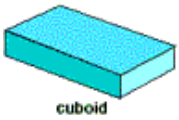
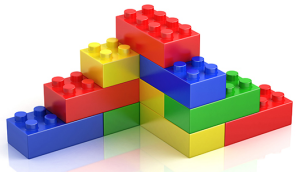
BRICKS, BLOCKS AND BOXES



Find a cuboid (with edges of integer values) that has a surface area of exactly 100 square units. Is there more than one?

It is quite easy to find a few solutions. The big challenge is to find all possible solutions.



What do you notice about this construction?

What do you notice about the dimensions of these building blocks? Look carefully at the blocks. How does the width of the block compare with the length?

Find the dimensions of a standard house brick in your country.

What do you notice about the width and the length of bricks?

Look at the way a wall is built.

Why do you think bricks are designed with these dimensions?

HELP

Try some dimensions for the cuboid and work out its surface area. Keep a record of everything you have tried.

Work with a partner or a group and between you split the work so you can check lots of surface areas and everyone does not have to check everything.

If you find a surface area near, but not exactly, 100 square units then try to find a solution by making small changes in the dimensions. For example for a 1 by 2 by 15 cuboid the surface area is 94 square units, so by changing one of these dimensions you might find a solution.

To answer the building questions, go and look at a wall and make some measurements.

If you can use some building blocks, then experiment by building some constructions that go around a corner as in the illustration shown. Take special notice of the ratios of the width and the length of the blocks.

NEXT

Cuboids with the same surface area

Find a convincing argument that all possible solutions have been found.

Once this has been answered, you might like to consider these extensions:

- Express the method for calculating surface area, algebraically.
- What surface area values generate lots of cuboids and which give none or just one?
- Could you set up a spreadsheet to help with the calculations?

Building Application

Investigate the dimensions for a double thickness wall with mortar 10 millimetres thick.

Investigate dimensions of paving slabs. When paving an area you are planning for 2 dimensions in contrast to building a 3 dimensional wall.

NOTES FOR TEACHERS

SOLUTION

Cuboids with the same surface area

METHOD 1 Using a method of trial and error we need to be systematic to avoid repeating calculations and to be sure of finding all the solutions. Label the lengths of the edges *x, y*, and *z* where $x \le y \le z$. The results for the surface area are given in the table below.

We don't try all values, just those that give a surface area around 100. Note that for any chosen *x* and *y* then the surface area increases as we increase *z* so we try some values and pick the values of *z* accordingly. Once we get a surface area of more than 100 we don't need to try larger values of *z*.

This method shows that there are only 2 solutions: 1, 2, 16 and 2, 4, 7.

No algebra is used, only the calculation of surface areas.

x=1 y=1		***x=1 y=2***		***x=1 y=3***		***x=1 y=4***		***x=1 y=5***		***x=1 y=6***		***x=1 y=7***	
z	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***
10	42	10	64	10	86	5	58	5	70	6	96	7	126
20	82	15	94	11	94	6	68	6	82	7	110		
30	122	16	100	12	102	7	78	7	94				
21	86					8	88	8	106				
22	90					9	98						
23	94					10	108						
24	98												
25	102												

x=2 y=2		***x=2 y=3***		***x=2 y=4***		***x=2 y=5***		***x=2 y=6***			
z	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***		
10	88	5	62	4	64	5	90	6	120		
11	96	6	72	5	76	6	104				
12	104	7	82	6	88						
		8	92	7	100						
		9	102								

x=3 y=3		***x=3 y=4***		***x=3 y=5***		***x=4 y=4***		***x=4 y=5***	
z	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***	***z***	***S Area***
3	54	4	80	5	110	4	96	5	130
4	66	5	94			5	112		

5	78	6	108										
6	90	7											
7	102												

METHOD 2

The use of algebra simplifies the calculations and speeds up the process.
Suppose we label the lengths of the edges x, y, and z where $x \leq y \leq z$ then the surface area is

$2(xy + yz + zx) = 100$

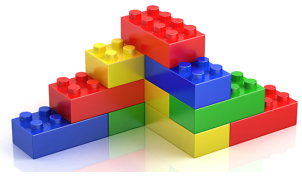
So $xy + yz + zx = 50$ and $z = (50 - xy)/(x + y)$.

For each pair of values of x and y we can calculate z.
We only get solutions where z is a whole number.
Calculations are recorded in the table below with numbers rounded to 1 decimal place. Notice that as x and y increase z decreases so we stop at the values for which z becomes less than x or y.

Checking the solutions in the table: $2(2 + 16 + 32) = 100$ and $2(8 + 14 + 28) = 100$
This method shows that the only two solutions are 1, 2, 16 and 2, 4, 7.

