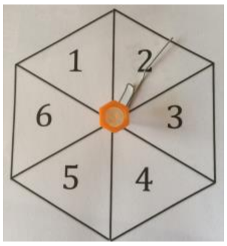
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

SUM DIFFERENCE GAMES Inclusion and Home Learning Guide is part of a Learning Pack downloadable from the AIMING HIGH website https://aiminghigh.aimssec.ac.za It provides related activities for home learning for all ages and learning stages from pre-school to school-leaving, together with guidance for home-learning and also for inclusion in school lessons, all on the **Common Theme OF COUNTING ALL CASES**. Guidance for school lessons is given in the separate Notes for Teachers documents.

Choose what seems suitable for the age or attainment level of your learners.

SUM DIFFERENCE GAMES

Roll the dice, add up the numbers on the two GOLD dice and then subtract the number on the WHITE. If the numbers on the gold dice are 6 and 1, and the number on the white is 4, the result is 3.



If you use a spinner then spin 3 times, add the first two scores and subtract the third.

Play a game against a friend. In the **Zero-Six game** you win if the final score is zero and she wins if the score is 6. Do this many times and score a point each time you win a round. Is this a fair game? How do you know?

Try it out. Play the game, roll these dice many times and see what numbers you make each time by doing the addition and subtraction.

Work in pairs and talk about what to do and how to record your results when you throw the dice. You will need two dice of one colour and one of another colour, or a 1-to-6 spinner. After experimenting, try to predict totals that will NOT be possible and those that will. Check these predictions. Can you decide whether the game is fair or not?

What about the **Even-Odd game** where you win if the final score is odd and your friend wins if it is even?

For an even more challenging game, decide on two operations then allow players, on their turn, to choose which to apply to the gold dice and which to the white.

Make up your own games with different rules and decide whether your game is fair or not.

HELP

You should also use a number line and count to the right and to the left to help you to work with the negative numbers.

If you want an easier challenge that does not involve negative numbers you can do a similar activity where you find the results for adding the scores on the 3 dice.

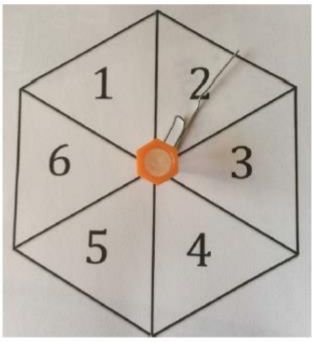
NEXT

If you want a bigger challenge, use two dice only and multiplication, or three dice as in the Zero-Six and Even-Odd games using two operations of the four: addition, subtraction, multiplication and division. After experimentating, try to predict totals that will NOT be possible, then check these predictions, and decide if the game is fair.

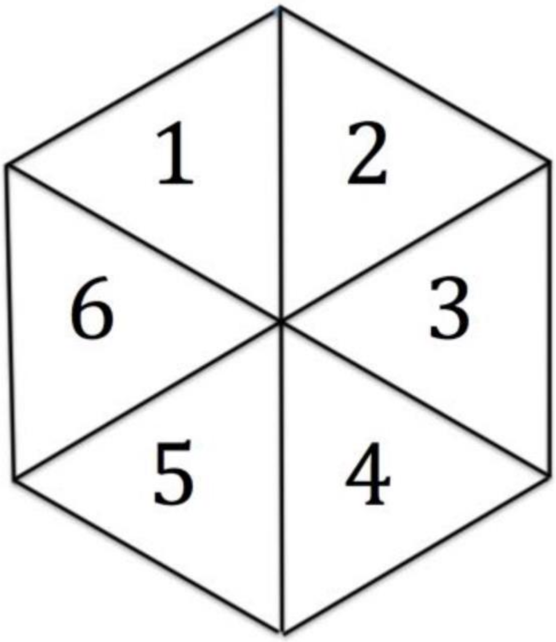
For an even more challenging game, decide on two operations then allow players to choose which applies to the gold dice and which to the white.

Use the probability scale from 0 to 1 and quantify the probability of getting particular results.

As a follow-up, create your own variations of the activity.
Ask and investigate 'What if ...?' questions.



