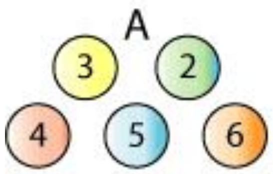
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

THE ODDS AND EVENS GAMES Inclusion and Home Learning Guide is part of a Learning Pack downloadable from the AIMING HIGH website https://aiminghigh.aimssec.ac.za/years-9-to-12-odds-and-evens/

This Guide provides **related activities for all ages and learning stages from pre-school to school-leaving**, together with guidance for home-learning and also for inclusion in school lessons, all on the Common Theme: Probability sample spaces. Guidance for school lessons is given in the separate Notes for Teachers document.

Choose what seems suitable for the age or attainment level of your learners.

ODDS AND EVENS



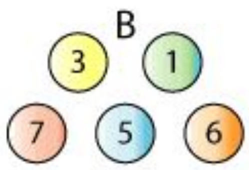
In Game A you pick out two balls at random from a bag containing 5 balls numbered 2, 3, 4, 5 and 6.

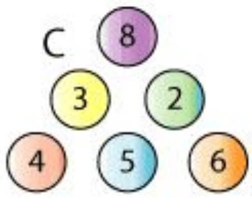
If the total is even you win. If it is odd you lose.

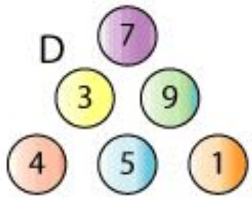
Are you equally likely to win or to lose (a fair game)?

Games B, C and D use the numbers shown below and the rules are the same.

Which set of balls would you choose to give yourself the best chance of winning?







Do your own experiments. Make number cards and put them in an envelope or bag.

To do interactive computer simulations of these games see http://nrich.maths.org/4308

HELP

	First choice					
Second choice	A	2	3	4	5	6
	2	X			7	
	3		X		8	
	4	6		X		
	5			9	X	
	6					X

Play Game A several times and record the results. You are conducting trials in a probability experiment.

Then fill in a table as shown colouring the odd and even totals in contrasting colours and putting in crosses because you can't choose the same ball twice.

This is called **a sample space diagram**.

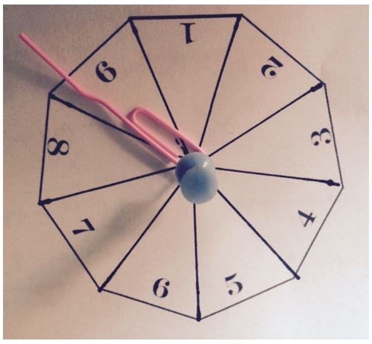
The sample space diagram shows that there are exactly 20 possible outcomes when you choose 2 cards at random. There are 20 elements in the sample space.

NEXT

None of the sets looked at so far (A, B, C and D) gives a fair game.
Can you find out whether there are any sets that would give a fair game?

