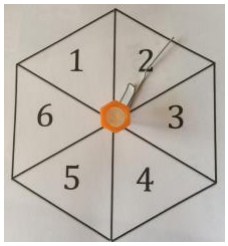
SUM DIFFERENCE GAMES



Roll the dice, add up the numbers on the two GOLD dice and then subtract the number on the WHITE. If the numbers on the gold dice are 6 and 1, and the number on the white is 4, the result is 3.

If you use a spinner then spin 3 times, add the first two scores and subtract the third.

Play a game against a friend. In the **Zero-Six game** you win if the final score is zero and she wins if the score is 6. Do this many times and score a point each time you win a round. Is this a fair game? How do you know?

Try it out. Play the game, roll these dice many times and see what numbers you make each time by doing the addition and subtraction.

Work in pairs and talk about what to do and how to record your results when you throw the dice. You will need two dice of one colour and one of another colour, or a 1-to-6 spinner. After experimenting, try to predict totals that will NOT be possible and those that will. Check these predictions. Can you decide whether the game is fair or not?

What about the **Even-Odd game** where you win if the final score is odd and your friend wins if it is even?

For an even more challenging game, decide on two operations then allow players, on their turn, to choose which to apply to the gold dice and which to the white.

Make up your own games with different rules and decide whether your game is fair or not.

HELP

You could use a number line and count to the right and to the left to help you to work with the negative numbers.

If you want an easier challenge that does not involve negative numbers you can do a similar activity where you find the results for adding the scores on the 3 dice.

You will need two dice of one colour and one of another colour. Work in pairs and talk about what to do and how to record your results when you throw the dice.

NEXT

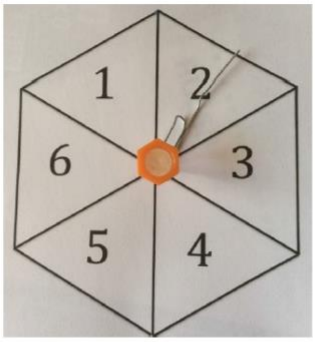
If you want a bigger challenge, use two dice only and multiplication, or three dice as in the Zero-Six and Even-Odd games using two operations of the four: addition, subtraction, multiplication and division. After experimentating, try to predict totals that will NOT be possible, then check these predictions, and decide if the game is fair.

For an even more challenging game, decide on two operations then allow players to choose which applies to the gold dice and which to the white.

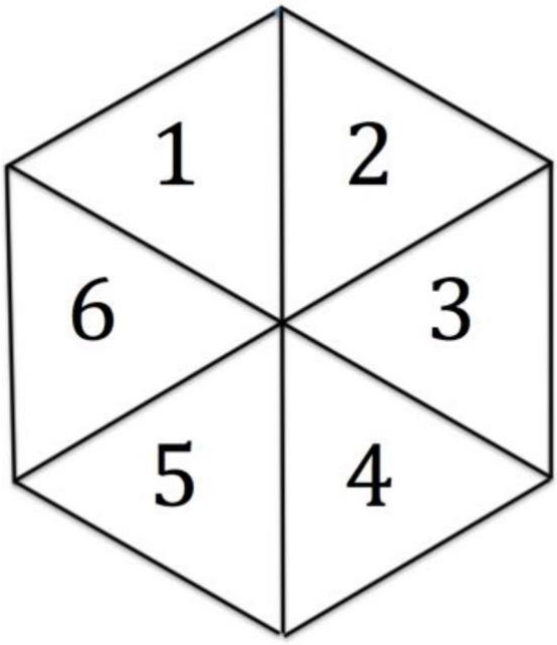
Use the probability scale from 0 to 1 and quantify the probability of getting particular results.

As a follow-up, create your own variations of the activity.

Ask and investigate 'What if ...?' questions.



To make your own spinner as shown in the picture you will need a paper clip and a pin. Straighten out one end of the paper clip, cut out the template and then pin the paper clip and the hexagon on a flat surface so that the spinner spins freely. Now you are ready to play the game.



NOTES FOR TEACHERS

SOLUTION

The largest score is 6 + 6 - 1 = 11 and the smallest score is 1 + 1 - 6 = -4. All scores between -4 and +11 can be made in many different ways because of the different combinations of scores on the 3 dice.