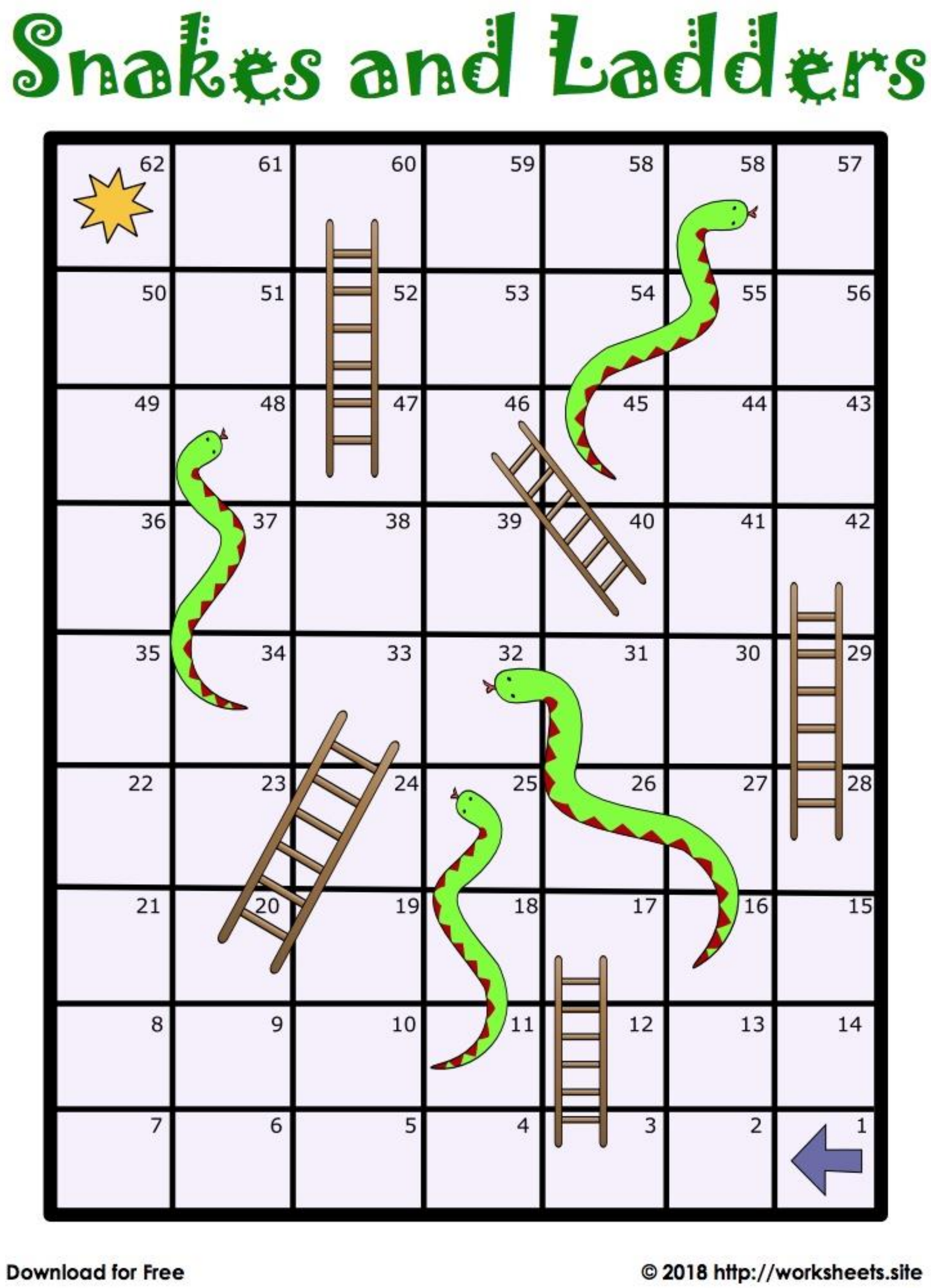
To make your own spinner as shown in the picture you will need a paper clip and a pin. Straighten out one end of the paper clip, cut out the template and then pin the paper clip and the hexagon on a flat surface so that the spinner spins freely. Now you are ready to play the game.