1	2	3
4	5	6
7	8	9

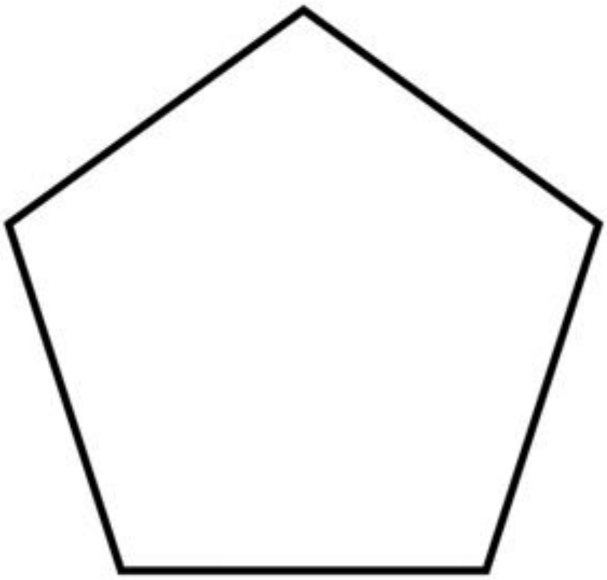
MAKING SPINNERS FOR THESE GAMES

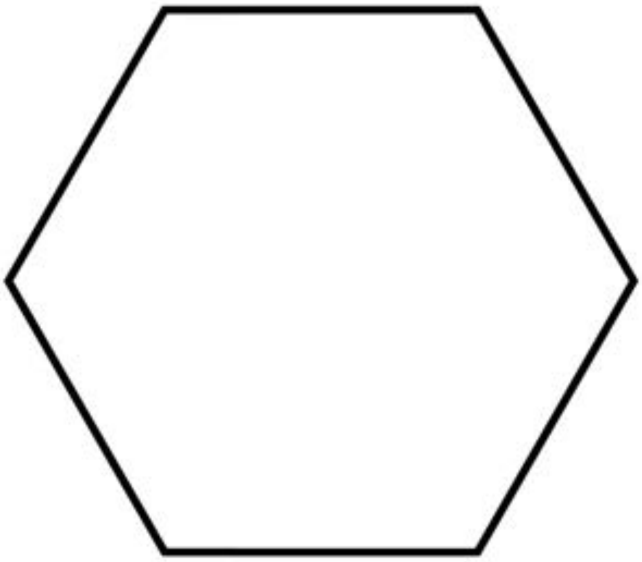


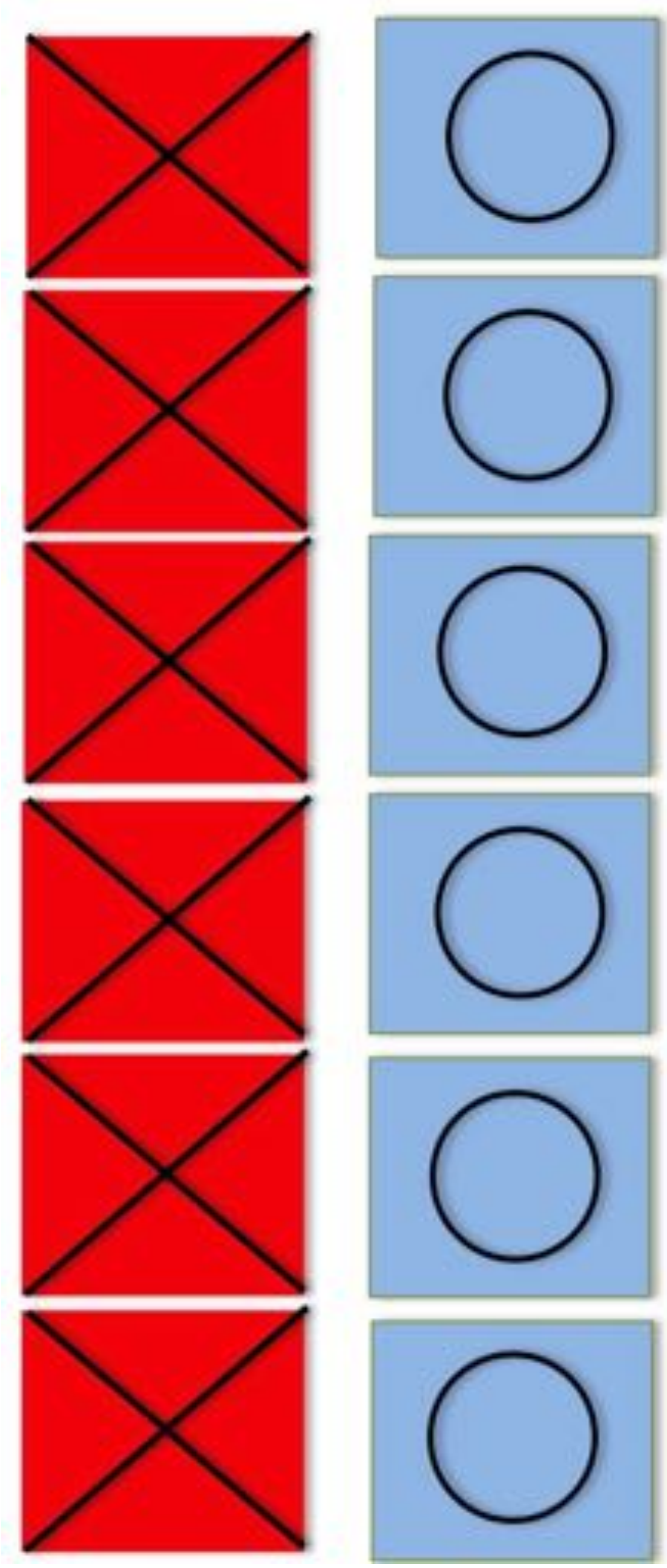
You can simulate the games using spinners made from the templates below, a pin and a paperclip.

Mark the segments on the spinners, and the 5 or 6 numbers for the game.

Cut out and pin on a flat surface through the paper clip and the template (as in the picture) making sure that the paper clip spins freely.







PROOF SORTING EXERCISE FOR YEAR 12 AND 13 STUDENTS.

This is the 1st statement. Cut out the strips and re-construct the proof by arranging the remaining statements in the correct order.

Odds and Evens

Imagine we play the game with a even numbered balls and b odd numbered balls.

If a and b are any two consecutive triangle numbers, the reverse argument holds.
We have proved that a necessary and sufficient condition for a game of this type to be a fair game is that the total number of balls ($a + b$) is a square number n^2, moreover a and b turn out to be consecutive triangle numbers.

For a fair game we must have the probability of an even sum equal to the probability of an odd sum; that is:

$(a^2 - a) + (b^2 - b) = ab + ab$

$\Leftrightarrow \quad a^2 - 2ab + b^2 = a + b$

$\Leftrightarrow \quad (a - b)^2 = a + b$

This suggests that a and b are consecutive triangle numbers n because the rule for generating the sequence of triangle numbers is that the difference between the n^{th} and $(n-1)^{th}$ number is equal to n.

Proof:

For a even and b odd numbers this is the sample space diagram, with blue for even totals and yellow for odd totals:

	a even	b odd
a even	$a^2 - a$	ab
b odd	ab	$b^2 - b$

We know that

$a + b = n^2$

and $a - b = n$ (or $b - a = n$ if $b > a$).

We have proved that: the game will be a fair game if and only if $(a - b)^2 = a + b$

This leads us to think that:

- the total number of balls (i.e. $a + b$) has to be a square number, say n^2;
- the difference between the number of odd and even balls is n.

Adding the two equations:

$2a = n^2 + n$

$\Rightarrow a = \frac{1}{2}(n^2 + n)$

$\Rightarrow a = \frac{1}{2}n(n+1)$

This formula gives a as the n^{th} triangular number.

Similarly, b is the $(n - 1)^{th}$ triangular number $\frac{1}{2}n(n - 1)$.

Conjecture: The game will be a fair game if and only if the number of balls is a square number, in which case the number of odd, and even balls, a and b, are two consecutive triangle numbers.