AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18
on the theme of SCALE

Choose what seems suitable for the age or attainment level of your learners

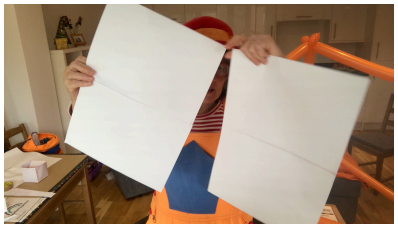
The original SCALE PAPER AIRPLANES was designed for Years 4 to 12

SCALE PAPER AIRPLANES

Watch the video http://bit.ly/ScalePaperAirplanes . The video gives instructions for making the planes but you can use your own design. It is more planet friendly to use scrap paper, either from old magazines, advertising material or printing paper.

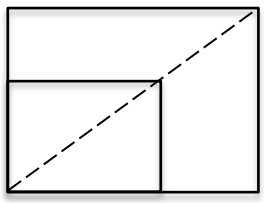

Use scrap paper.

The pictures show one paper airplane made with ordinary A4 printing paper and another bigger airplane.

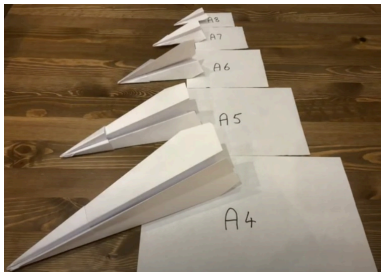


Two sheets of A4 paper make a mathematically similar rectangle with double the area which we call A3. Two sheets of A3 double the area again making A2 size paper, so the area scale factor is 2 every time. What is the linear scale factor?

To check whether or not two rectangles are similar (in the mathematical sense)



place them with one vertex coinciding and draw the diagonal as shown in the diagram. The diagonal of the larger rectangle must go through two vertices of the smaller rectangle. **To check whether your paper is suitable for this work**, fold a sheet in half; if this method shows that the half sheet is similar to the full sheet then the paper is suitable.



Fold an A4 sheet in half to make a similar A5 sheet and then fold again and again and again to make A6, A7 and A8 paper. Make each sheet into a smaller and smaller and smaller scale model airplane, following the directions on page 3 as demonstrated in the video.

What are the scale factors for area and for edge lengths?

Make your own airplanes and investigate how far they fly. Change the design and find out how the change affects the performance of your planes. For example, you could add a small paper clip in different places to weight the plane and see what happens when you fly it. Try flying your planes outdoors and aim them upwards to try to catch a gust of air and make the plane fly further.

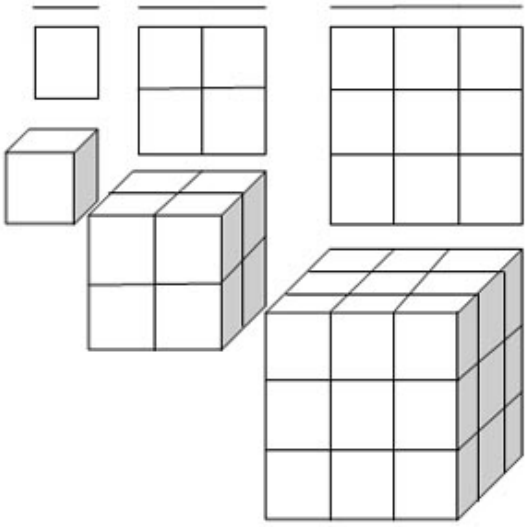
Check, as explained above, whether your paper is suitable for this work. If not then you need to cut your paper down so the edges are in the ratio $\sqrt{2} : 1$. For example, the edge lengths of A4 paper are 297 by 210 millimetres. US standard letter paper measures 279 by 216 millimetres so, for this work, cut 279 by 19 millimetre strips off the longer edge of US letter sized paper to make sheets measuring 279 by 197 millimetres and work to the nearest millimetre. Call them U4 and always use the 279 by 197 mm U4 paper whenever the instructions in this document ask for A4.

HELP

If you are not sure how to work out the linear scale factors then measure the corresponding edges to find out. You do not double or halve the lengths; what happens to the lengths? The challenge is to understand WHY you get that ratio between edge lengths (linear scale factor).

