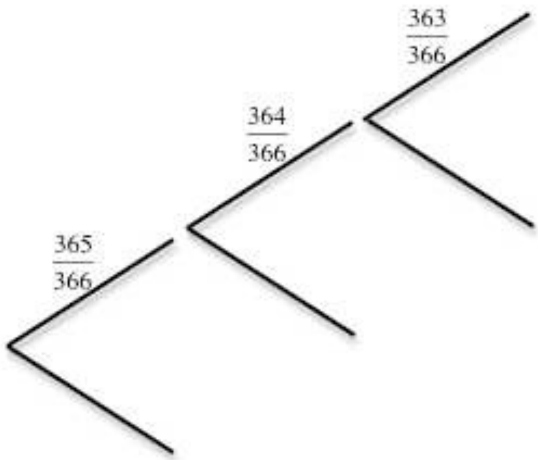
SAME BIRTHDAY



In a group of 23 people there is more than a 50% chance that two will have the same birthday.

Does that surprise you?

Including February 29th, how many different birthdays can there be? Start with 3 people A, B and C. What is the probability that B has a different birthday from A? What is the probability that A and B have different birthdays and C has a different birthday from A and from B?

In a group of 3 people what is the probability that two will have the same birthday?

What about a group of 4 people? Or a bigger group?

Can you use the unlabelled tree diagram above and do the calculations to work out the probabilities?

HELP

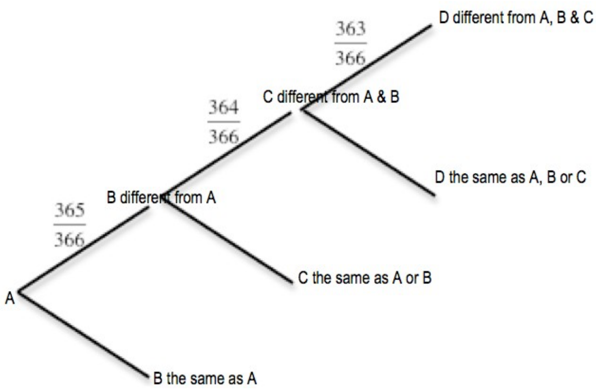
One of the 'golden rules' of problem solving is to work on simple cases when a problem seems difficult. Learners who find this problem difficult can do the At Least One (n=2) https://aiminghigh.aimssec.ac.za/at-least-one/
and Same Sweets (n=5)
https://aiminghigh.aimssec.ac.za/same-sweets/
and Same Birth-month (n=12) problems first
https://aiminghigh.aimssec.ac.za/same-birth-month/.

NEXT

Some learners could work on calculating the answers on a spreadsheet. Alternatively, learners could think of different contexts for posing problems like the Birthday Problem, but for different numbers of possibilities, and they could create and solve their own problems.

NOTES FOR TEACHERS

SOLUTION



Including 29th February there are 366 birthdays in a year.

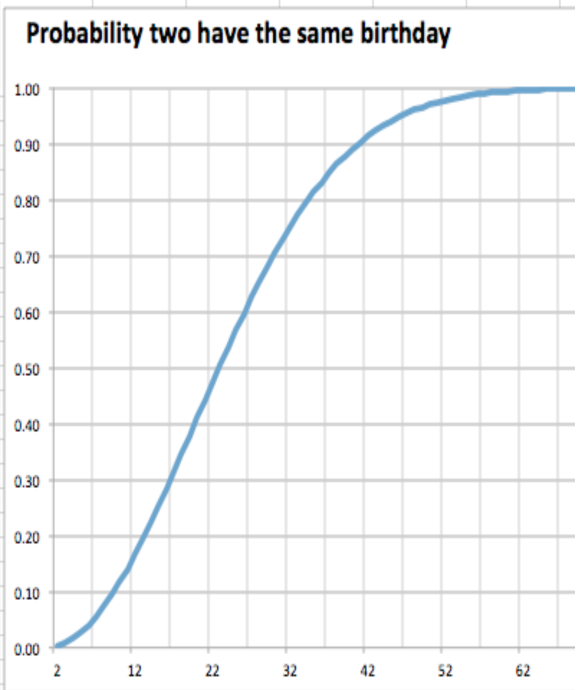
If there are more than 366 people in a group then, by the pigeon- hole principle, it is certain that at least one pair will have the same birthday.

However this probability is very close to 1 for much smaller groups.

For 3 people, A, B and C, the probability that A and B have different birthdays is 365/366 and the probability that C has a different birthday from both A and B is

$\frac{365}{366} \times \frac{364}{366} \times \frac{363}{366} = 0.98$ The calculations can be done on a spreadsheet.

Probability all have different birthdays	Number of people in group	Probability 2 have the same birthday
1.00	2	0.00
0.99	3	0.01
0.98	4	0.02
0.97	5	0.03
0.96	6	0.04
0.94	7	0.06
0.93	8	0.07
0.91	9	0.09
0.88	10	0.12
0.86	11	0.14
0.83	12	0.17
0.81	13	0.19
0.78	14	0.22
0.75	15	0.25
0.72	16	0.28
0.69	17	0.31
0.65	18	0.35
0.62	19	0.38
0.59	20	0.41
0.56	21	0.44
0.53	22	0.47
0.49	23	0.51
0.46	24	0.54
0.43	25	0.57
0.40	26	0.60
0.37	27	0.63
0.35	28	0.65
0.32	29	0.68
0.29	30	0.71



We reason in the same way for a group of 5 or more people. The probability that a 5th person has a different birthday from the first 4 people is 362/366 so we multiply the probability given above by 362/366 to get the probability that all 5 people have different birthdays which gives 0.97.

