PROOF SORTING EXERCISE FOR YEAR 12 AND 13 STUDENTS.

This is the 1st statement and the other statements are jumbled up. Cut out the strips and re-construct the proof by arranging the remaining statements in the correct order. **(A)**

RED/BLUE and ODDS/EVENS GAMES Imagine we play the game with different objects (counters, cards, balls) of two types (two colours red and blue or numbered even and odd).

We have proved that: the game will be a fair game if and only if $(a - b)^2 = a + b$ **(B)**

- the total number of objects (i.e. $a + b$) has to be a square number, say n^2;
- the difference between the number objects of the two types (i.e. $a - b$) is n.

That is: $a + b = n^2$ (1)

and $a - b = n$ (2)

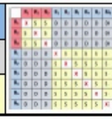
We have proved that a necessary and sufficient condition for a game of this sort to be a fair game is that the total number of objects ($a + b$) is a square number n^2, moreover a and b must be consecutive triangle numbers. **(C)**

This suggests that a and b are consecutive triangle numbers as the rule for generating the sequence of triangle numbers is that the difference between the n^{th} and $(n-1)^{th}$ triangle number is equal to n, and their sum is the square number n^2. **(D)**

(e.g. $T_7 - T_6 = 7$ and $T_7 + T_6 = 7^2$)

To Prove This is a sample space diagram (2-way table) It shows the total numbers of possible outcomes, grey for one outcome (e.g. different or odd) yellow for the other outcome (e.g. same or even). **(E)**

	a	b
a	$a^2 - a$	ab
b	ab	$b^2 - b$



Notation: Label the number of objects of the different types as a and b with $a > b$. **(F)**

The symbol ⇔ means that the logical argument works both ways, the statement and its converse are both true, it's an 'if and only if' argument.

Adding the equations (1) and (2) : $2a = n^2 + n \Leftrightarrow a = \frac{1}{2}(n^2 + n) \Leftrightarrow a = \frac{1}{2}n(n+1)$ **(G)**

This formula gives a as the n^{th} triangular number.

Similarly, b is the $(n - 1)^{th}$ triangular number $\frac{1}{2}n(n-1)$.

If a and b are any two consecutive triangle numbers, the reverse argument holds.

For a fair game we must have the probability of one outcome equal to the probability of the other outcome; that is: $(a^2 - a) + (b^2 - b) = ab + ab$ *(from the sample space diagram)* **(H)**

$\Leftrightarrow a^2 - 2ab + b^2 = a + b$ *(by rearranging this expression)*

$\Leftrightarrow (a - b)^2 = a + b$

Conclusion: Isomorphic games. The proof applies to all games that involve randomly picking two objects from a bag of objects of two different types where winning or losing corresponds to picking objects of the same or different types, for example, with red corresponding to odd and blue corresponding to even. The games are fair if and only if the number of objects of each type are consecutive triangle numbers a and b (so that $a + b = n^2$ and $a - b = n$). **(I)**

Conjecture: The game will be a fair game if and only if the numbers of objects of each type are consecutive triangle numbers and the total number of objects is a square number. **(J)**