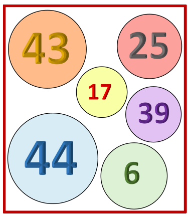
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

MATHOPIA LOTTERY



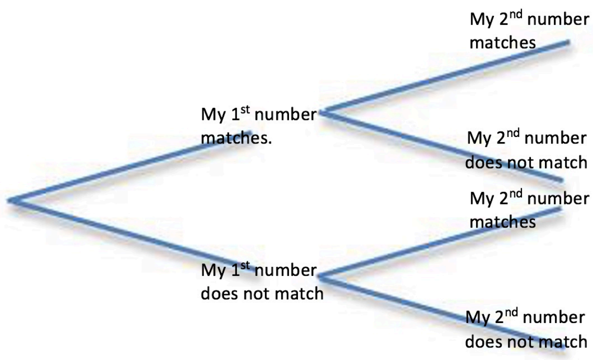
In the Mathopia Lottery, 49 balls are numbered 1 to 49 and 6 balls are chosen at random without replacing any of the balls so that 6 different winning numbers are chosen.

Each lottery ticket has 6 numbers and you win a top prize if your 6 numbers match the 6 numbers chosen that week.

Is buying lottery tickets a waste of money? What is your chance of winning the top prize?

A good problem-solving technique is to try simple cases if the problem seems difficult. The Lucky Numbers problem provides a simple case to try.
https://aiminghigh.aimssec.ac.za/lucky-numbers/

HELP



Think about a tree diagram that starts like this. Perhaps work with a partner so that you can help each other.

To solve the problem you need more branches. Ask yourself which branches you need and don't draw them all, just draw the branches that you need.

NEXT

The number of balls differs from country to country and the South African National Lottery has 52 balls, to read about it see:
http://en.wikipedia.org/wiki/South_African_National_Lottery .

You could discuss the good and bad aspects of having a National Lottery, including addiction to gambling.

Students could consider the probability of matching 3, 4 or 5 numbers with the six numbers drawn.

You would need to buy 7 million tickets in order to have a better than even chance of winning. Suppose you buy one ticket each week from the age of 18, and live to the age of 98, calculate how many lifetimes you would have to go on buying tickets for in order to have a better than even chance of winning?

NOTES FOR TEACHERS

SOLUTION

The probability that my first number matches is 6/49.
To win I must have 6 matches so next I work out the probability that both my first and my second numbers win, that is 6/49 x 5/48 (as we have already used one of the 49 numbers).
The probability of my first three numbers matching is 6/49 x 5/48 x 4/47
The probability of all my six numbers matching is
6/49 x 5/48 x 4/47 x 3/46 x 2/45 x 1/44 = 7.15 x 10^{-8}
or 1 in 13 983 816 million.

If you buy one ticket each week from the age of 18, and live to the age of 98, the number of lifetimes you would have to go on buying tickets for in order to have a better than even chance of winning is 1683 lifetimes.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

A bag contains 4 blue counters and 2 yellow counters.
The counters will not be replaced each time one is picked.
What is the probability of getting two different coloured counters?

A	B	C	D
$\frac{16}{30}$	$\frac{8}{30}$	$\frac{16}{60}$	$\frac{10}{30}$

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: **A.** (4/6×2/5)+(2/6×4/5)
Common Misconceptions
B. Learners have only considered one of two possibilities.
C. or **D** Confused about probability or about fractions or just guessing

Why do this activity?

This activity offers an engaging context in which to develop students' understanding of experimental and theoretical probability. They can calculate theoretical probabilities, perhaps by first starting a tree diagram (my first number matches or my first number does not match) then moving towards multiplying fractions based on conditional probabilities. The activity also provides a basis for discussion about gambling in the light of it being a waste of money. Other aspects of gambling, such as addiction, can be discussed at the teacher's discretion.

Learning objectives

In doing this activity students will have an opportunity to develop a deeper understanding of probability, especially of compound events.

Generic competences

In doing this activity students will have an opportunity to relate what they learn in school to the real world and in this case to make them aware of the odds against the gambler.

Suggestions for teaching

You could simulate the lottery in class by having 49 numbered cards in a bag. Each member of the class can choose 6 numbers. Then you draw 6 numbers from the bag. Does anyone in the class win? Does anyone have 5 matching numbers? Or 4?

Learners may not be able to start calculating the probabilities. Starting with simple cases is a good problem solving strategy. For a simple case, just put a few numbered cards in the bag (say 6) and draw out 2 cards. Ask learners to predict the chance of winning Then play the game. How many winners now? "Did we win as often as you expected? How could we calculate the probability of winning?" The class should now try to calculate the probability without the teacher suggesting how to do it. Give learners some time to work on this. Some students may list combinations (systematically or otherwise), whereas others may use tree diagrams. Move students on to consider the chances of winning with four balls (from six). Ideally, they will work on this using both listing and tree diagram approaches.

Bring the class together to share methods. Highlight anyone who has listed systematically to discuss the importance of making sure every combination is considered. Take time to discuss the symmetry that emerges from choosing a number on either side of three, and ask students to consider why this happens. If necessary, move students towards a tree diagram approach.

What if there were 20 cards and if players had to match 4 numbers? Ask learners to predict the probability of winning. Play the game as a class again (in effect a trial or class experiment). Then learners should calculate the probability of winning.

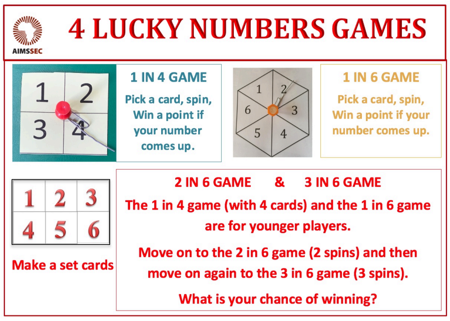
Working on this should give them enough confidence using tree diagrams to be able to solve the original problem.

Key questions

- Did we win as often as you expected?
- How could we calculate the probability of winning?
- In the simple case of a lottery with 6 balls rather than 49, why is the probability of winning the two numbers from six lottery the same as the probability of winning the four numbers from six lottery?

Preparation and Follow up

Lucky Numbers – a simple case
https://aiminghigh.aimssec.ac.za/lucky-numbers/



In a Box https://aiminghigh.aimssec.ac.za/years-6-12-in-a-box/

At Least One https://aiminghigh.aimssec.ac.za/years-8-10-at-least-one/

Epidemic https://aiminghigh.aimssec.ac.za/years-10-12-epidemic/

Two Aces https://aiminghigh.aimssec.ac.za/years-9-12-two-aces/

In the Bag https://aiminghigh.aimssec.ac.za/years-9-12-in-the-bag/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13