 5 means move West while 2, 4, or 6 means move East. For example, a score of (3, 4) gives 4 steps West.

The other die gives the number of steps to move. The winner is the first player to get to their destination.

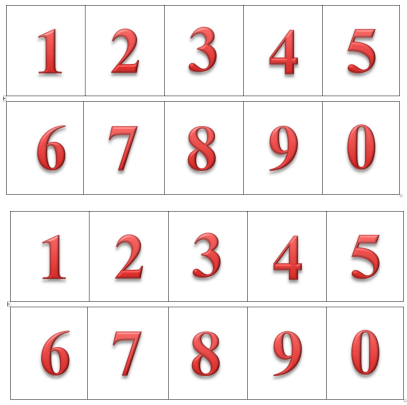
INCLUSION AND HOME LEARNING GUIDE

THEME: INPUT AND OUTPUT

Early Years - WHAT AM I DOING? GAME

When you first introduce this game you need 2 sets of dotty cards printed and cut out, or copied, from page 4. Later when the children can play the game with the dotty cards you can use either 2 sets of digit cards printed and cut out, or copied, from page 3, or one set of cards of each sort. You should judge what 'rules' you use to suit the child or children involved. With subtraction only do '-1' so that you avoid negative numbers. After each round return all the cards to the central 'pool'.

Start with 2 sets of cards



GAME FOR 2 PLAYERS. You give me one card from the pool and I give you one card. I say "What am I doing?"

Suppose you give me a 4 and I give you a 5, then you give me a 2 and I give you a 3… you would probably say "Your number is I more than mine" or something like that. You might need me to do this several times with different numbers.

If instead, I want to double your number, and you give me a six (6) then I would use a 1 and a 2 to make a twelve (12).

Every time you guess correctly what I am doing you score a point. To win you must score 5 points. If the child wants to do so you might reverse roles in the next round.

GAME FOR A TEACHER WITH A GROUP If the teacher plays this game with a group, then the children take it in turns to give her a number, and the teacher holds up her number (or two numbers) and says "What am I doing?" The teacher keeps doing exactly the same operation on different numbers until someone guesses what she is doing. The winner is the first child to guess correctly what the teacher is doing.

To give the 'rule' in words, the children have to think mathematically and they have to make the first (and most difficult) step in stating the function. This lays the foundation for later work.

You want the children to enjoy the game. Just play it a few times and come back to it another day. They will get better and better at it.

Lower Primary - WHAT AM I DOING? GAME

Play this game as described for Early Years but make the operations you do on the numbers more complicated as the children get better at the game. If the learners have not played the game before then use 2 sets of dotty cards printed and cut out, or copied, from page 4. Later when the children can play the game with the dotty cards you can use either 2 sets of digit cards printed and cut out, or copied, from page 3, or one set of cards of each sort.

Start with 2 sets of cards

1	2	3	4	5
6	7	8	9	0

1	2	3	4	5
6	7	8	9	0

GAME FOR 2 PLAYERS. You give me one card from the pool and I give you one card (or perhaps 2 cards).

I say "What am I doing?"

Suppose you give me a 4 and I give you a 9, then you give me a 2 and I give you a 5… you might say "You are doubling my number and adding 1" or something like that. You might need me to do it a few more times with different numbers before you can guess what I am doing.

If instead, I just want to double your number, and you give me a six (6) then I would use a 1 and a 2 to make a twelve (12).

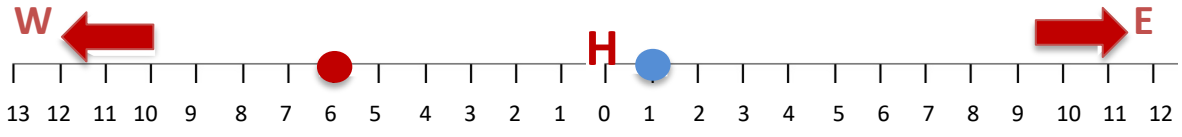
Every time you guess correctly what I am doing you score a point. To win you must score 5 points. The players could reverse their roles in the next round.

GAME FOR A TEACHER WITH A GROUP If the teacher plays this game with a group, then the children take it in turns to give her a number, and the teacher holds up a number (or two numbers) and says "What am I doing?" The teacher keeps doing the same operation on different numbers until someone guesses what she is doing. The winner is the first child to guess correctly what the teacher is doing.

EAST WEST GAME

You will need an East West Number Line as shown, 2 counters and 2 dice or spinners.

The teacher should explain that, in this game, you go some steps East or West in the direction shown by the arrow. Players start at H (the zero point). Player W wants to get to 12West. Player E wants to get to 12East. They take turns to throw 2 dice. One score (say on the red die) gives the number of steps. The other score (say on a green die) gives the direction: 1, 3 or 5 means move West while 2, 4, or 6 means move East. For example, a score of (3, 4) gives 4 steps West. The winner is the first player to get to their destination.



