AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

PRIME SIEVE

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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

On the 100 square grid, choose a colour, circle the number 2 and make a line through 4, 6, 8, 10 and the rest of the 2 times table like this:

You do not cross through the first number in the table but it may already be crossed out.

Change colour, circle the number 3 and make a line through 6, 9, 12. 15 and all the rest of the 3 times table.

Do the same for the 5 and 7 times table.

What do you notice about the multiples of 8, 9 and 10?

What can you say about the numbers that are not crossed out?

The number 4 has 3 factors: 1, 2 and 4. How many factors does 5 have? What about the other circled numbers?

Put a circle around all the other numbers that are not crossed out **except** the number 1. How many factors do they have? What can you say about them?

HELP

Working in pairs to share ideas and support each other.
If you are having trouble with this question you should just colour multiples of 2, on the first grid below then just multiples of 3 on the next grid and multiples of 4, 5, 6, …, 10 on the other grids. What do you notice about the patterns of these multiples.

Then follow the instructions for How to Shade the Prime Sieve given below on page 2.

1	2	3	4	5	6	7	8	9	10
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51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 8

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 9

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

PRIME SIEVE

HOW TO SHADE THE PRIME SIEVE

1. Put a circle around the number 2. Cross out all the **other** multiples of 2.
2. Put a circle around the number 3. Cross out all the **other** multiples of 3.
3. Put a circle around the number 5. Cross out all the **other** multiples of 5.
4. Put a circle around the number 7. Cross out all the **other** multiples of 7.
5. Now make a list of all the numbers that are NOT crossed out except the number 1.
6. What do you notice about the numbers on your list?

NEXT

Now do the Further Questions below.

After that, what can you say about making a 1 to 400 sieve? Which multiples will you cross out to be sure that you are left with the primes?"

We're used to working with grids with ten columns, but you might find an interesting result if you use this six-column grid instead. See page 8. Can you **predict** what you will see?

FURTHER QUESTIONS

To get all the prime numbers between 1 and 100, why is it only necessary to cross out all the multiples up to multiples of 7 and not multiples of 1?

If you had to find all the prime numbers up to 200 by the sieve method what multiples would you need to cross out?

Use this grid to find all the prime numbers between 1 and 200.

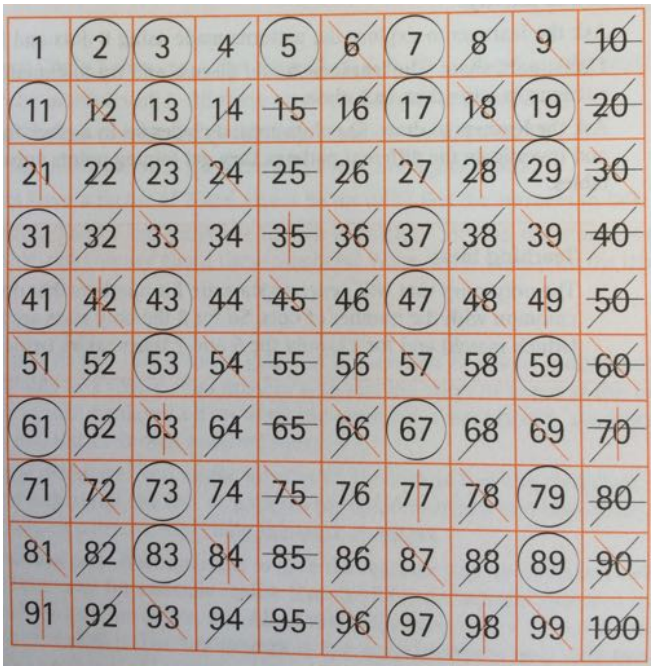
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

NOTES FOR TEACHERS

SOLUTION

The prime numbers between 1 and 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.



The circled numbers have only 2 factors, the number itself and 1. Numbers with exactly 2 factors are called **PRIME NUMBERS**.

The number 1 is not a prime number as it has only one factor.

Numbers crossed out twice have 2 prime factors, numbers crossed out 3 times have 3 prime factors etc.

It is not necessary to cross out multiples of 11 because they are 22, 33, 44, 55, 66, 77, 88 and 99 which are already crossed out as multiples of 2, 3, 4 etc. Each is the product of a number smaller than 10 and a number bigger than 10.

For the 1 to 200 sieve, as 14 × 14 = 196, it is only necessary to cross out multiples of 2, 3, 5, 7, 11 and 13. All numbers up to 200 that have 2 factors will have one factor less than 14 and one factor more than 14.

The primes between 1 and 200 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197 and 199.

Do this after the learners have completed the prime sieve activity to give them the opportunity to discover prime numbers for themselves.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

1, 2, 4, 5, 9, 31, 33

A. 2
B. 3
C. 4
D. 5

The correct answer is B. 2, 5 and 31 are prime numbers, the other numbers are composite.
A. Perhaps the students did not recognise that 31 is a prime number.