NEXT

It's obvious that the area scale factor is 2 and you have found the linear scale factor. Now what about the volume scale factor? This is more tricky.



This diagram shows 1-dimensional lines, 2-dimensional areas and 3-dimensional volumes. It shows a line one unit long, an area 1 square unit and a volume 1 cubic unit; it also shows lines, areas and volumes for which the linear scale factor is 2 and 3.

Explain what the area and volume scale factors are in the 3 enlargements shown in the diagram.

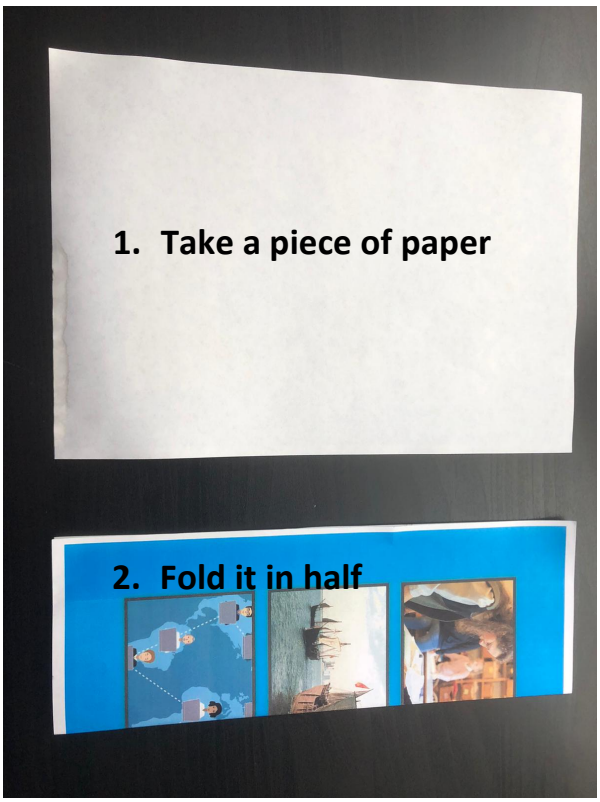
Explain how and why the scale factors for the sequence of paper airplanes are different.

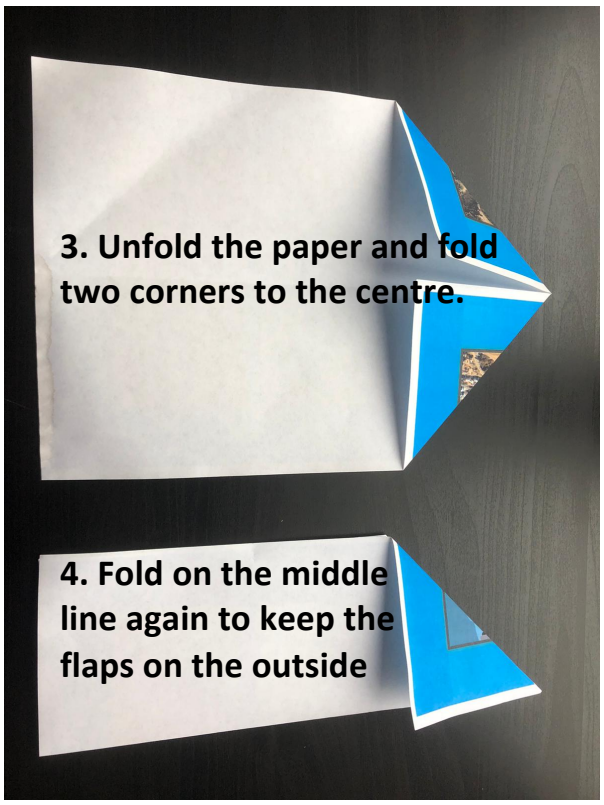
Suppose we change the paper material in a true enlargement before we construct the paper airplanes. For example, the A2 paper used to make the A2 model airplane has edge lengths 594 by 420 mm, so it is an enlargement of A4 paper by a linear scale factor 2. Because we double the lengths we must also double the thickness of the paper used. Fix eight A4 sheets together with sticky tape to make A1 sized paper, and then fold it in half to make a double thickness A2 sized sheet. Then fold the double thickness A2 sheet to make your A2 model airplane which will be a true scaled up version of the A4 model (an enlargement in the full sense).

