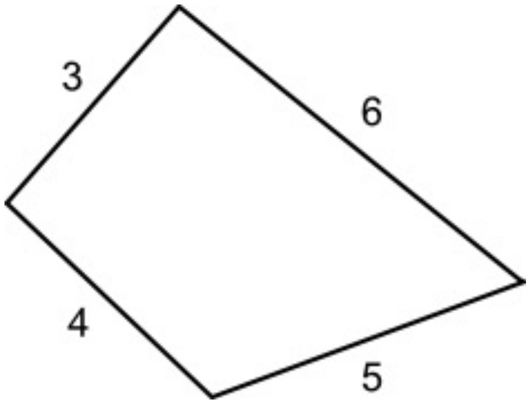
BENDY QUADS

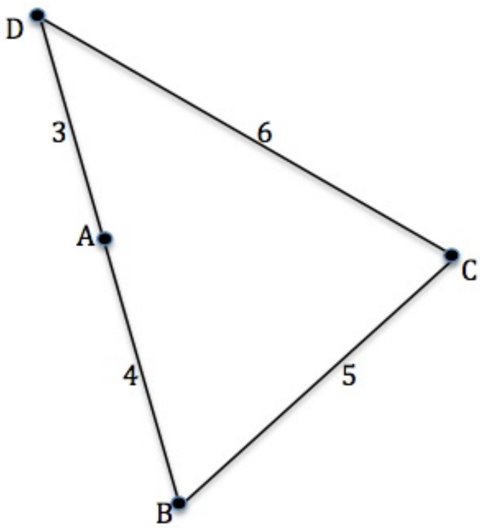
See the Bendy Quads video https://bit.ly/BendyQuadsVideo



Four rods are hinged at their ends to form a convex quadrilateral with sides of length 3, 4, 5 and 6 (in that order). Investigate the different shapes that the quadrilateral can take if the polygon is always convex.

How do the angles change as the bendy quad changes shape?

Can any of the angles reduce to zero degrees?

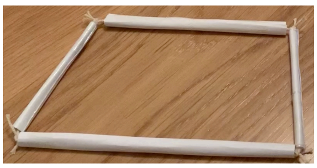


Can any of the angles increase to 180 degrees?

Calculate the size of angle C when the rods form a triangle as shown. If the polygon remains convex, can angle C get any smaller than shown in this diagram? What is the smallest size of angle C and what is the largest?

Find the smallest and largest values that the other angles can take in a similar way.

HELP



You might make a model that you can manipulate and experiment with, changing the angles. You could use 4 paper sticks of lengths 3, 4, 5 and 6 units choosing your own scale. For example, your sticks could be 6 cm, 8 cm, 10 cm and 14 cm (linear scale factor 2).

The special quads in the two pictures, with edge lengths 2, 3, 2 and 5, can both form a symmetric trapezium.

For the stiff quad model cut 4 strips of card and join them to form a quadrilateral of the given dimensions using split pins to link the strips of card.

The final calculations only require the use of cosine and sine rules.

NEXT

You could investigate non-convex quadrilaterals.

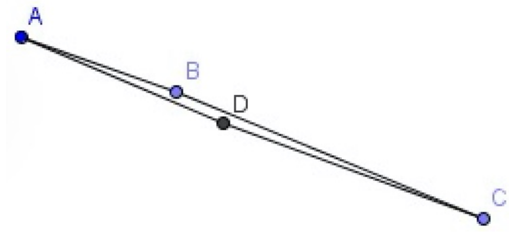
You could investigate the area of the quadrilateral and how this changes.

Can you make all the types of quadrilateral with 4 rods, for example a trapezium or a cyclic quadrilateral?

Try a quadrilateral with edges of lengths: 3, 5, 8 and 6. What is special about this quadrilateral?

NOTES FOR TEACHERS

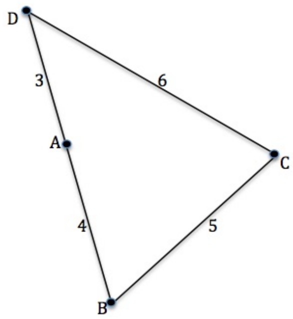
SOLUTION



As AB +BC = CD + DA = 9

we see that angles A and C can reduce to 0^o.