The table and the graph shown are copied from an Excel spreadsheet where the values have been given to 2 places of decimals.

For example, for 15 people, the probability that all have different birthdays (calculated to 4 decimal places) is: $(365 \times 364 \times 363 \times 362 \times \ldots \times 352)/366^{14} = 0.7477$.

So, the probability that there is at least one pair with the same birthday is:
1 – 0.7477 = 0.2533 and the probability to 2 places of decimals is 0.25 as can be seen from the table and the graph.

Why do this activity?

The well known birthday problem provides a context to which learners can easily relate and one in which they can engage in problem solving and discover answers for themselves.

One of the most important problem solving skills is to be able to find and solve simple cases of the same problem and then to extend the method used in the simple cases to solve the more difficult problem. The simplest case of this problem involves just two possibilities (n=2) rather than 366 possibilities (n=366, the number of different birthdays.

Learning objectives

Learners should come to understand that the probability of an event happening is 1 minus the probability of it not happening.

Learners will gain a deeper understanding of the use of tree diagrams.

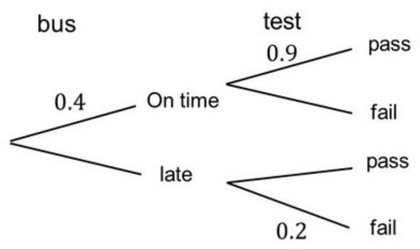
DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

Ben travels to school by car, the probability of the him being on time is 0.4. If the car is on time he has a probability of passing the maths test of 0.9. If the car is late his probability of failing the test is 0.2.



What is the probability he passes the test?

A 0.72 | B 0.84 | C 0.1728 | D 1.7

1.It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.

2.Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

3.Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

4.If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

B. is the correct answer 0.4×0.9 + 0.6×0.8=0.84

Common Misconceptions

A. 0.9×0.8= 0.72 (just multiplying the probabilities of passing). It shows that the learner does not understand the structure and use of a probability tree diagram.

C. 0.4×0.9×0.6×0.8 = 0.1728. The learner has some idea that he should multiply the probabilities but does not understand the structure and use of a probability tree diagram.

D. This learner understands almost nothing about probability as probabilities always lie between 0 and 1. https://diagnosticquestions.com

Generic competences

Through working in pairs or groups to solve this problem learners have the opportunity to:

- gain problem solving skills and apply whet they have learned in a real life situation;
- practise using a spreadsheet and develop skills at using a spreadsheet to carry out calculations;
- develop team working skills.

Suggestions for teaching

It is helpful to think about a small part of the tree diagram, the first few branchings. But drawing a whole tree diagram is a much too complicated a task and not in any way necessary.

The teacher could start by posing the Birthday Problem and then suggest that the learners first try to solve the simple problem for 2 possibilities: “In a family what is the probability that there are two children of the same sex?” The class should solve this problem for families of 2 children and of 3 children and they should discuss why it is actually a simple case of the Birthday Problem with n=2 instead of n=366.

Next the class could work on the Same Sweets problem (n=5) from https://aiminghigh.aimssec.ac.za/same-sweets/

and the same Birth Month problem (n=12)

https://aiminghigh.aimssec.ac.za/same-birth-month/

It is important to let learners discuss the problems and suggest ways of solving the problems themselves. You could use the ‘One–two–four–more’ strategy where learners work individually, then in pairs, then in fours, then the whole class discusses the problem with learners presenting their ideas on the chalkboard. That way the teacher can help all the groups of learners to work at a pace that suits them.

If computers are available then the teacher might help the learners to calculate the answers using a spreadsheet.

Key questions

1. How many different birthdays are there?
2. Can 367 people all be born on different birthdays?
3. For 2 people what is the probability that the second person has the same birthday as the first person? Why?
4. For 2 people what is the probability that the second person has a different birthday from the first person? Why?
5. Suppose two people have different birthdays, what is the probability that a third person has a different birthday from both of the first two people? Why?

6. If you know the probabilities of two events, how do you work out the probability that both events happen?
7. If you have worked out the probability that an event happens how do you find the probability that the event does NOT happen?

If you have worked out the probability that 3 people all have different birthdays, how do you find the probability that two have the same birthday?

In summary, at the end of the lesson: RULES OF PROBABILITY

All events have a probability of 0 or 1 or between 0 and 1
The probability of an event NOT happening is 1 – the probability the event happens.

The probability of an event is the fraction:
the number of ways the event can happen
the total number of different possible events

If S and T are two events,
Probability(S and T) = Probability(S) × Probability (T given that S has already happened) so we can draw a tree diagram and multiply the probabilities along the branches.

INDEPENDENT EVENTS
If and only if S and T are independent events
Probability(S and T) = Probability(S) × Probability(T)

PIGEON HOLE PRINCIPLE, explained by an example:

If there are 366 post boxes, and more than 366 letters are put in the boxes, then at least one box must have 2 or more letters put in it.

FOLLOW-UP ACTIVITIES

Same Sweets https://aiminghigh.aimssec.ac.za/same-sweets/
Same Birth-Month https://aiminghigh.aimssec.ac.za/same-birth-month/
At Least One https://aiminghigh.aimssec.ac

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339 Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12				
	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13