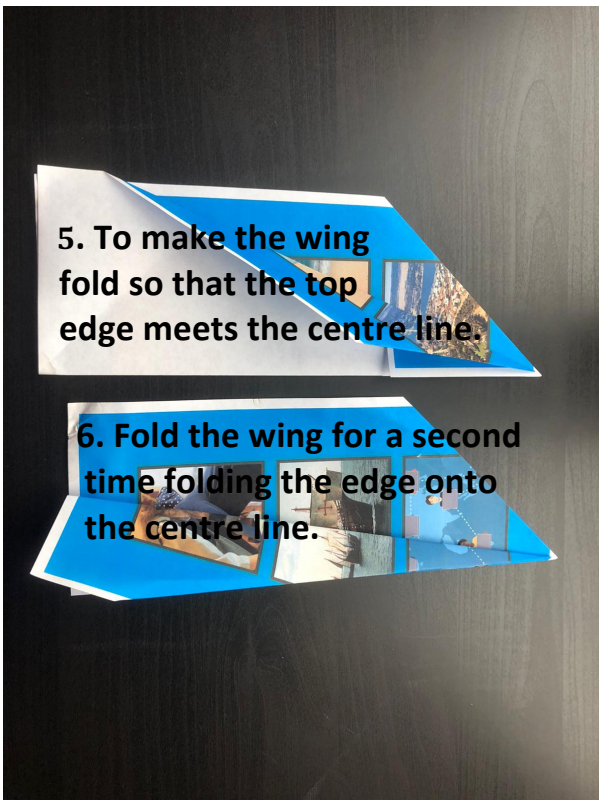
What effect does this design have on the performance of the model? Try it!

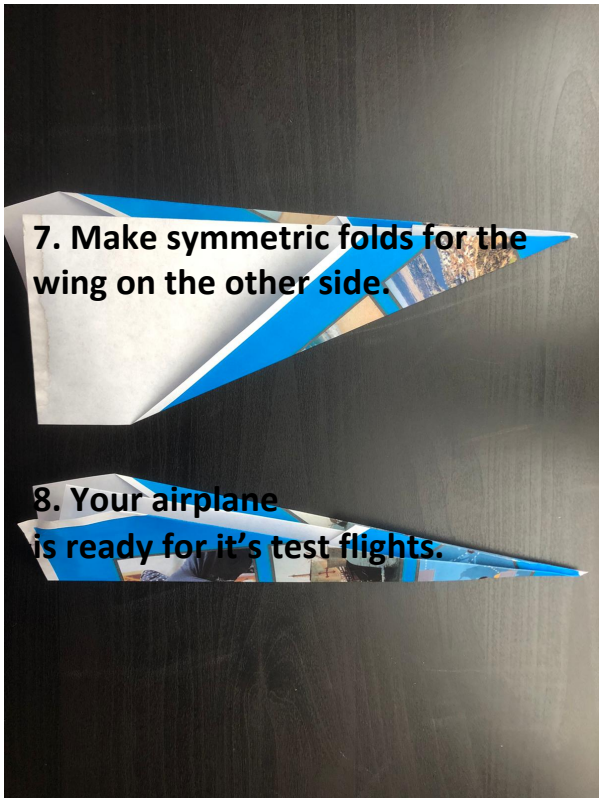
MAKE MODEL AIRPLANES

Make planes using this method as demonstrated in the video
http://bit.ly/ScalePaperAirplanes and to your own design.





