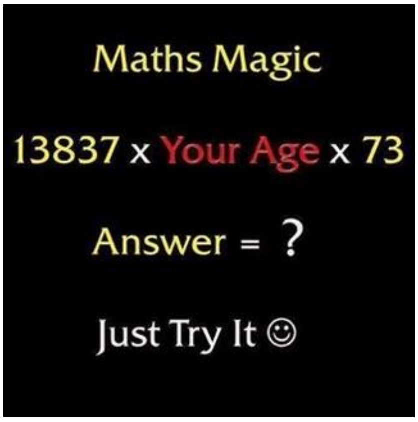
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

MAGIC 13837

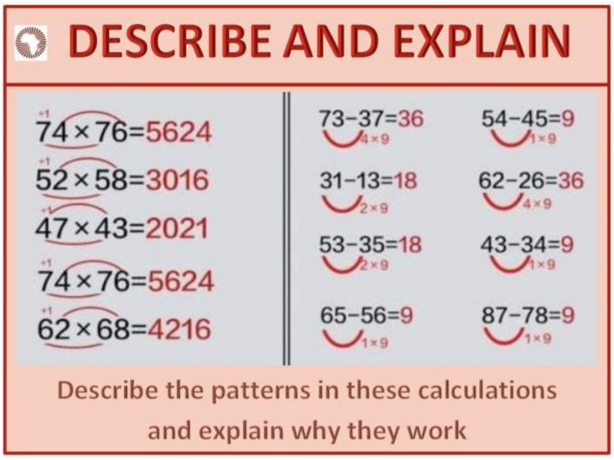


Try the ages of your family and friends.

What happens?

Can you explain it?

Does this work for very old people over the age of 99? Why or why not?



The product of two 2-digit numbers can be written as (10a+b)(10c+d). Notice that
(1) in all the examples on the left, the tens digits are repeated so a = c
(2) the units digits add up to 10 so b + d = 10
(3) the product is given by 100a(a+1)+ bd, e.g. 100(7x8) + 4x6.
Can you prove this result?
(4) Can you prove that the quick calculation method shown on the right always works?

HELP

Work out:

a. 101 × 23
b. 10 101 × 23
c. 101 × 45
d. 10 101 × 45
e. 1 010 101 × 45

What do you notice? Can you explain what happens?

This should help you to explain the magic of 13837.

NEXT

The oldest verified age of a living person was 122 years and 164 days: Jeanne Calment of France, who died in 1997. Would this number trick work for her age? Why or why not? Work out

(a) 1001 × 234
(b) 10001 × 4567
(c) 1001001 × 432
(d) 1001001001 × 765

What happens? Why?

Can you make up some similar multiplications like this?

NOTES FOR TEACHERS

SOLUTION

13837 x 73 = 1 010 101

and 1 010 101 multiplied by any 2-digit number gives that number repeated 4 times.

Suppose your age is 10p + q then

1 010 101(10p + q)

= 1 000 000(10p + q) + 10 000(10p + q) + 100(10p + q) + 1(10p + q)

= 10 000 000p + 1 000 000q

+ 100 000p + 10 000q

+ 1 000p + 100q

10p + q

= pq pqp qpq

1 001 101 is 1 million 10 thousand one hundred and one.

Suppose you are 12 years old, then

1 million 10 thousand one hundred and one × 12

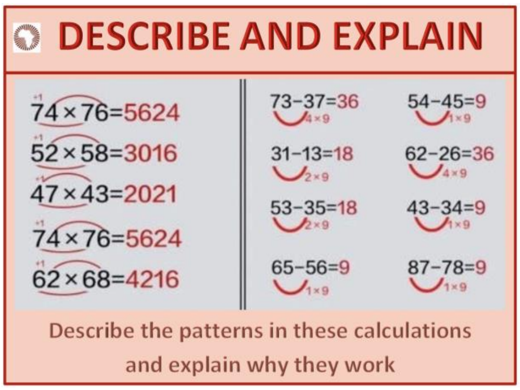
= 12 million 120 thousand 12 hundred and 12

= 12121212

NEXT Answer: the trick only works for 2 digit numbers. To get a similar result for a 3 digit number you must multiply by 1 001 001 001.

For Jeanne Calment's age:

1 010 101 × 122 = 122 million + 1 million 220 thousand + 12 thousand 2 hundred + 122 = 123 232 322)



The product of two 2-digit numbers on the left can be written as (10a+b)(10c+d). Notice that

(1) in all the examples, the tens digits are repeated so a = c

(2) the units digits add up to 10 so b + d = 10

(3) the product is given by 100a(a+1)+ bd, e.g. 100(7x8) + 4x6

PROOF

(10a + b)(10c+d) = (10a + b)(10a +d) which reduces by simple algebra to the form given in (3) above.