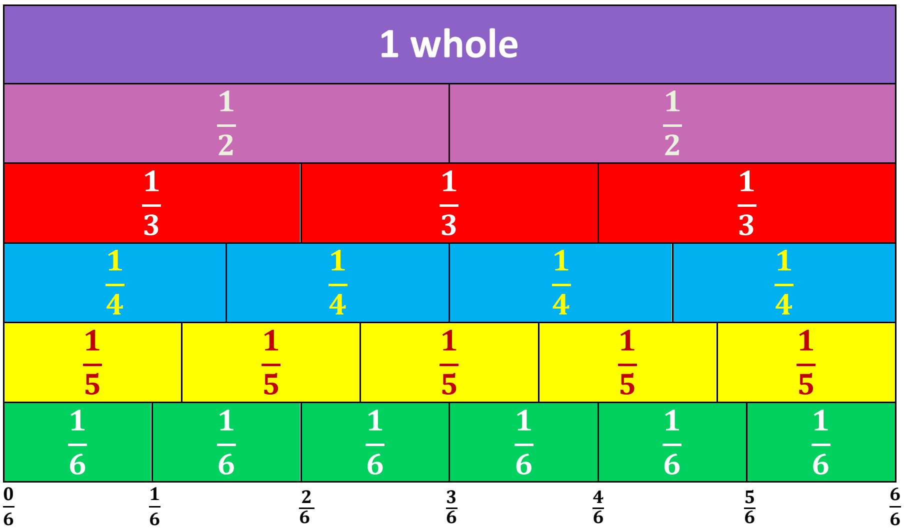
INCLUSION AND HOME LEARNING GUIDE
THEME: COUNTING ALL CASES

Early Years

Each player needs a counter. Take it in turns to roll 2 dice (or use a spinner), add the numbers on the dice and move forward that number of squares. If you land at the bottom of a step ladder you go up. If you land on a snake's head you go down. The first to get to 62 is the winner.



Lower Primary Fraction Game



Players throw 2 dice or use a spinner. They must make a fraction with the numbers on the dice putting the bigger number on top. If the two numbers are the same then the player scores 1. For example, 2 and a 3 make the fraction $\frac{2}{3}$ and 3 with a 4 make the fraction $\frac{3}{4}$. Use the fraction wall to compare the two fractions. The bigger fraction wins the round.

Improper Fraction Variation

Players throw 2 dice or use a spinner. They must make a fraction with the numbers on the dice putting the bigger number on top. If the two numbers are the same then the player scores 1. For example, 2 and a 3 make the fraction $\frac{3}{2}$ and 3 with a 4 make the fraction $\frac{4}{3}$. The bigger fraction wins the round.

Upper Primary & Lower Secondary Years 4 - 9 Great Race Game

HORSE	RACE TRACK										FINISH
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											
12											

Make your own 'game board' as shown with tracks for 12 runners numbered 1 to 12. Which is the best number to choose? Why?

To give each player one of the horses, either use a 1-12 spinner or put 12 cards numbered 1 to 12 in a bag and each player picks a card giving them a horse.

Throw two fair dice and add the numbers to decide which horse goes forward on that throw.. Mark an 'x' in the next box or use a counter. The winner is the first to move 11 times to cross the finish line.

Play the game as a group. Make your game board as in the diagram above. Use two large dice. If there are fewer than 11 players then people can have more than one runner.

Alternatively, this could be a **People Maths Game** if it's possible to mark out the board on the ground, and the players could actually act as the runners and take one step forward if their number comes up.

Explain the game (use humour or local knowledge to make the game more meaningful to learners). Let learners experience and make sense of what actually happens when they play the game. The games are like experimental trials.

Play the game a few times and record the numbers of the winning horses. Ask learners to comment on the results. Do some numbers come up more often than others? If so, why? Ask learners to make conjectures about the favourite horses with the best numbers, who win most often, and the horses that don't often win. Then continue playing to see if their conjectures are born out. **This is sufficient for Year 6.**

For Year 7 and older learners ask:

"**Why** do certain horses win more often than others?"

"**Why** use a cube for a die, rather than a different cuboid?"

Having played the game a few times learners should try to make a list of all the different possible scores.

It's important for learners plan systematic methods of finding all possible cases. Teachers should bear in mind that the generic skill of planning and working systematically is **much more valuable** than being able to find answers to specific maths questions. So, over time, teachers should plan to give less and less guidance on how to do this but rather expect the learners to think out for themselves how to plan to work systematically.