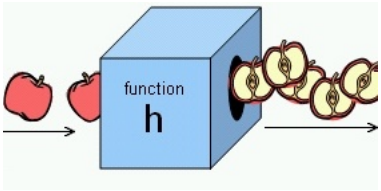
Discuss different rules, like for example, the rule 'Go 7 steps to the East'. You should use lots of simple rules before you make it complicated.

Suppose the chosen rule is
'**Go half-way back towards H for HOME then go 4 steps to the East**'.
From the position of 6West as shown by the red counter in the picture, 'half-way towards Home' goes to 3West, and then 4 steps East goes to 1East as shown by the blue counter.

From the red position of the counter listed below the rule 'Go half-way back towards H for HOME then go 4 steps to the East', the counter goes to the blue position listed:

4 East from Home	6 East from Home
4 West from Home	2 East from Home
10 East from Home	9 East from Home
10 West from Home	1 West from Home
12 West from Home	2 West from Home
7 East from Home	Between 7 East and 8 East from Home ($7\frac{1}{2}$ East from Home)

Upper Primary

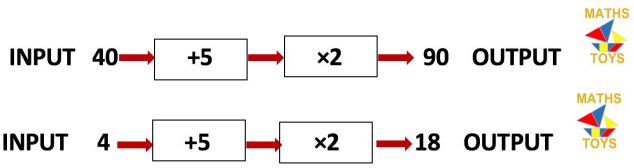


What is happening in this function machine? Perhaps it's labelled as h, for **HALF,** because the fruit is cut in half.

Don't put your hand in this machine! 😂

With your learners make your own function machine using any old cardboard box. Make holes in two opposite faces, label the hole on the left INPUT and label the other hole OUTPUT. Suppose your machine halves numbers rather than fruit. You'll need some scrap paper cut into small pieces. Ask a learner to write a number on a scrap of paper and put it into the INPUT hole. You should take it out through the OUTPUT hole, look at it and write the output on another scrap of paper. For example, if 20 is the input then your output should be 10.

You should be able to make your machine do many different calculations, so it would be a good idea to draw a dial on the front face that you can pretend to turn while you stick a different label on the front of the machine. Change your machine to an +5 machine. Get a second box and make a ×2 machine, so your two boxes together make a Double Function machine. These two flow chart diagrams both show the machine called **'Add 5 and double the number'**



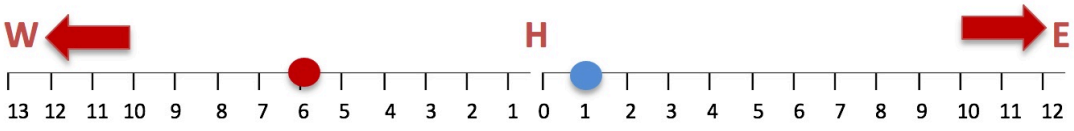
You might like to watch this YouTube video https://youtu.be/sBTkYoDdC0g where a group of AIMSSEC teachers model an **'Add 5 and double the number'** machine in a different way. If you do this with your learners, then it's better to use scraps of paper and, instead of the teacher giving numbers to the learners, the 'People Machines' actually write the numbers on scraps of paper and pass them on. So, the +5 machine passes the OUTPUT from the +5 machine on as the INPUT to the ×2 machine.

After acting out your own function machine your learners are ready to play the Function Game (or Guess my Rule Game) as described on page 1 with your whole group. It can be played with any number of players.

Key questions

What am I doing?
If I give you the number 5 can you do the same and give me the answer?
Can you explain in words what I am doing with your numbers?

In another lesson PLAY THE EAST WEST GAME as described for Lower Primary.

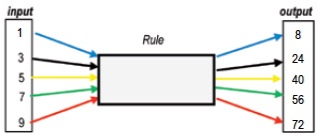


Lower Secondary

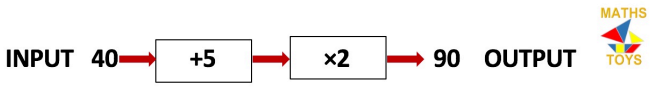
Play the Function Game as described on page 1. With younger learners play it several times before gradually introducing the mathematical language. Also with younger learners play the game often, and only introduce the variation with inverse functions in a later lesson.

You could enact your own function machine as a 'People Maths' activity, or make one from old cardboard boxes as described in the Upper Primary section.

Ask the learners to make up their own rule or function and draw a flow chart for it like this one for **×8**:



or like this double (combined) function machine:



They could work in pairs or small groups. After they have invented their own function charts they can exchange them with other learners who have to find the rule. Or they can come to the board in pairs and play the role of the teacher in leading the Function Game with an older person or teacher acting as referee.

Finally, everyone should work together to provide a summary of what has been learned giving the mathematical terms: rule, input, output, function and inverse function.

