



**AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)**

AIMING HIGH

THE FACTORS AND MULTIPLES GAME Inclusion and Home Learning Guide

is part of a Learning Pack downloadable from the AIMING HIGH website

<https://aiminghigh.aimssec.ac.za/years-6-12-factors-and-multiples-game/>

It provides related activities for home learning for all ages and learning stages from pre-school to school-leaving, together with guidance for home-learning and also for inclusion in school lessons, all on the **Common Theme FACTORS AND MULTIPLES** Guidance for school lessons is given in the separate Notes for Teachers documents.

Choose what seems suitable for the age or attainment level of your learners.

FACTORS AND MULTIPLES GAME

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
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This is a game for one or two players.

The first player chooses a positive even number that is less than 50 and crosses it out on the 100 square grid.

The second player chooses a number to cross out. The number must be a factor or multiple of the previous number.

Players continue to take it in turns to cross out numbers, at each stage choosing a number that is a factor or multiple of the number just crossed out by the other player.

The first person who is unable to cross out a number loses.

For one player, or as a group competition, follow the same rules and see who can make the longest chain of numbers.

HELP

Use a smaller number board, for example an 8 by 8 board showing 1– 64. This makes the mental calculations much easier, without watering down the mathematics.

Using 2 different colours will help you to show the difference between factors and multiples.

NEXT

Switch the challenge from winning the game to covering as many numbers as possible. You can again work in pairs trying to find the longest sequence of numbers that can be crossed out.

Can more than half the numbers be crossed out?

This challenge could run for an extended period. The longest sequence can be displayed on a noticeboard and everyone can be challenged to improve on it; any improved sequences can be added to the noticeboard.

Explain why your choice of numbers is good.

It is possible to get a chain of 77 numbers.

Can your group find a chain 77 numbers long or longer?

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INCLUSION AND HOME LEARNING GUIDE

THEME: FACTORS AND MULTIPLES

All these activities are connected so mixed-age groups can learn together at home. Everyone can join in the common starter. Older siblings learn from helping young ones.

Starter for All People Maths *An activity for all ages in a group of 5 or more.*

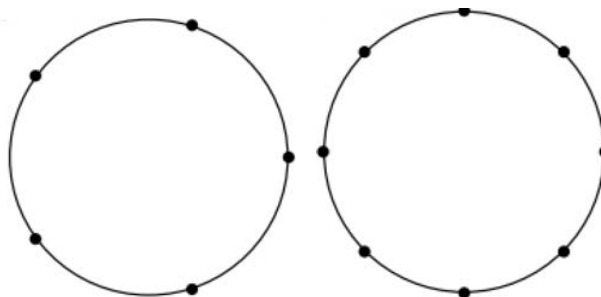
With fewer people use a pencil and paper and join points on a circle instead of passing the string from person to person.

Start with 5 people standing in a circle numbered 0, 1, 2, 3, and 4. A ball of string is given to 0 who holds the end of the string and passes the ball to 1.

Pass the ball across the circle from 0 to 1 to 2 to 3 to 4 then back to 0 unrolling the string and pulling it taut. What

shape has been made? (Answer: a pentagon).

What was the rule? (Answer: pass to next person).



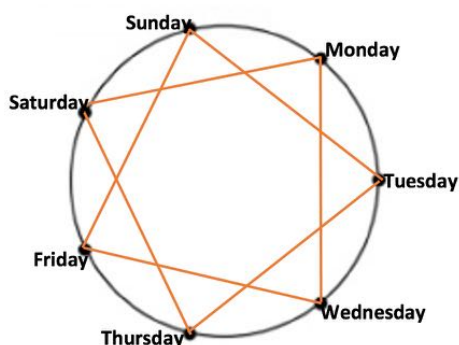
Now ask the group to start again.

Each person should point to 2 other people on the other side of the circle as shown in the picture.

0 passes the ball of string to 2 and it then goes to 4, then to 1 then to 3 then back to 0.

What shape is this? Answer: a star)

What is the rule? (Answer: pass to every alternate person in the circle).



Do the same with 7 people in a circle or 7 dots on a piece of paper numbered 0 to 6 for the days of the week. Talk about the cycle of 7 alternate days (each time from one day to the day after tomorrow) and how this cycle takes 2 weeks.

Do the same with 8 dots on a circle or with a circle of 8 people if you have that many.