Ask questions to explore ideas about the **36 equally likely outcomes** such as (3, 5) and (6, 2). Then ask "how many other ways can you get a total of 8? After playing the game the learners will realise that the totals (2 to 12) are not equally likely. The next step is to discover why.

+	1	2	3	4	5	6
6						12
5		7	8			
4				8		
3						
2						8
1	2					

The 36 possible different ways the total scores occur by adding the scores on two dice can be recorded in a 2-way table as shown.

The table shows the number of different ways that each of the scores 2 to 12 occur, why 7 has a better chance of winning than other numbers, and why 2 and 12 have the least chance of winning.

Generally this is sufficient for Year 7. Older learners should play the game, complete the table and work out the probabilities for each of the players to win. Care must be taken in defining the situations precisely so it is clear what outcomes might occur.

The list of all possible outcomes is called the sample space. In the Great Race the sample space is the set of 36 ordered pairs of numbers (x, y) where x and y belong to the set {1, 2, 3, 4, 5, 6}; these give scores of 1 up to 12.

Ask learners to fill in the remaining scores in the 2 -way table and work out the probabilities of winning for runners 2 to 12.

This analysis only works because the two dice scores are **independent** (the outcome on one does not affect the outcome on the other) and each outcome is **equally likely**.
You could ask learners whether the score on one die affects the score on the other, and also whether it affects scores on later throws**. T**his lays foundations for later work on independent events.

Key Questions

- Where have we used the fact that the outcomes are equally likely?
- What do you think would happen if the playing board were longer, say with 20 spaces rather than 10? Why?
- What if you used two fair spinners, each with the numbers 1 to 4?
- What if the spinners each had the numbers 1 to 5, but an outcome of 3 on a spinner was twice as likely as any other number?
- If you threw one die twice and added the scores, rather than using two different dice, would anything in your analysis change? Why or why not?

Lower Secondary   Years 7 – 11   SUM DIFFERENCE GAME

You could introduce this 3-dice problem using real dice and modelling the calculation a few times so that learners get a feel for it. It is important not to rush this and learners need a good understanding of the results for 2-dice before analyzing the 3-dice possibilities, so you should start by playing the Great Race Game, see the section for upper primary, if learners have not met it before.

Once some results have been written down, invite the learners to speculate on how many different results there might be.

CHOOSE A GAME for example the **Zero-Six game** described on page 1 or the **Even-Odd game** where Player 1 wins if the final score is even and Player 2 wins if the final score is odd. The learners should play the game in pairs or teams and record the number of wins for each final score.

Say very little else at this stage, but after a short time, bring everyone together again to share insights so far. Then collect all the results and discuss whether the game seems to be a fair game.

Discuss the range of answers that learners have found up to that point, and make sure they are happy with subtracting one number from a smaller number. Using a number line which includes negative numbers might be helpful at this point.

When talking about whether the game is a fair game you need to find a way to **record the different possible final scores to be sure no results are left out.** Ask learners to suggest ways of doing this, then give them time to work on this.

After a while, invite the learners to describe how they are working. Some may be throwing real dice, others may be listing numbers. **Ask learners to explain how they are recording their findings.** Encourage some sort of system so that they can be sure no results are left out. This could be in the form of a table or chart. Allow learners to choose a way that suits them.

The **Even-Odd game** is most suitable for experiments because a single throw decides the result and you can quickly record the results of many trials. The **Zero-Six game** is a bit more exciting and finding out whether it is fair or unfair could be more of a challenge. After playing the games the group should discuss reasons why one of the games Even-Odd and Zero-Six might be a fair game and the other one not.

- Learners could **vary the rules of the game**, ask their own questions and be creative.
- Older learners could **analyse the different theoretical probabilities**.

Upper Secondary Years 12 and 13

It's important for you to have the skill to make plans for systematic methods of solving problems of all sorts because this is one of the skills that employers value.

The skill of working systematically is much more valuable than being able to find answers to specific maths questions because this skill is applicable in many situations, some involving mathematics and others not, but all involving logical reasoning.

In the Zero-Six Game and the Odd-Even Game, described on page 1, the challenge is to find all possible cases when you throw 3 dice, add the scores on the first two dice and subtract the score on the third.

Either play the games against a friend and record the scores or repeatedly throw the dice yourself as trials in an experiment. Devise a system for listing all possible outcomes. Play repeatedly until either (1) you decide you have found all possible triples or (2) you are bored!