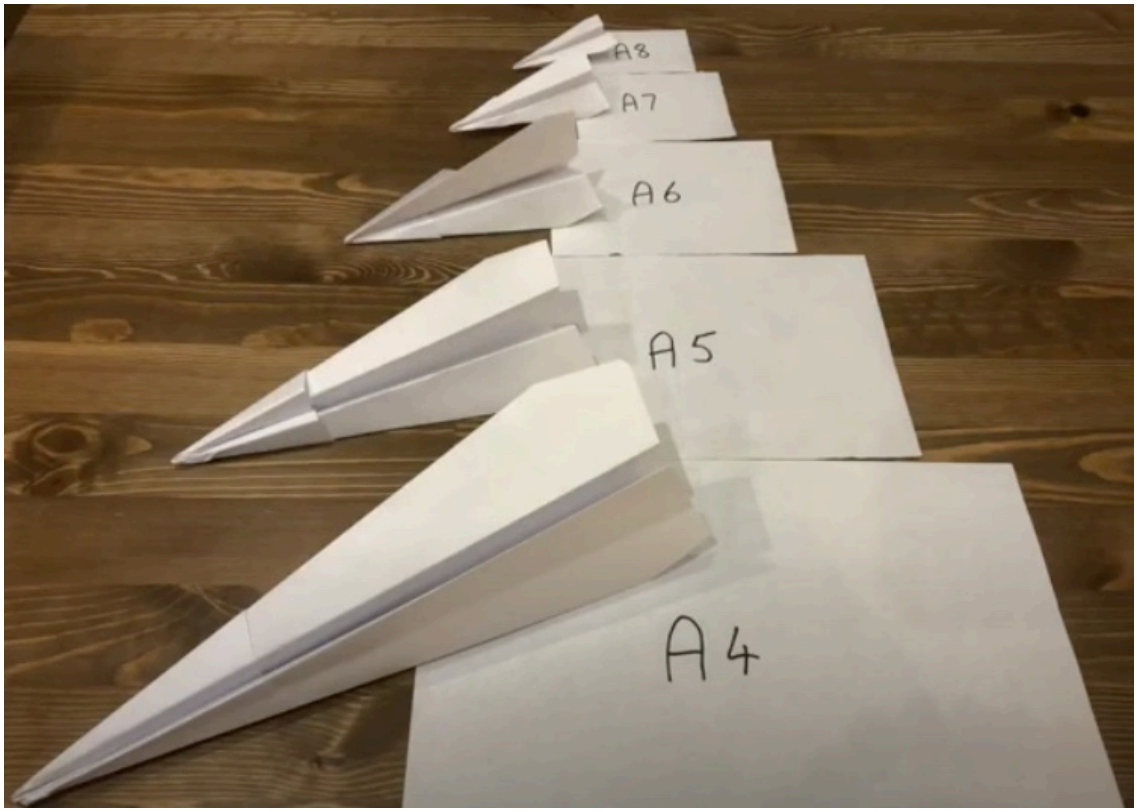




INCLUSION AND HOME LEARNING GUIDE
THEME: POSITIVE AND NEGATIVE

Early Years

Make model airplanes of different sizes and test fly them indoors and outdoors.



Make paper airplanes for your children.

Start by making an airplane from an A4 sheet.

Let them see you fold an A4 sheet of paper in half to make an A5 sheet and make a smaller airplane. Then repeat the process folding an A5 sheet in half to make an A6 sheet, folding A6 in half to make A7, and A7 in half to make A8 and then making a set of 5 airplanes.

Talk about what you are doing using the word 'half'. You are not formally teaching about fractions but rather the children will learn this language in the same way as they have learned language from babyhood. With 6 and 7 year olds you should use the word 'quarter' and also even 'eighth'.

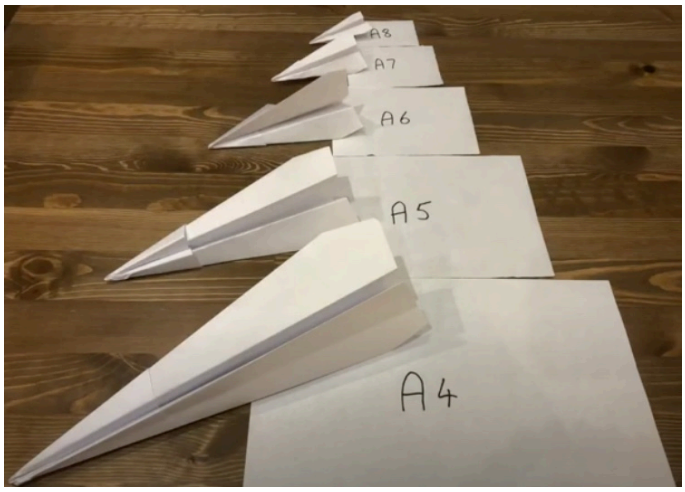
Let the children play with the airplanes. They should fly them to see which plane goes farthest, fly them outdoors as well as indoors, launch them on a level flight path and then aim them on flight paths slightly up at different angles.