C. Perhaps the students thought that 1 is prime.

D. These students do not understand prime numbers or do not know that 33 = 3 × 11 so it is not prime.

https://diagnosticquestions.com

Why do this activity?

This activity enables learners to discover prime numbers for themselves. It emphasises the fact that some numbers are prime, others have a few factors and others have many factors. Learners can experience visual learning. It can be used to introduce or to reinforce the concept of prime numbers.

Learners should do the Patterns of Multiples Sheets 1, 2 and 3 first SEE PAGE 2. Spread this over several weeks with other activities like Orchestra and Clap Hands so there are both active and sitting activities and not too much of the same thing in a session.

This activity offers opportunities to explore multiples in more depth than usual, in particular looking at the links between multiples of different numbers. It also encourages students to see the connection between primes and multiples.

Learning objectives

In doing this activity students will have an opportunity to:

- to know the vocabulary associated with multiplication, division and prime numbers;
- to discover that certain numbers have the special property of having exactly 2 factors and are called prime numbers;
- to engage in mathematical thinking about the process and why, to find the prime numbers less than 100, it is only necessary to mark multiples of 2, 3, 5 and 7 and not of any larger numbers.

Generic competences

In doing this activity students will have an opportunity to:

- **visualize** patterns and develop the skill of interpreting and creating visual images to represent concepts;
- **persevere and work systematically** to investigate all possible cases.

Suggestions for teaching

Resources (1) Learners should have a copy of the worksheet on page 1, or a 1-100 number grid, or squared paper, and colouring pens. Alternatively you could use one large grid on paper or on the board.
(2) Patterns and Multiples worksheets page 2.
(3) For high flyers: Six column grid and 1 to 400 grid.

Leave the diagnostic quiz until AFTER the learners have completed the prime sieve activity.

LESSON STARTER: You might start the lesson by the Counting and Clapping activity. Everyone counts in multiples of 2, speaking loudly and clapping on the even numbers and speaking softly and **not** clapping on the odds. Then half the class clap on multiples of 3, saying those numbers loudly while the other half of the class simultaneously repeats the clapping on 2's. Ask 'What do you hear?" "What numbers will be loud?" "What numbers does everyone clap on?". You might want to repeat this for another pair of multiples and then talk about '*common multiples'*.

MAIN LESSON *Encourage discovery.* The worksheet deliberately avoids mentioning the words *prime numbers* so that, whether or not learners have met prime numbers before, they will discover the prime numbers by themselves using this sieve method. So just ask questions to guide the learners to notice patterns and encourage them to give reasons for what they observe.

You might wonder why, unlike the clapping activity, you don't colour in the first number of a table. That is because eventually the learners discover that the uncoloured numbers have exactly 2 factors, they are the PRIME numbers, those numbers that have no FACTORS other than themselves and 1.

There are lots of patterns to look for – numbers coloured twice have two factors, numbers coloured three times have three factors etc. You might want to look for the way the numbers make a pattern on the grid.

If you don't have 1-100 number grids for the learners to colour individually, you could do this as a whole class activity. You will need one big sheet of paper (say flip chart paper) at the front on which you have drawn the 1 – 100 grid. Alternatively you could draw the grid on the chalkboard or the learners could start by making their own number grids.

It is important to give the learners time to talk about what they notice. This helps them to get a 'feel' for the way numbers fit together. We call this '*number sense'*. They might for example realise that when they cross out all the multiples of 2 then, at the same time, they are crossing out multiples of 4, 6, 8 and 10.

Finally make sure that the learners know the mathematical words 'multiple', 'common multiple', 'factor' and 'prime number'. At the end of the lesson summarise what has been learned in the lesson.

Key questions

When multiples of 2, 3 5 and 7 have been crossed out:

- Which numbers get crossed out more than once, and why
- Which numbers don't get crossed out at all, and why?
- What do you notice about the multiples of 8, 9 and 10?
- What can you say about the numbers that are not crossed out?
- The number 4 has 3 factors: 1, 2 and 4. How many factors does 5 have? What about the other circled numbers?
- Put a circle around all the other numbers that are not crossed out except the number 1. How many factors do they have? What can you say about them?

- Why do you get all the prime numbers between 1 and 100 by marking all multiples up to multiples of 7?
- Why is it not necessary to cross out multiples of 11?
- If you had to find all the prime numbers up to 200 by the sieve method what multiples would you need to cross out?
- Which possible factors do we need to consider in order to decide if a number is prime?

Follow up

Find the prime numbers using the 6-column grid on page 8.

See Chapter 1 of the AIMSSEC Book Mathematical Thinking in the Lower Secondary Classroom
http://aimssec.ac.za/our-work/mt-book-series/

For more information about Eratosthenes and his sieve method see:
http://www.mathsisgoodforyou.com/artefacts/sieve.htm
There is more about this activity on http://nrich.maths.org/7520

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA. **New material will be added for Secondary 6.** **For resources for teaching A level mathematics see https://nrich.maths.org/12339**				
	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**

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