Stand in a circle.

Number the people 0, 1, 2, ... up to 7.

Give the ball of string to 0 who must hold one end of the string. Then the ball is passed around the circle to 1, then 2, then 3 ... back to 0 unrolling it and pulling the string taut. What shape has been made?

(Answer: an octagon).



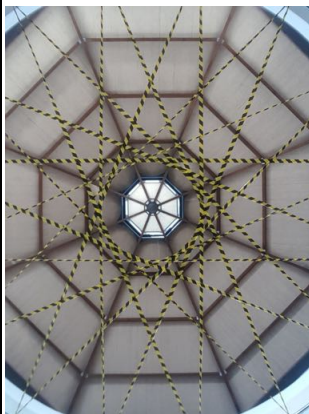
What was the rule? (Answer: pass to next person).

Now ask the group to start again. Pass the string from 0 to 2 to 4 and so on missing out one person each time until the string gets back to 0.

What shape has been made (Answer: a square).

Now ask the group to start again. Pass the string from 0 to 3 to 6 and so on missing out two people each time until the string gets back to 0.

What shape has been made (Answer: a star).



The same activity can be done with different numbers of points around a circle and counting in different multiples.

In this picture the lines join every 7th point giving the sequence:

0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112 ...

or, with the original numbering:

0, 7, 14, 5, 12, 3, 10, 1, 8, 15, 6, 13, 4, 11, 2, 9, 0 ...

You might like to try this 16-point star (or 16-cycle) yourselves.

These photos are from The Catalan Math Museum <https://mmaca.cat/>

MMACA - Museum **MA**thematics **CA**talonia



Early Years



Clapping on Multiples Do this often for a few minutes. It is an activity that can be done by 2 people or in a group. The purpose is to build understanding of multiples and common multiples, and to build confidence generally about number. This experience prepares children for learning multiplication facts and multiplication tables.

Everyone counts rhythmically 1, 2, 3, 4, 5... . Very young children start by counting up to 10. They can progress to counting up to 20 and then up to 30, clapping on the **multiples of 2**. Clap and speak loudly on the even numbers and don't clap on the odd numbers.

1 clap 3 clap 5 clap 7 clap 9 clap 11 clap 13 clap 15 ...

Next everyone counts and claps on the **multiples of 3** up to 30. Clap and speak loudly on the multiples of 3 and don't clap on any of the other numbers.

1 2 clap 4 5 clap 7 8 clap 10 11 clap 13 14 clap ...

If this has become easy for the children then next you all count together, but one person or group claps on multiples of 2: 2, 4, 6, 8, ... and the other simultaneously on multiples of 3: 3, 6, 9, 12, ...

Talk about the numbers when everyone claps together and what is special about them.

Lower Primary

Clapping on Multiples Do this often for a few minutes. This activity can be done by 2 people or in a group. The purpose is to build understanding of multiples and common multiples, and to build confidence generally about number. This experience prepares children for learning multiplication facts and multiplication tables.

Everyone counts rhythmically 1, 2, 3, 4, 5... . Very young children start by counting up to 10. They can progress to counting up to 20 and then up to 30, clapping on the **multiples of 2**. Clap and speak loudly on the even numbers and don't clap on the odd numbers.

1 clap 3 clap 5 clap 7 clap 9 clap 11 clap 13 clap 15 ...

Next everyone counts and claps on the **multiples of 3** up to 30. Clap and speak loudly on the multiples of 3 and don't clap on any of the other numbers.

1 2 clap 4 5 clap 7 8 clap 10 11 clap 13 14 clap ...

Then clap on multiples of other numbers, for example:

1 2 3 4 clap 6 7 8 9 clap 11 12 13 14 clap ...

Split into 2 groups and count in unison, but one person or group claps on multiples of 2: 2, 4, 6, 8, ... and the other simultaneously on multiples of 3: 3, 6, 9, 12, ...

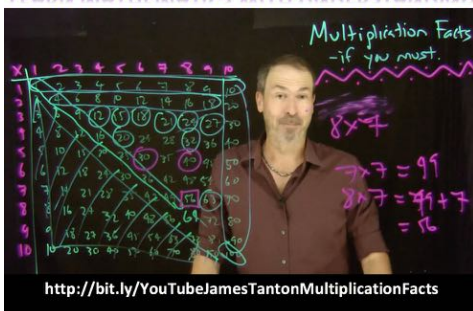
Talk about the numbers when everyone claps together and what is special about them. (They are common multiples). Do the same with 3 multiples.