Ask them to line up the planes in order of size.

It's important to make this experience enjoyable for the children, so come back to it another time if interest falls away.

Lower Primary

Explore ideas of halves, quarters and eighths by paper folding, make model airplanes of different sizes and test fly them indoors and outdoors.



Make paper airplanes for your children.

Start by making an airplane from an A4 sheet.

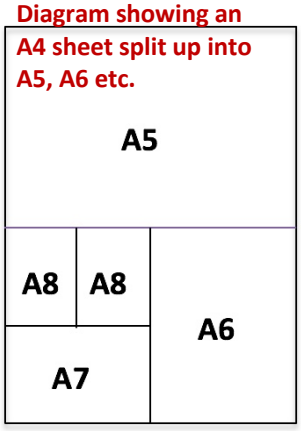
Check if your paper is suitable as described on page 2.

Let the children see you fold an A4 sheet of paper in half to make an A5 sheet and make a smaller airplane.

Then repeat the process folding an A5 sheet in half to make an A6 sheet, folding A6 in half to make A7 and A7 in half to make A8 and then making a set of 5 airplanes.

Talk about what you are doing using the word 'half'. You are not formally teaching about fractions but rather the children will learn this language in the same way as they have learned language from babyhood. With 6 and 7 year-olds you should use the word 'quarter' and also even 'eighth'.

Diagram showing an A4 sheet split up into A5, A6 etc.



Let the children play with the airplanes. They should fly them to see which plane goes farthest, fly them outdoors as well as indoors, launch them on a level flight path and then aim them on flight paths slightly up at different angles.

Ask them to line up the planes in order of size.

It's important to make this experience enjoyable for the children, so come back to it another time if interest falls away.

Ask them to compare the A4 and the A6 planes. What do they notice?

(You will find that the A4 plane is twice as long as the A6 plane.)

Find out if the same is true when you compare the A5 and A7 planes and again when you compare the A6 plane to the A8 plane.

Upper Primary

Make model airplanes extending the range of sizes from an eighth size sheet to four sheets and test fly the airplanes indoors and outdoors.

Start by watching the video if possible http://bit.ly/ScalePaperAirplanes. Then give everyone two pieces of A4 scrap paper and show them how to make a paper airplane with the first piece (follow the instructions on page 3 as demonstrated in the video). Then tell them to work with a partner and join their second sheets of paper together and use the double sheet to make a bigger airplane so now the pair have made 3 airplanes between them.

If possible, arrange for the learners to work in groups of 4 and give them 4 sheets. Tell them that the ordinary sheet of printing paper with which they made the first airplane is called A4 paper. Ask each group to join 2 sheets together and tell them that the double sheet is called A3. Ask them "When you compare one sheet of A4 paper with the larger A3 sheet made by joining two single sheets together, what can you say about the areas?" With some discussion the learners should be able to understand that the A3 sheet has double the area of the A4 sheet.

Ask the learners to measure the edges of the A4 sheet and the A3 sheet of paper and to write down the measurements. When they have done that ask them if the edge lengths of the A3 are double the edge lengths of the A4 paper. They should be convinced from their own measurements that the answer is "No, the edge lengths are not twice as long".

The picture shows two A3 sheets that will be joined to make an A2 sheet The next task for the learners is to join 4 sheets of paper, which makes A2 size paper, to measure the edges, write down the measurements and compare the edge lengths of A4, A3 and A2 paper. What do they notice?

Learners should notice that the edge lengths of A2 paper are double the edge lengths of A4 paper. When learners make an A2 airplane and compare the dimensions of the A4, A3 and A2 airplanes they will discover that the length of the A2 plane is double the length of the A4 plane (a linear scale factor 2).