Then work out the probabilities of the different scores and decide whether or not the Zero-Six and Odd-Even games are fair games.

Think about the difference between **dependent and independent events**. Suppose Event A is 'firstly, pick up a stone and hold it in your hand' and

Event B is 'secondly, without using the other hand put the stone in the right-hand pocket of your trousers or skirt'.

Is the success of Event B dependent or independent of the outcome of Event A?

Describe some pairs of independent events and some pairs of dependent events.

Key questions

- What are all the different possible triples of numbers?
- What are the final answers by doing the addition and subtraction each time?
- Is there a good way of making sure you find all the possibilities?
- How will you record what you've found out?
- How do you know whether a game is fair or not?
- Do you think this game is fair?
- Do you think that it's certain you would get all possible triples of 3 numbers if you carried out 100 trials in the experiment?

SOLUTIONS

THE GREAT RACE

The table shows the 36 possible different totals on adding the scores on two dice (red and blue say) and it shows the number of ways that each of the scores 2 to 11 occur.

+ →↓	1	2	3	4	5	6
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7

Table 1. *Notice the symmetries.*

Probability of 2 = Probability of 12 = $\frac{1}{36}$

Probability of 3 = Probability of 11 = $\frac{2}{36}$

Probability of 4 = Probability of 10 = $\frac{3}{36}$

Probability of 5 = Probability of 9 = $\frac{4}{36}$

Probability of 6 = Probability of 8 = $\frac{5}{36}$

Probability of 7 = $\frac{6}{36}$

ZERO-SIX AND EVEN-ODD GAMES

Care must be taken in defining the situations precisely so it is clear what outcomes might occur. The list of all possible outcomes is called the **sample space**. In the Great Race the sample space is the set of 36 ordered pairs of numbers (x, y) where x and y belong to the set {1, 2, 3, 4, 5, 6}; these give scores of 1 up to 12.

The theoretical probabilities for the 36 equally likely outcomes of throwing two dice (say a red and a blue die) can be analysed using the **2-way table** shown above exactly as in the Great Race Game.

With 3 dice, the largest score is 6 + 6 - 1 = 11 and the smallest score is 1 + 1 - 6 = -4. All scores between -4 and +11 can be made in many different ways because of the different combinations of scores on the 3 dice.

For example the score of 9 can be made in 6 ways:

6 + 6 – 3 = 9; 6 + 5 – 2 = 9; 5 + 6 – 2 = 9;

6 + 4 – 1 = 9; 4 + 6 – 1 = 9; 5 + 5 – 1 = 9.

Table 2. Occurrences of scores -4 to +11.

TOTALS ON TWO GOLD DICE→		2	3	4	5	6	7	8	9	10	11	12
Number of ways to get gold total		1	2	3	4	5	6	5	4	3	2	1
	SUM ON TWO GOLD DICE MINUS NUMBER ON WHITE DIE											
WHTE DIE with black spots↓	1	1	2	3	4	5	6	7	8	9	10	11
	2	0	1	2	3	4	5	6	7	8	9	10
	3	-1	0	1	2	3	4	5	6	7	8	9
	4	-2	-1	0	1	2	3	4	5	6	7	8
	5	-3	-2	-1	0	1	2	3	4	5	6	7
	6	-4	-3	-2	-1	0	1	2	3	4	5	6

This table shows for example that the number 1 occurs as:

2 – 1, or as 3 - 2 or 4 – 3 or 5 – 4 or 6 – 5 or 7 – 6.

2 occurs as the sum on the gold dice in 1 way so 2-1 = 1 occurs once.

3 occurs as the sum on the gold dice in 2 ways so 3-2 = 1 occurs twice.

4 occurs as the sum on the gold dice in 3 ways so 4-3 = 1 occurs 3 times.

5 occurs as the sum on the gold dice in 4 ways so 5-4 = 1 occurs 4 times.

6 occurs as the sum on the gold dice in 5 ways so 6-5 = 1 occurs 5 times.

7 occurs as the sum on the gold dice in 6 ways so 7-6 = 1 occurs 6 times.

So the final answer of 1 occurs in 1 + 2 + 3 + 4 + 5 + 6 = 21 ways.

The number of times each final score occurs is given in Table 3 below.