Skip Counting

×	1	2	3	4	5	6	7	8 ...
2	2	4	6	8	10	12	14	16 ...
3	3	6	9	12	15	18	21	24 ...
5	5	10	15	20	25	30	35	40 ...

Also do skip counting in sequences of multiples of 2 until this becomes easy for the learners, then progress to skip counting in 3's, then in 4's, then 5's etc. Write the counting sequences in a 2-way table as shown, and point to the numbers along the rows while the learners skip count. This lays the basis for learning and understanding multiplication facts and using 2-way tables in many other ways.

LEARN MATHEMATICS WITH UNDERSTANDING



Watch this video and use some of these ideas.

Here is a live link:

<https://bit.ly/YouTubeJamesTantonMultiplicationFacts>

Upper Primary

Learners should do all the activities described for Lower Primary until they are confident that they know all multiplication facts up to 12 times 12. Also play the FIZZ BUZZ game, see https://en.wikipedia.org/wiki/Fizz_buzz.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
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Use 8 of the 100 squares on page 2 for learners to colour all the multiples of 2, 3, 4... up to 9.

See for example this diagram showing multiples of 6.

Encourage the learners to describe and explain the patterns formed by the shaded squares.

<https://aiminghigh.aimssec.ac.za/years-5-8-multiple-patterns/>

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
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81	82	83	84	85	86	87	88	89	90
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Use another blank 100 square. Circle 2 and then draw a diagonal line to cross out all multiples of 2 up to 100. Do the same for 3 and multiples of 3 as shown in this diagram. Then circle the next number not crossed out which is 5. Circle 5 and cross out all further multiples of 5 as shown in the diagram. Repeat for 7 and multiples of 7. The next number not crossed out is 11. Circle 11 and all other numbers not already crossed out. Notice all multiples of 11 have been crossed out. **You have circled the PRIME NUMBERS.**

See <https://aiminghigh.aimssec.ac.za/years-6-9-prime-sieve/>

Lower Secondary – Years 7 to 10

Start with the Diagnostic Quiz. You might spend a whole session on this alone as you should not make these homelearning sessions too long anyway if you want to keep your children motivated and interested in learning more. Older children who understand factors and multiples can play the game before you have this homelearning session.

Venn diagrams are very important in mathematics and often not well understood by learners. They appear in school leaving exams and can often be found to illustrate a point in a report.

Work through the learning activities on pages 1 and 2. To introduce the rules, play the game with your learners a few times. At the end of each game, ask the learners to analyse why the last few moves led to the end of the game - working out better moves that could have been made to avoid losing.

To start with you could choose **not** to mention the first rule that restricts the starting number to a positive even number that is less than 50. Wait until learners discover that the first player can win after just three numbers have been crossed out before discussing the need to restrict the initial number.

Encourage learners to consider the probable next few moves. Game strategies form a natural context for developing deductive logic.

You might focus on accuracy of calculation – encouraging learners to share, and to explain to each other, their mental strategies.

In another session, switch the challenge from winning the game to covering as many numbers as possible. Try to find the longest sequence of numbers that can be crossed out. Can more than half the numbers be crossed out?

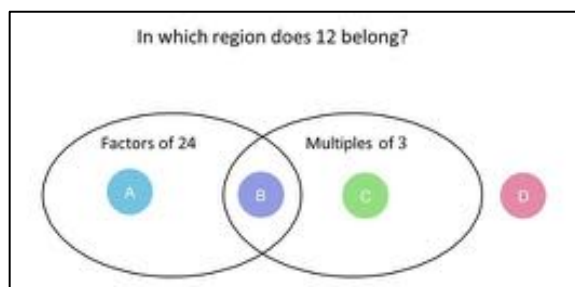
This challenge could run for an extended period: the longest sequence can be displayed on a noticeboard and learners can be challenged to improve on it; any improved sequences can be added to the noticeboard. Ask learners to explain why their choice of numbers is good. It is possible to get a chain of 77 numbers. Can your group find one that long or longer?

Before leaving this learning activity use the Diagnostic Quiz again and ask a few more questions to remind the learners of the basic properties of factors and multiples.