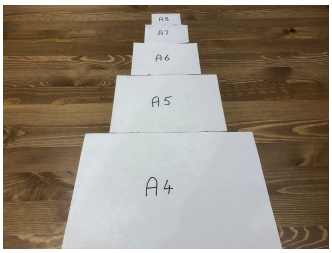
Let the children play with the airplanes. They should fly them to see which plane goes farthest, fly them outdoors as well as indoors, launch them on a level flight path and then aim them on flight paths slightly up at different angles.

If time allows, the learners could make the smaller scale models as described for Lower Primary if they have not already done so.

Lower Secondary

Learn about LINEAR AND AREA SCALE FACTORS and the relationship between them by making model airplanes and test flying them indoors and outdoors. Extend the range of sizes from an eighth size sheet to sixty-four sheets

Start by watching the video if possible http://bit.ly/ScalePaperAirplanes . Then give everyone two pieces of A4 scrap paper and show them how to make a paper airplane with one piece, then tell them to work with a partner, join their two sheets of paper together and make a bigger airplane.



Ask Key Questions 1, 2 and 3 and make sure that the learners give reasons for their answers.

Give each pair a sheet of A4 scrap paper a ruler and a pair of scissors. Show them this diagram and tell them to fold their sheet of paper in half and to make smaller pieces of paper as shown in the diagram.

Show everyone this picture and explain that it shows two A3 size sheets of paper made from joining A4 sheets and that these two A3 sheets can be joined to make an A2 sheet.

Organise the class into groups of 4 learners.

A Groups should make a sequence of paper airplanes from A5, A6, A7 and A8 paper and work out all the scale factors. They will only need 1 sheet of A4 paper.

B Groups should make paper airplanes from A3 and A2 paper and work out all the scale factors. They will need 6 sheets of A4 paper.

C Groups should make an A1 sheet and make it into a paper airplane. They will need 8 sheets of A4 paper.

Make a display of models from A1 to A8 in increasing size and discuss the answers to Key Questions 4 & 5. Draw this table on the board and ask the learners to say what should go in each of the boxes.

Type of paper	A8	A7	A6	A5	A4	A3	A2	A1	A0	A -1	A -2
Number of sheets of A4	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64

The learners should then test the models, preferably outdoors, and fly their models to see how far they can make them go. They should aim their planes upwards on different flights paths at different angle to try to catch a gust of air and make the plane fly further.

Follow by learners designing different style airplanes from different sized paper and investigating how far they fly. Encourage them to change the design and find out how the change affects the performance of their planes. For example, they could add a small paper clip in different places to weight the plane and see what happens when they fly it.

Key Questions

1. When you compare one sheet of A4 paper with a larger sheet made by joining two single sheets together, what can you say about the ratio of the areas?
2. What is the linear scale factor and what is the area scale factor of the two airplanes made from A4 and A3 paper? If you are not sure you can make some measurements.
3. Measure the edges of an A4 sheet of paper and divide the length of the longer edge by the length of the shorter edge? What do you notice about the answer (the ratios of the lengths of the edges).
4. What is the linear scale factor between the successive lengths of corresponding edges in the sequence of paper sizes: A4, A3, A2 and A1 so on?
5. What is the area scale factor between the successive areas of the sequence of paper sizes: A4, A3, A2 and A1 so on?
6. What is the linear scale factor between the successive lengths of corresponding edges in the sequence of paper sizes: A4, A5, A6, A7 and A8?
7. What is the area scale factor between the successive areas of the sequence of paper sizes: A4, A5, A6, A7 and A8?
8. What is the linear scale factor from A4 to A2 and from A5 to A3?
9. Measure the diagonals of your paper, what is the ratio of the diagonal lengths?

Upper Secondary

Conduct your own engineering experiments and learn about scale factors and enlargements in mathematics. Make model airplanes with a range of sizes from an eighth of a sheet to sixty-four sheets and test fly the airplanes indoors and outdoors.

Watch the video http://bit.ly/ScalePaperAirplanes and work through the activities. If possible work with a group to make a set of models from A8, A7, A6, A5, A4, A3 and A2 paper and then compare and test them in test flights, both indoors and outdoors.