FOR OLDER LEARNERS - VARIATION TO INVERSE FUNCTIONS (whole group game)

The game can be varied by sometimes using the numbers suggested by learners as inputs and sometimes as outputs, the teacher writing the numbers in the appropriate input and output columns. This can lead to a class discussion of inverse functions.

For example, for the function $x \rightarrow 3x+5$, if the output is 41 how do you find the input? You undo the "multiply the number by 3 then add 5" by first subtracting 5, then dividing by 3 which gives the input 12. The inverse function is $x \rightarrow (x - 5)/3$.

PLAYING THE FUNCTION GAME IN PAIRS

First the players must agree on what numbers are allowed as inputs, and what types of function are allowed. Players might agree on whole number inputs and +, -, × and ÷ or choose a different set of input numbers and allow more functions. Players take turns to create their own function for the other player to guess. The winner is the player who guesses the rule correctly with the smallest number of clues.

KEY QUESTIONS

What am I doing?

If I give you the number 5 can you do the same and give me the answer?

Can you explain in words what I am doing with your numbers?

For older learners: Now can you explain that with a formula?

If the rule is x → 3x + 5 and the input is 6 what is the output?

Give different rules & inputs.

If the rule is x → 3x + 5 what does 6 map to? Give different rules and inputs.

If the rule is x → 3x + 5 what do you do to the input to get the output?

If the rule is x → x + 5 what do you do to the output to get the input?

Upper Secondary

Follow the instructions on pages 1 and 2. Challenge a friend to play the **FUNCTION GAME**. Perhaps use all the functions you know about.

Start playing with a few functions and then raise the level of challenge by including more and more functions and different sets of input numbers. You could use calculators because the skill is not in doing the calculations but rather in understanding functions so well that you can confidently use and apply them.

You can combine 3 or more functions and also play the **INVERSE FUNCTION GAME**.

Play the game with younger people restricting the choice of the set of input numbers, and the functions allowed, so that they are using functions that they know about.

Why do this activity?

By playing this game learners will learn about functions and inverse functions, and they will practice finding the formulas in an enjoyable way. By asking "What am I doing?" the teacher can introduce functions ***without using any technical language*** and then later she can introduce and use the words 'rule', 'input', 'output', 'function' and 'inverse function'.

After that frequent use of these terms while playing the game will help the learners to learn the mathematical language.

These games can be played many times for a short while each time, to provide a gentle introduction to functions and also, for older learners, as a way to revise previous work on functions and inverse functions. Learners need a lot of practice in thinking algebraically and these activities provide practice in an undemanding and playful way.

Learning objectives

In doing this activity students will have an opportunity to:

- learn about functions in a playful way;
- experience a ***gradual*** introduction to the mathematical language used in discussing functions;
- **learn about East and West** (in one version of the game and lay a foundation for learning about negative numbers and vectors;
- practise thinking algebraically;
- practise visualization and 'thinking in pictures' using flow charts, mapping diagrams and the Number Line;
- practise making and testing conjectures.

Generic competences

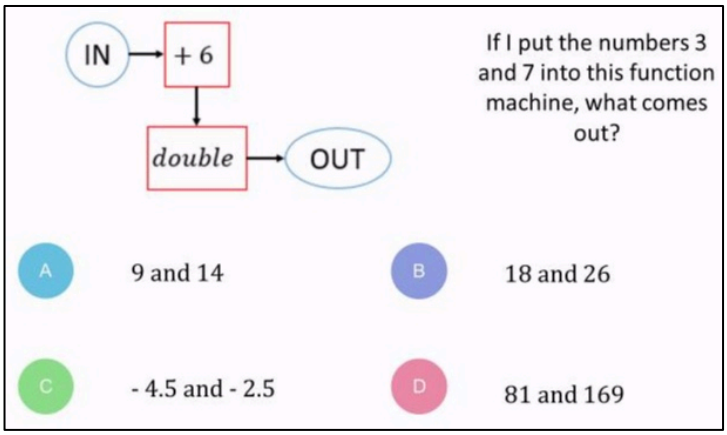
In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **learn about East and West** (in one version of the game);
- **exchange ideas**, think critically, present ideas to others and record ideas effectively.

DIAGNOSTIC ASSESSMENT This can be done before or after the lesson and as a group as described below, or the question can be answered individually.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is B. **3 → 9 →19 and 7→13 →26**

A. These learners have added 6 to the first number and doubled the second number.
C. Perhaps some halving has been done here instead of doubling.
D. These learners have squared instead of doubling. https://diagnosticquestions.com

Follow up

Function Flow https://aiminghigh.aimssec.ac.za/function-flow/
Swop https://aiminghigh.aimssec.ac.za/swop/
Building Functions https://aiminghigh.aimssec.ac.za/building-functions/
Undoing https://aiminghigh.aimssec.ac.za/undoing/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
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