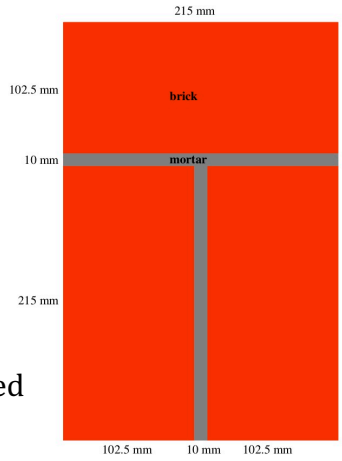
x = 1		***x = 2***		***x = 3***		***x = 4***		***x = 5***	
y	***z***	***y***	***z***	***y***	***z***	***y***	***z***	***y***	***z***
1	49/2=24.5	2	46/4=11.5	3	41/6=6.8	4	34/8=4.25	5	25/10=2.5
2	48/3=16	3	44/5=8.8	4	38/7=5.4	5	30/9=3.3	6	
3	47/4=11.75	4	42/6=7	5	35/8=4.4	6		7	
4	46/5=9.2	5	40/7=5.7	6		7		8	
5	45/6=7.5	6	38/8=4.75	7		8			
6	44/7=6.3	7		8					
7	43/8=5.4	8							

Building Application



The height of a **LEGO brick** is around 9.6 millimetres. The length and width of a 1x1 **LEGO brick** are 8 millimetres, which means that for every additional part of a **brick** (e.g. 2x1), every part adds up 8 more millimetres, which means that the width is 16 millimetres and the length is 8 millimetres.

With these dimensions walls can be strengthened by cross bricks and they can be built to turn corners.

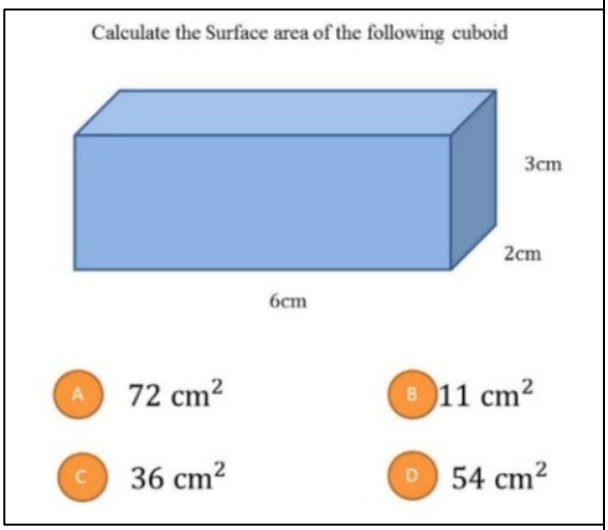


The usual thickness of the mortar used to hold bricks together is 10 mm.

The standard dimensions of 215 mm × 102.5 mm × 65 mm mean that because 102.5 + 102.5 + 10 = 215 walls can be built with the length of a brick equal to two widths + the thickness of the mortar.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class: **"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".**
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is A :
the surface area is $2(18 + 12 + 6) = 72$ cm²
Possible misconceptions:
B. Students giving this answer probably added 6 + 3 + 2

C. Students giving this answer have multiplied the dimensions that would give the volume, not the surface area.
D. This answer could have been calculated as 3 × 3 × 6 or it could have been a guess.https://diagnosticquestions.com

Why do this activity?

This activity requires a lot of calculations of surface areas, within a rich problem solving context, where it is necessary to work systematically to record the values that have been tried to avoid repeating calculations and to be sure of finding all the solutions.

The building application can follow on in or it can be introduced at a different time. It offers scope for the students to do some investigations for themselves and to apply their school mathematics in a practical situation.

Learning objectives

In doing this activity students will have an opportunity to:

- learn to calculate the surface area of a cuboid;
- work systematically to consider and possible cases and check that all the solutions have been found.
- Older students could use algebra to find the solutions.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in real life situations or in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.
In doing this activity students will have an opportunity to:

- think mathematically and to reason logically;
- interpret problems and work systematically to find all possible solutions;
- develop the skill of interpreting and creating visual images;
- apply their knowledge in a real life situation.

Suggestions for teaching

Work with a specific cuboid, eg 2×3×5, or a breakfast cereal box, to establish how to calculate surface area of cuboids. Learners could practise working out surface area mentally on some small cuboids.

Present the problem and ask learners to keep a record of things that they tried that didn't work (and what was wrong) as well as things that did work. In this initial working session, try to ensure that learners are calculating surface area correctly. Don't give them either of the tables above to fill in. It is much better for learners to experiment with their own ways of recording and, later in the lesson, for the whole class to discuss the different ways that different learners have recorded their results and the advantages and disadvantages of the different methods.

This activity could be turned into a game.
In groups, or as a class, keep a record of all cuboids whose surface areas have been calculated.

Award ten points for a bulls eye "100", five points for each surface area between 95 and 105, and two points for 90 to 95 or 105 to 110.

Miscalculated results could lose points, providing motivation for peer checking, and helping each other.

At the end of this lesson you might ask students to reflect on what they have achieved, which methods and ideas were most useful, and what aspects of the problem remain unanswered.

Key questions

- Have you found none/one/some or all of the solutions
- Is there a cube that will work?
- How might you organise a systematic search for the cuboids with surface area 100?

Follow up

See: **Same Volume** https://aiminghigh.aimssec.ac.za/years-6-8-same-volume/
For **Max Box** you have to vary the dimensions to find the box that has the maximum volume: https://aiminghigh.aimssec.ac.za/years-9-12-max-box/