You should discover for yourself the area scale factors and linear scale factors of your models corresponding to the scale factors of the different sizes paper.

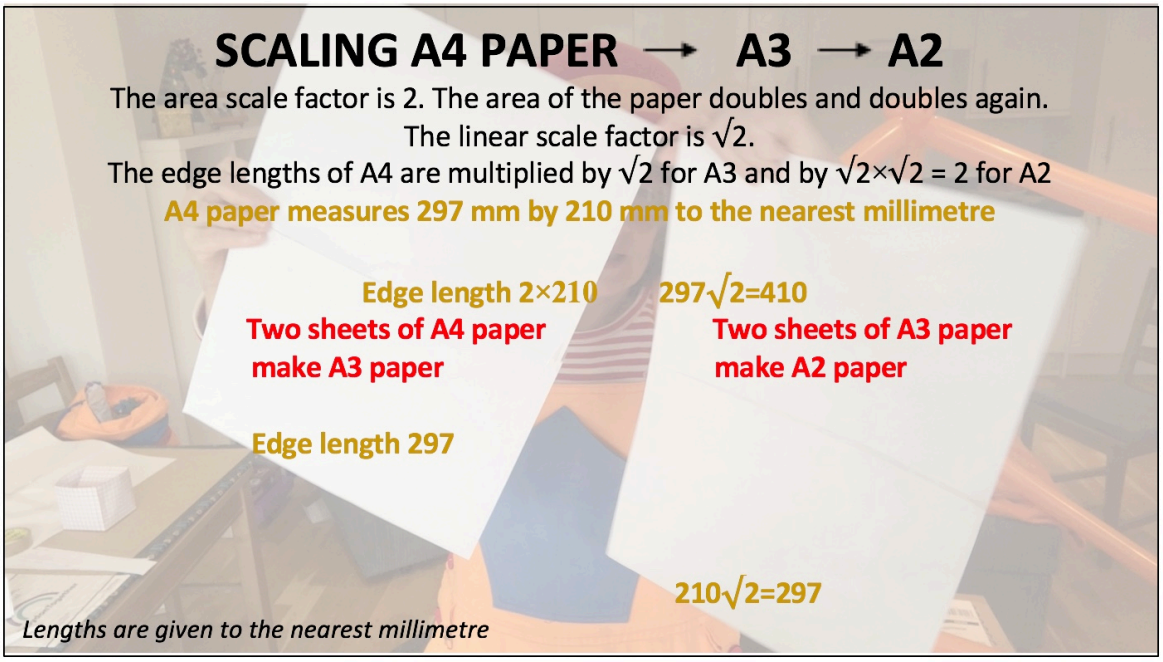
Answer all the Key Questions.

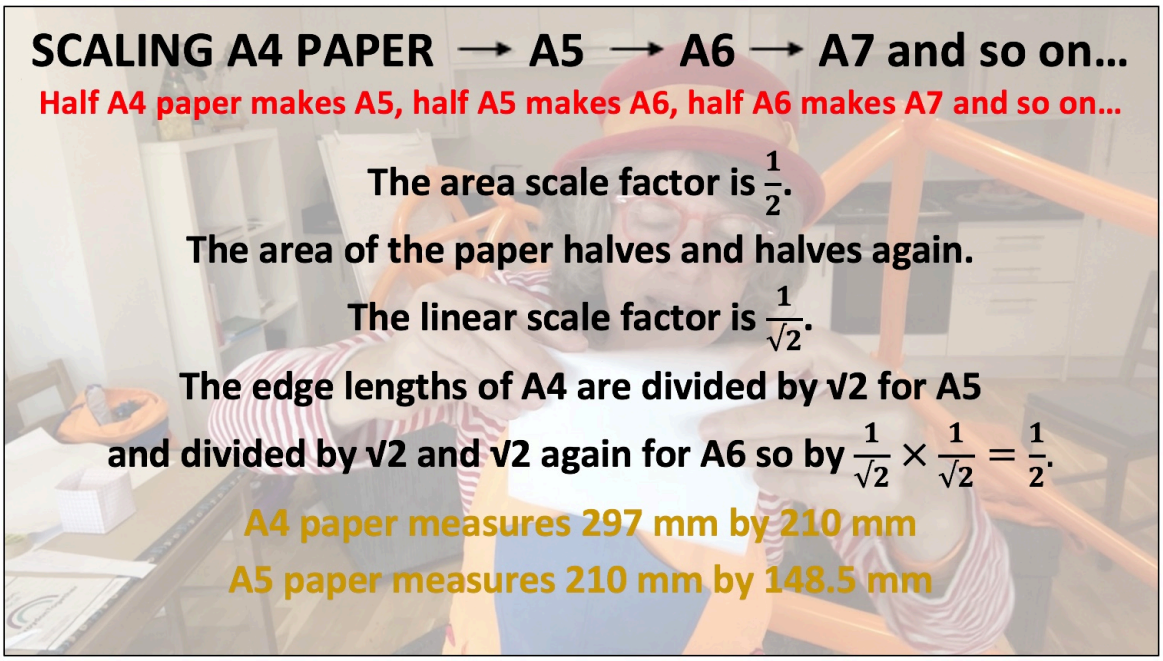
Answer the questions in the 'NEXT' box on page 2. Make an A2 plane with a double thickness of paper and test it in flight.