For example the score of 9 can be made in 6 ways:

6 + 6 – 3 = 9; 6 + 5 – 2 = 9; 5 + 6 – 2 = 9;

6 + 4 – 1 = 9; 4 + 6 – 1 = 9; 5 + 5 – 1 = 9

Table 1 TOTALS ON 2 DICE

		FIRST DIE					
		1	2	3	4	5	6
SECOND GOLD DIE	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

There are 6 possible outcomes each time a die is thrown and so there are 6 x 6 = 36 outcomes when 2 dice are thrown as summarized in this table.

We can see from this table how many times each of the totals from 2 to 12 occur. For example the total 7 occurs 6 times and 8 occurs 5 times.

There are 6 x 6 x 6 = 216 different outcomes when 3 dice are thrown. It would be tedious to list them all but we record below how many times each score occurs.

Results for the analysis of the 3-dice results are given below for completeness but only a minority of older learners would get that far.

Table 2. Occurrences of scores -4 to +11.

TOTALS ON TWO GOLD DICE →		2	3	4	5	6	7	8	9	10	11	12
Number of ways to get gold total		1	2	3	4	5	6	5	4	3	2	1
	SUM ON TWO GOLD DICE MINUS NUMBER ON WHITE DIE											
WHTE DIE with black spots ↓	1	1	2	3	4	5	6	7	8	9	10	11
	2	0	1	2	3	4	5	6	7	8	9	10
	3	-1	0	1	2	3	4	5	6	7	8	9
	4	-2	-1	0	1	2	3	4	5	6	7	8
	5	-3	-2	-1	0	1	2	3	4	5	6	7
	6	-4	-3	-2	-1	0	1	2	3	4	5	6

This table shows for example that the number 1 occurs as:

2 – 1, or as 3 - 2 or 4 – 3 or 5 – 4 or 6 – 5 or 7 – 6.

2 occurs as the sum on the red dice in 1 way so 2-1 = 1 occurs once.

3 occurs as the sum on the red dice in 2 ways so 3-2 = 1 occurs twice.

4 occurs as the sum on the red dice in 3 ways so 4-3 = 1 occurs 3 times.

5 occurs as the sum on the red dice in 4 ways so 5-4 = 1 occurs 4 times.

6 occurs as the sum on the red dice in 5 ways so 6-5 = 1 occurs 5 times.

7 occurs as the sum on the red dice in 6 ways so 7-6 = 1 occurs 6 times.

So the final answer of 1 occurs in 1 + 2 + 3 + 4 + 5 + 6 = 21 ways.

The number of times each final score occurs is given in Table 3 below.

Table 3 Calculations for the total number of occurrences of each final score																
FINAL SCORES	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
Number of occurrences calculation	1	1+2	1+2+3	1+2+3+4	1+2+3+4+5	1+2+3+4+5+6	2+3+4+5+6+5	3+4+5+6+5+4	4+5+6+5+4+3	5+6+5+4+3+2	6+5+4+3+2+1	5+4+3+2+1	4+3+2+1	3+2+1	2+1	1
Number of occurrences of each score	1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1
Total number of occurrences	1 + 3 + 6 + 10 + 15 + 21 + 25 + 27 + 27 + 25 + 21 + 15 + 10 + 6 + 3 + 1 = 216															

Which games are fair?

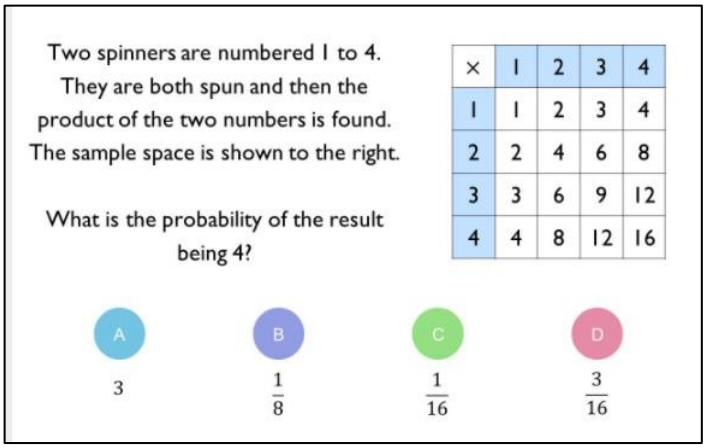
Now we can see that the game where one player wins with a zero and the other wins with a six is NOT a fair game. The chance of throwing a zero is 15/216 = 6.9% whereas the chance of throwing a 6 is 21/216 = 9.7%.

DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: D that is 3/16

Possible misconceptions:

A. This is the number of occurrences of 4 (frequency), not the probability.

C. This is the probability of each result in the sample space.

https://diagnosticquestions.com

Why do this activity?

This activity offers a context for different tasks working towards different learning objectives for different ages or for different attainment within the same class. (See the Inclusion and Home learning Guide for more activities). Teachers might use this activity as the same starting point for all learners and then use it to provide differentiated work according to the individual needs of the learners.

1. It offers practice in addition and subtraction, including negative results, where it is important for learners to concentrate on making sure that they find all the different ways of getting the totals from rolling the three dice. This will need some sort of system and you could focus on how answers could be recorded. For younger learners you could change the rule to finding the sum on the 3 dice (from 3 to 18).
2. It offers opportunities for probability experiments.
3. It offers opportunities for learners to vary the rules of the game, to ask their own questions and to be creative.
4. It offers opportunities for older learners to analyse the different theoretical probabilities.

Learning objectives

In doing this activity students will have an opportunity to:

- conduct probability experiments;
- practise addition and subtraction, including negative results;
- work systematically to find all the different ways of getting the totals;
- vary the rules of the game, ask their own questions and be creative;
- analyse the different theoretical probabilities.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- develop the **skill of planning a system** to cover all possible cases that can arise and **working systematically** to solve a problem;
- interpret and **solve problems**;
- **develop life skills and consideration for others** – playing games and working as a team to understand the theoretical basis of the game and discover winning strategies.

Suggestions for teaching

You could introduce the problem using real dice and modelling the calculation a few times so that learners get a feel for it. Once a few results have been recorded on the board, invite the learners to speculate on how many different results there might be.

1. Ask learners to work in pairs or small groups on the problem. Say very little else at this stage, but after a short time, bring them together again to share insights so far.

Discuss the range of answers that learners have found up to that point and make sure they are happy with subtracting one number from a smaller number. (Using a number line which includes negative numbers might be helpful at this point.) Invite some pairs to describe how they are working. Some may be throwing real dice, others may be listing numbers. **Ask**

learners to explain how they are recording their findings. Encourage some sort of system so that they can be sure no results are left out. You could ask children to suggest ways of recording which would help them to be sure no results are left out. This could be in the form of a table or chart. Allow learners to choose a way that suits them.

2. **Choose a game**, for example the **Zero-Six game** described above or the **Even-Odd game** where Player 1 wins if the final score is even and Player 2 wins if the final score is odd. Get the players to play the game in groups and record the number of wins for each final score. Then collect all the results from the whole class and discuss whether the game seems to be a fair game. The **Even-Odd game** is suitable for class experiments because a single throw decides the result and the learners can quickly record the results of many experiments. The **Zero-Six game** is a bit more exciting and finding out whether it is fair or unfair is more of a challenge.
3. Learners could **vary the rules of the game**, ask their own questions and be creative.
4. Older learners could **analyse the different theoretical probabilities**.

It is important not to go on to stages 3 & 4 too soon as learners need a good understanding of the results for 2 dice before analyzing the 3 dice possibilities. It will help learners if they are introduced to the **Great Race game** before trying to analyse the probabilities for 3 dice. (See the Inclusion and Home learning Guide).

Key questions

- What are all the different possible numbers?
- What are the final answers by doing the addition and subtraction each time?
- Is there a good way of making sure you find all the possibilities?
- How will you record what you've found out?
- How do you know whether a game is fair or not?
- Do you think this game is fair?
- Do you think that the results for the whole class will be similar to the results for your group?

Follow up *See the NEXT box on page 2*

Nine and Tens https://aiminghigh.aimssec.ac.za/nines-and-tens/

Odds and Colours Games https://aiminghigh.aimssec.ac.za/odds-and-colours-games/