## AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES <br> SCHOOLS ENRICHMENT CENTRE (AIMSSEC) AIMING HIGH

## HOW MANY FACTORS?



How many factors does 72 have?
Investigate numbers that are products of a square number and a cube number. How many factors do they have?
How does the number 1 behave in the world of factors?
What can you say about numbers that have exactly 2 factors?
What can you say about numbers that have exactly 3 factors?
Give some examples of numbers with 4 factors. What do they have in common?
Give some examples of numbers with 5 factors. What do they have in common?
What about numbers with 6 factors?
Give some examples of numbers with 12 factors.
What is the smallest number with exactly fourteen divisors?

## HELP

How many factors does 72 have?
Can you find a connection between the number of factors of 72 and the exponents in the expression giving the factorisation of 72 into primes?
$72=2^{3} \times 3^{2}$.
You might find it easier to explore the problem in the context of finding all possible rectangles with whole number edge lengths that have the same area. For example you could draw all the rectangles with area 24 and you will find that there are four of them and they are: 1 by 24,2 by 12, 3 by 8 and 4 by 6 . Notice not 5 by anything and when you go up beyond 5 you just get the same rectangles again.

Then think about the problem of working out how many such rectangles there would be without drawing them all.


#### Abstract

NEXT What is the smallest number with exactly 100 factors? Which number less than 1000 has the most factors? These, and other similar questions, could be explored with paper and pencil using prime factorisation or it could be an opportunity for you to use a spreadsheet or simple coding (programming). If you have an interest in programming you might wish to consider how to write a simple program to find all the factors of a number. For very large numbers, the realisation that you only need consider potential factors less than the square root of the number speeds up a program considerably!