(4) The examples on the right are all of the form (10a+b) - (10b+a) where (10a +b)>(10b+a)

(5) The differences are all of the form 9(a - b)

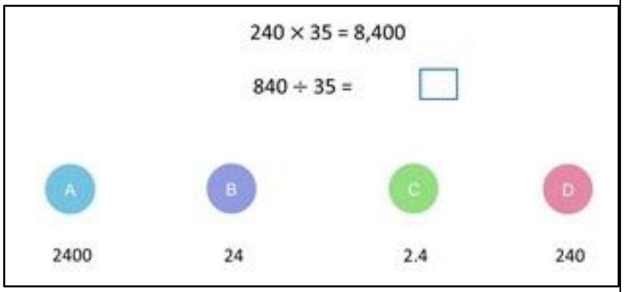
PROOF

(10a+b) - (10b+a) = 9a - 9b = 9(a - b)

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 fingers for D".



1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer to get practice in communication of mathematical ideas.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
5. The concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is B

A. This answer may arise from observing that 840 is on tenth of 8400 then multiplying 240 by 10 instead of dividing it by 10.
C. This answer may arise from observing that 840 is on tenth of 8400 then dividing 240 by 100 instead of dividing it by 10.
D. This answer shows the learner did not understand that 240 is wrong because this calculation involves 840 and not 8400. https://diagnosticquestions.com

Why do this activity?

This will seem like a trick to learners that they can play on their family and friends even without doing long multiplication as most cellphones these days have a calculator app. Explaining why this trick works will help learners to get a better understanding of place value, to appreciate the importance of the commutativity of multiplication and to develop their abilities to think logically and to explain mathematical ideas.

Learning objectives

Learners will gain:

- practice in using the properties of place value and the commutativity of multiplication;
- a deeper understanding of the process and operation of multiplication;
- development of reasoning and communication skills;
- (For older learners) practice in proving results by using algebra.

Generic competences

In doing this activity students will have an opportunity to interpret and **solve problems.**

Suggestions for teaching

Write the question on the board and ask the learners to work on it individually. This will give them practice in tackling questions in the way that they have to do for assessment. You may choose to let them use calculators or tell them to work it out by long multiplication.

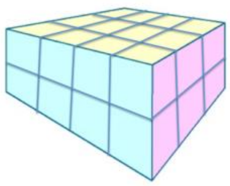
After most of them have found an answer and are trying to find an explanation ask them to work in pairs to compare answers and to find an explanation.

Then have a whole class discussion of the reasons. Accept all explanations without comment and keep asking "can anyone explain this in a different way?" This will encourage learners to speak up and not to be afraid of being told that their answer is wrong.

Then ask learners if

13837 × their age × 73 is the **same or different** from 13837 × 73 × their age.

You might write these on the board and ask the learners to vote yes or no.



If there are any learners who think that these are different then ask them to work out some examples like 2 × 3 × 4 and 2 × 4 × 3 and other similar examples. You might use a 2 by 4 by 3 cuboid made from unit cubes and discuss why the volume is 24 cubic units given by 2 layers of 3 by 4 cubes or 3 layers of 2 by 4 cubes or 4 layers 2 by 3 cubes.

Then ask them to say the number 1010101 aloud. Most learners will say **one zero one zero one zero one** or even **one o one o one o one**. Explain to the class that although this language is commonly used for serial numbers, telephone numbers and dates it is **not the correct mathematical language**.

Ask the class what the first 1 in 1010101 means. You might write it as **1** 010 101 to make this clear. (Answer 1 million)

Then ask them what the second 1 in 10**1**0101 means. You might write it as 1 0**1**0 101 to make this clear. (Answer 10 thousand)

Then ask them what the third 1 in 1010 **1**01. You might write it as 1 010 **1**01 to make this clear. (Answer 1 hundred)

Then ask them what the last 1 in 1010 10**1**. You might write it as 1 010 10**1** to make this clear. (Answer 1 unit)

Again ask the learners to say the number 1010101 aloud using the correct mathematical language. Keep the class discussion going until the learners understand that this is

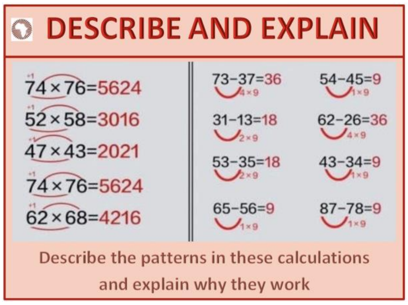
1 million 10 thousand 1 hundred and 1.

Then tell the class that you are going to give them another 5 minutes to work in pairs to find an explanation and follow this by a class discussion in which, if necessary, you keep asking key questions to guide learners towards an explanation.

With older learners you could ask them to use algebra to explain the trick.

Key questions

- Is 13837 × your age × 73 is the **same or different** from 13837 × 73 × your age? Why or why not?
- Is 2 × 3 × 4 the **same or different** from 2 × 4 × 3? Why or why not?
- What is the answer to 13837 × 73?
- How would you say the number 1 010 101?
  (You'll need to write this down and not say it yourself)
- How old are you? What is 1 million times your age?
- What is 10 thousand times your age?
- What is 1 hundred times your age?
- What is 1 times your age?
- If you know: 1 million times your age, and 10 thousand times your age, and 1 hundred times your age, and 1 times your age, then how would you find 1 010 101 times your age? What is the answer?



Follow up

The second question Describe and Explain is a good example of using place value to explain and prove simple number relationships.

Legs Eleven https://aiminghigh.aimssec.ac.za/years-10-12-legs-eleven/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13