Table 3 Calculations for the total number of occurrences of each final score

FINAL SCORES	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
Number of occurrences calculation	1	1+2	1+2+3	1+2+3+4	1+2+3+4+5	1+2+3+4+5+6	2+3+4+5+6+5	3+4+5+6+5+4	4+5+6+5+4+3	5+6+5+4+3+2	6+5+4+3+2+1	5+4+3+2+1	4+3+2+1	3+2+1	2+1	1
Number of occurrences of each score	1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1
Total number of occurrences	1 + 3 + 6 + 10 + 15 + 21 + 25 + 27 + 27 + 25 + 21 + 15 + 10 + 6 + 3 + 1 = 216															

Which games are fair?

We can see that the **Zero-Six Game** where one player wins with a zero and the other wins with a six is NOT a fair game. The chance of throwing a zero is 15/216 = 6.9% whereas the chance of throwing a 6 is 21/216 = 9.7%.

The **Even-Odd Game** is a fair game because the probability of an even score is

(1 + 6 + 15 + 25 + 27 + 21 + 10 + 3)/216 = 108/216 = $\frac{1}{2}$

And the probability of an odd score is:

(3 + 10 + 21 + 27 + 25 + 15 + 6 + 1)/216 = 108/216 = $\frac{1}{2}$

Why do this activity?

This activity offers a context for different tasks working towards different learning objectives for different ages or for different attainment within the same class. Teachers might use this activity as the same starting point for all learners and then use it to provide differentiated work according to the individual needs of the learners.

1. It offers practice in addition and subtraction, including negative results, where it is important for learners to concentrate on making sure that they find all the different ways of getting the totals from rolling the three dice. This will need some sort of system and you could focus on how answers could be recorded. For younger learners you could change the rule to finding the sum on the 3 dice (from 3 to 18).
2. It offers opportunities for probability experiments.
3. It offers opportunities for learners to ask their own questions, to vary the rules of the game,

and to be creative.
4. It offers opportunities for older learners to analyse the different theoretical probabilities.

Learning objectives

In doing this activity students will have an opportunity to:

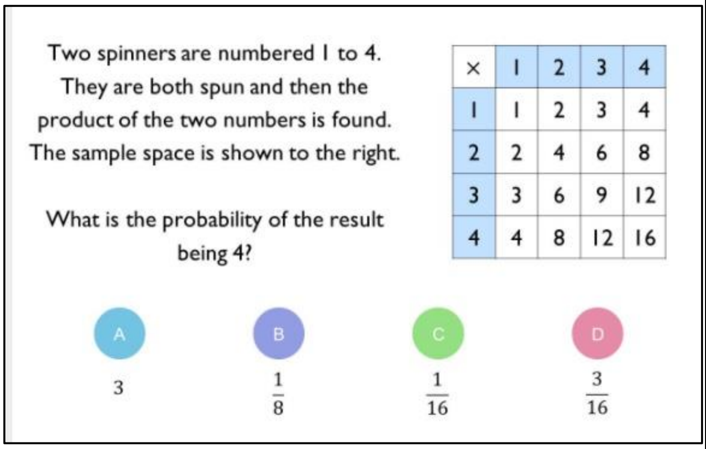
- conduct probability experiments;
- practise addition and subtraction, including negative results;
- work systematically to find all the different ways of getting the totals;
- vary the rules of the game, ask their own questions and be creative;
- analyse the different theoretical probabilities.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- develop the **skill of planning a system** to cover all possible cases that can arise and **working systematically** to solve a problem;
- interpret and **solve problems**;
- **develop life skills and consideration for others** – playing games and working as a team to understand the theoretical basis of the game and discover winning strategies.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.
Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".


Two spinners are numbered 1 to 4. They are both spun and then the product of the two numbers is found. The sample space is shown to the right.

What is the probability of the result being 4?

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

A: 3
B: $\frac{1}{8}$
C: $\frac{1}{16}$
D: $\frac{3}{16}$

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: D that is 3/16
Possible misconceptions:
A. This is the number of occurrences of 4 (frequency), not the probability.
C. This is the probability of each result in the sample space.
https://diagnosticquestions.com

Follow up *See the NEXT box on page 2*

Nine and Tens https://aiminghigh.aimssec.ac.za/nines-and-tens/

Odds and Colours Games https://aiminghigh.aimssec.ac.za/odds-and-colours-games/