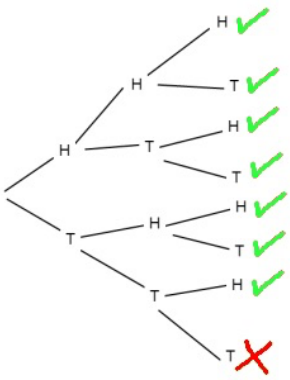
AT LEAST ONE

If Busi has 3 children what is the probability that at least one will be a girl?

Imagine tossing a coin three times.
What's the probability you will get a head on at least one of the tosses?

Suppose you drew this tree diagram.



How would it help you to find the answers to these questions?

What is the connection between the questions?

How could you work out the probability of Busi having 3 boys?

What do you notice about the probabilities of having at least one girl and having 3 boys?

HELP

One method you could try starts with listing all the outcomes.
Start your list as follows:
The eldest child could be a boy or a girl – 2 outcomes B or G
The second child could be a boy or girl – 4 outcomes BB, BG, GB or GG
…
Then trace each outcome along 3 successive sub-branches of the tree.

NEXT

You could extend the question to 4 tosses (4 children in the family) or more.

NOTES FOR TEACHERS

SOLUTION

Sample space diagram for family of 3		
1st	2nd	3rd
G	G	G
G	G	B
G	B	G
B	G	G
G	B	B
B	G	B
B	B	G
B	B	B

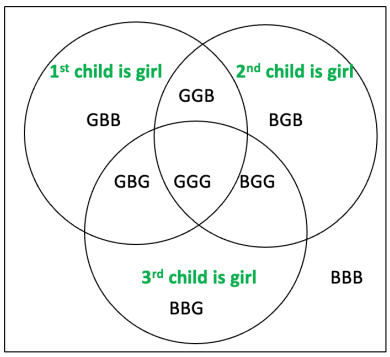
The probability of a girl is ½ and the probability of a boy is ½.

This is the same as the probability of a head or a tail when you toss a coin.

So the coin tossing is a way of mathematically modelling the real life situation of probabilities for different numbers of boys and girls in a family.

We can list the possibilities as HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

We can list the families: GGG, GGB, GBG, GBB, BGG, BGB, BBG and BBB. Similarly the probability of each outcome is ⅛.

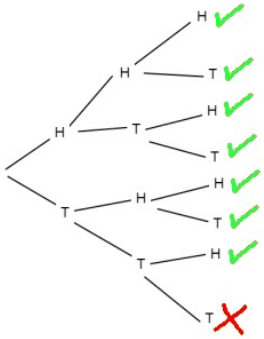


The Venn diagram is another representation of the same data. It shows the 8 possible outcomes for a 3-children family.

The tree diagram shows the results for 3 tosses of a coin with a tick against the branches showing at least one head and a cross on the branch showing no heads (all tails).

We can write the probability ½ on each of the 14 small branches of the tree which gives the probability of each outcome for the 3-children family as ½×½×½=⅛.

So there is at least one head for 7 out of 8 of the outcomes and the probability of getting at least one head is ⅞.



The probability of getting 3 tails is ½×½×½=⅛ which is
1 – probability of at least one head
and these two probabilities add up to 1.

By the same reasoning, the probability of at least one girl is ⅞.

Why do this activity?

This activity can be used to introduce tree diagrams or to develop a better understanding of them. Learners can easily relate to the situations of children in a family and tossing a coin and this question offers an opportunity for mathematical talk. Even if learners have not seen a tree diagram before, they can be given the diagram with the seven ticks and one cross and asked how they might use to answer the questions.

Rather than starting by the class tossing a coin to get an experimental probability they are encouraged to think mathematically about using this idea to model the real-life situation. The activity offers opportunities to use different methods.

Learning objectives

In doing this activity students will have an opportunity to:

- meet or review the idea of tree diagrams and their applications;
- appreciate and find meaning in the definition and concept of probability.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations and proofs;
- **apply** knowledge and skills;
- develop the skill of **interpreting visual images** to represent concepts and situations.

Suggestions for teaching

Start with the diagnostic question and a class discussion about it. This question is a bit more difficult than the question about families and coin tossing as the probability changes because balls are not replaced in the bag. However in both cases the tree diagram and probabilities are given so all the learners have to do is to read and interpret the tree diagram.

Teachers could draw the tree diagram and write the questions on the board and ask the class to try to answer the questions. The class could work in pairs or small groups. After allowing sufficient time the teacher can lead a discussion asking learners to make suggestions about using the tree diagram. Then, based on what the learners have done, and what they say about it, the teacher can give her explanation and summarize what they have learned.

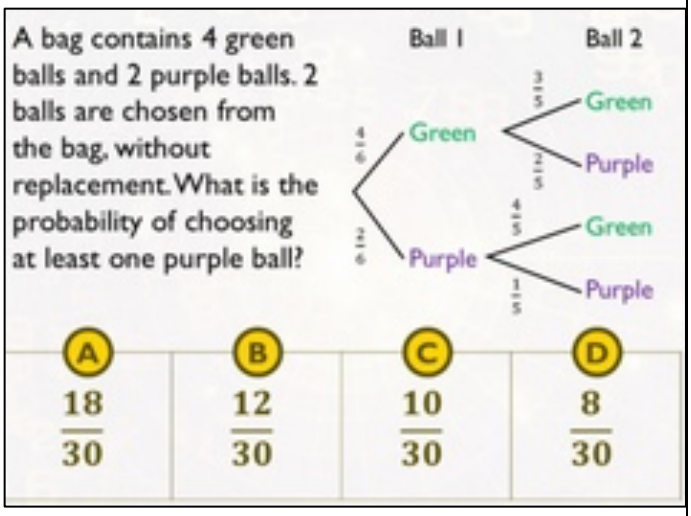
Key questions

- What is the probability of a baby being a girl?
- What is the probability of the coin landing as a head?
- What happens for two tosses of the coin one after the other?
- Can you list the outcomes?
- How does the tree diagram show the results for tossing a coin 3 times?

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer to develop communication skills and because voicing thoughts helps people to think more clearly.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is A.
Possible misconceptions:

B. This is the probability of choosing no purple balls. The correct answer is 1 – 12/30 = 18/30

C. This is the probability that the second ball chosen is purple. It seems learners giving this answer may not have understood the question.

D. Learners may have thought like this: I think D is the right answer is because 2/6*4/5=8/30

https://diagnosticquestions.com

Follow up

In a Box https://aiminghigh.aimssec.ac.za/years-6-12-in-a-box/

Out in the Snow https://aiminghigh.aimssec.ac.za/years-10-12-out-in-the-snow/

Epidemic https://aiminghigh.aimssec.ac.za/years-10-12-epidemic/

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