So angle A can change in size from 0^o to 180^o



However, if we just consider convex polygons, then angle C cannot get smaller than shown in this diagram where the rods form a triangle.

The smallest possible sizes of angles C, B and D are found from this diagram.

By the cosine rule: $7^2 = 6^2 + 5^2 - 60 \cos C$

So cos C = 12/60 = 1/5 and angle C = 78.5^o to the nearest tenth of a degree.

Angle C can change from 0^o to 78.5^o.

Using the sine rule:

sinB = 6/7 (sin C) = 5/7 (sin D) so angle B = 57.1^o and angle D = 44.4^o

Angle B changes from 57.1^o to 180^o.

Angle D changes from 44.4^o to 180^0.

Why do this activity?

This activity involves the interpretation of a very simple concrete structure, a linkage of 4 rods with the joints between the rods at the vertices totally flexible. Experiment and investigation lead to ideas about the angles that can be formed in these bendy quadrilaterals. Different cases can be considered, including convex and non-convex bendy quads in 2D and even in 3D. The conjectures need justification and proof by forming convincing arguments.

Solutions use the cosine and sine rules. To find the constraints on the angles in the general case requires an argument using inequalities.

Learning objectives

In doing this activity students will have an opportunity to:

- investigate a range of geometrical possibilities for a quadrilateral;
- practise applying the sine and cosine rules.

Generic competences

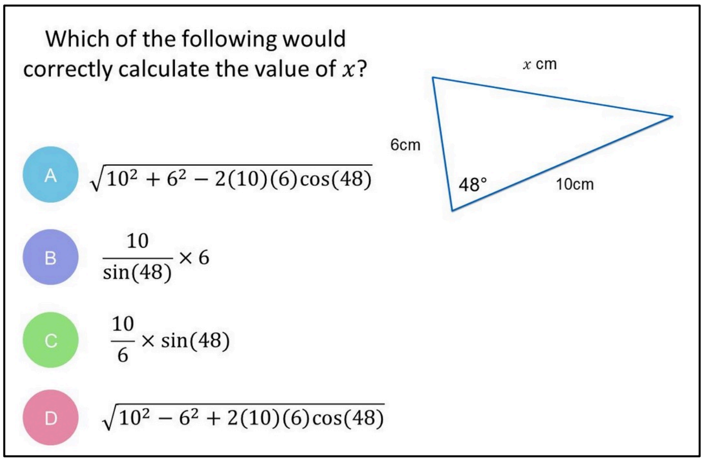
In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer so that they develop their communication skills and deepen their understanding by putting their thoughts into words.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
5. The concept is needed for the lesson to follow, so explain the right answer or give a remedial task.

The correct answer is A using the cosine rule.
Students giving answers B and C are incorrectly trying to use the sine rule.
Students giving answer D are misusing the cosine rule getting that signs wrong.
https://diagnosticquestions.com

Suggestions for teaching

You might make a demonstration model by cutting 4 strips of card and joining them to form a quadrilateral of the given dimensions using split pins to link the strips of card.

Allow time for learners to explore the different shapes that the quadrilateral can take using a model as described or dynamic geometry software such as Geogebra. This will help them to identify what can be varied and how much variation is possible.

Discuss with the class what it means for a quadrilateral to be convex and discuss how the angles change. Give some time for the learners to think about the range of possibilities and to make suggestions.

You can make it more challenging by simply showing the original diagram and leaving it to the learners to discover how some angles can reduce to zero or increase to 180^{o}, and how the triangle forms the limiting shape if the quadrilateral remains convex (so that the quadrilateral is **not** an arrowhead).

You might make it easier for the learners by suggesting that they consider the configurations where ABC and ADC become straight lines or where DAB becomes a straight line.

If you let the learners decide how to do the calculations some learners will use the cosine rule for calculating all the angles and some will also use the sine rule. When you summarize the work and check answers you can discuss how both methods apply.

Key questions

- If you flex the quadrilateral can the angles be any size?
- Can any of the angles reduce to 0^{o}?
- Can any of the angles increase to 180^{o}?
- Can the rods form a triangle?

Follow up

A challenging question that requires the setting up and solution of a quadratic equation: https://aiminghigh.aimssec.ac.za/years-11-12-solve-the-triangle/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

MATHS TOYS

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13