Key questions

- Do you have any winning strategies?
- Are there any numbers you shouldn't go to?

Upper Secondary



Start with this Diagnostic Quiz based on a Venn diagram to check that you understand Venn diagrams and the definitions of Factors and Multiples. Venn diagrams are very important in mathematics. They appear in school leaving exams and can often be found to illustrate a point in a report.

Work through the learning activities on pages 1 and 2. If you are working in a group play the Factors and Multiples game and try to devise winning strategies.

If you are working alone take up the challenge to make a sequence (chain) of factors and multiples to include as many numbers as possible. If possible, use the interactivity <https://nrich.maths.org/factorsandmultiples>.

For example, a chain of length 35 could be 36, 18, 54, 27, 81, 9, 45, 90, 30, 6, 42, 84, 21, 63, 7, 56, 28, 14, 70, 35, 5, 25, 75, 3, 12, 48, 96, 32, 16, 80, 20, 40, 8, 24, 72. At this point all the factors of the last number 72 have been used and all multiples of 72 are greater than 100.

Try to find your own chain of 77 or more numbers.

List some strategies for making long chains.

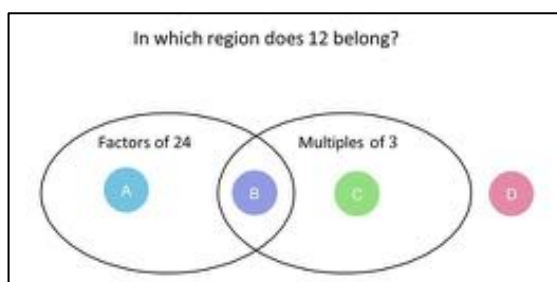
Diagnostic Assessment

This should take about 5–10 minutes.

Show the question to your child or group. This is a **Venn diagram** and some children may not have seen them before. Let them see if they can work out what the diagram represents and if they can answer the Quiz Question. Then discuss the four cases A, B, C and D. Say:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 for D”.

1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain reasons for their answer to develop communication skills and mathematical thinking by having to put thoughts into words.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask them again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** Numbers in the ellipse on the left are factors of 24, that is 1, 2, 3, 4, 6, 8, 12 and 24.



The correct answer is **B** because 12 is both a factor of 24 and also a multiple of 3.

Numbers in the ellipse on the right are multiples of 3, that is 3, 6, 9, 12, 15, ... There are infinitely many of them. Numbers in region D are neither factors of 24 nor are they multiples of 3, that is 5, 7, 10, 11, ... There are infinitely many of them.

Learners giving answer **A** do not understand that, as 12 is a multiple of 3, it does not belong in this region which is outside the ellipse on the right.

Learners giving answer **C** do not understand that, as 12 is a factor of 24, it does not belong in this region which is outside the ellipse on the left.

Learners giving answer **D** do not understand Venn diagrams and don't know that D is the region containing numbers that are not factors of 12 and also not multiple of 3.

<https://diagnosticquestions.com>

Why do this activity?

This activity can replace standard practice exercises on finding factors and multiples. In order to play strategically, learners must start to think of numbers in terms of their factors, utilising primes and squares to develop winning moves.

Learning objectives

In doing this activity students will have an opportunity to develop number sense and a deeper understanding of factors and multiples.

Generic competences

In doing this activity students will have an opportunity to:

- develop logical and strategic thinking skills;
- be creative and innovative and apply knowledge and skills.

Key questions

- Do you have any winning strategies?
- Are there any numbers you shouldn't go to?

Follow up

Patterns of multiples: <https://aiminghigh.aimssec.ac.za/years-5-8-multiple-patterns/>

Finding prime numbers: <https://aiminghigh.aimssec.ac.za/years-6-9-prime-sieve/>

Multiples and cog wheels: <https://aiminghigh.aimssec.ac.za/years-6-10-turning-cogs/>

Special properties of multiples of 37:

<https://aiminghigh.aimssec.ac.za/years-4-to-7-magic-of-37/>

Multiply 3 numbers for the volume of a cuboid:

<https://aiminghigh.aimssec.ac.za/years-6-to-8-same-volume/>

Patterns of certain multiples of 9:

<https://aiminghigh.aimssec.ac.za/years-6-9-times-nine/>

About multiples: <https://aiminghigh.aimssec.ac.za/years-6-10-minimising-output/>