Design your own airplane and you might use different materials. Aim for a better performance than the design explained in the video.

SOLUTION

Type of paper	A8	A7	A6	A5	A4	A3	A2	A1	A0	A -1	A -2
Number of A4 sheets	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64
Dimensions to the nearest millimetre	74 52.5	105 74	148 105	210 148	297 210	420 297	594 420	840 594	1188 840	1680 1188	2376 1680





Why do this activity?

This activity makes learning about scale enjoyable. Scale is one of the most important concepts in mathematics. Scale is used in maps and design and, by all of us unconsciously every day in judging distances and the size of objects around us. Scale is critical in our everyday life because we need to judge the relative sizes of objects close up and at a distance. When we see something unfamiliar we judge its size by looking for something nearby as a 'frame of reference'.


These two animals, the rock hyrax (or dassie) and the elephant, are closely related genetically. Do you know how big the rock hyrax is compared to an elephant? Look up details of these animals and compare their sizes.

Learning objectives

In doing this activity students will have an opportunity to:

- deepen their understanding of linear and area scale factors and the relationship between them;
- investigate volume scale factors;
- make and test conjectures.

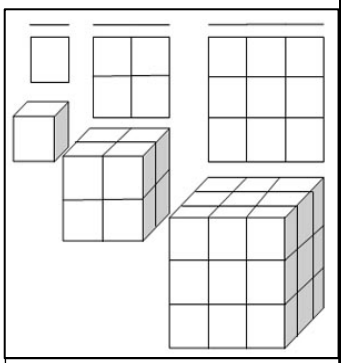
Generic competences

In doing this activity students will have an opportunity to:

- develop visualization skills;
- develop skills in design and model making;
- think like an engineer and to modify the model airplanes and test their performance.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes. It can be used at the start or at the end of the session and by individuals as well as by the whole group.
Show the question to the learners, including the diagrams, and say to the group:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.



The volumes shown in this diagram are in the ratio:

A. 1 : 4 : 9
B. 1 : 2 : 3
C. 1: 8 : 27
D. 1 : 9 : 27

The correct answer is **C**. The volumes are 1 cubic unit, 8 cubic units and 27 cubic units.

Common Misconceptions

A. Learners are mistaking area scale factors for volume scale factors.
B. Learners are mistaking linear scale factors for volume scale factors.
D. Again a poor understanding of scale factors, here they are giving the number of units of length, area and volume instead of the three volumes.
https://diagnosticquestions.com

Follow up

Little Man https://aiminghigh.aimssec.ac.za/years-4-6-little-man/

Sierpinski Number and Shape Patterns
https://aiminghigh.aimssec.ac.za/grades-6-12-sierpinski-number-and-shape-patterns/

Enlargement https://aiminghigh.aimssec.ac.za/years-9-10-enlargement/

Trisquares https://aiminghigh.aimssec.ac.za/years-9-12-trisquares/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13