AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC) AIMING HIGH

PROJECTION OF IMAGES is the theme

for this INCLUSION AND HOME LEARNING GUIDE

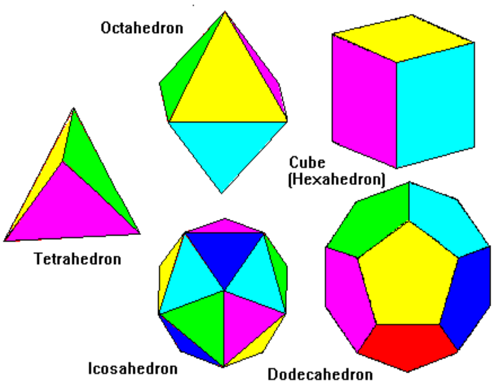
This Guide suggests related learning activities for all ages from 4 to 18+

Just choose whatever seems suitable for your group of learners

The SHADOWS ACTIVITY was designed for both Primary and Secondary

SHADOWS ACTIVITY

The picture shows some shadows cast by solid (3 dimensional 3D) objects as well as some flat (2 dimensional shapes. What could have made these shadows?



What sort of shadows can be made with these 3D objects?

Make models from the templates on pages 2 to 6 and use them to make shadows. Turn them around and make different shadows with the same object. Draw and label the shadows to record your discoveries.

HELP

Try making shadows of your hands and other objects.

NEXT

1. Which of these shapes could be the shadow of a cube? What objects could make these shadows?
2. Hold the solids at **different angles** to the wall. For example, it is possible to make a square shadow by holding a rectangular box at an angle to the wall.
3. Think about **ENLARGEMENT**. Cast smaller or larger shadows of the **same object** by using a torch. If you move the object nearer to the light source the image gets larger. If you move the object further from the light source the image gets smaller. Try it! This is how images are projected by a data projector. At the cinema you see very large images on the screen. Those images are projected from a projection room at the back of the cinema quite far away from the screen.

When lengths in the images are **twice the lengths** in the original object there is an enlargement of **SCALE FACTOR 2**. Investigate scale factors and how they are used.

(Adapted from a problem from the UKMT Maths Challenges)

RESOURCES

You can use sunlight or a torch or lamp to cast the shadows.

It is important to have solid objects for this activity. DO NOT do this activity using only pictures of solids. You must use actual solid objects. You can use anything in your house to make shadows, for example a mug, a coke can, a ball, a book. Collect solid objects from the kitchen or wherever you can find them, for example: packets and tins of various shapes.

Make models from scrap cardboard (see pages 2 - 6) then write their names on the solids to help you to remember the names. Print the names large and clear. You might hang your models from the ceiling. If you can't do that try to store them so that you can use them again.

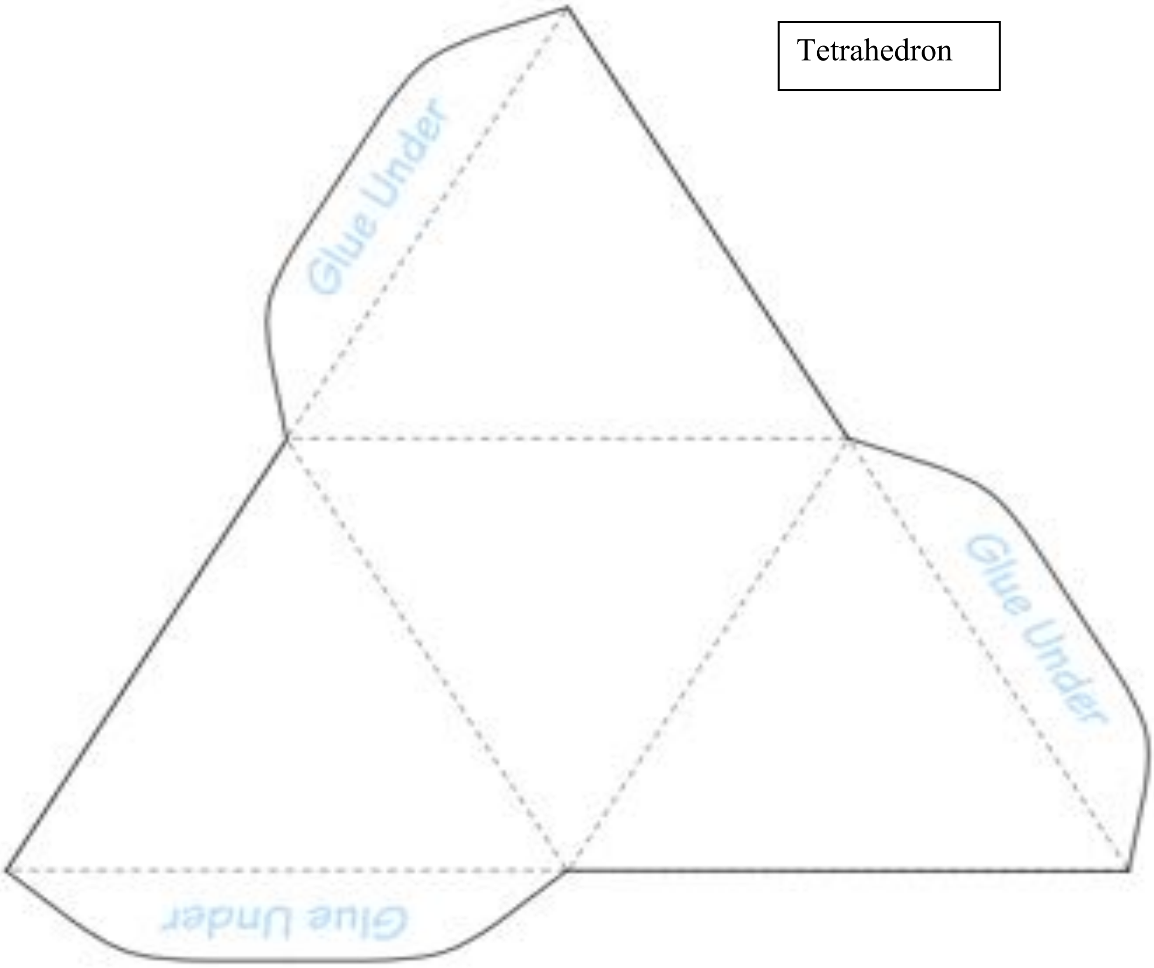
MAKING MODELS OF THE 5 REGULAR SOLIDS

http://www.mathsisfun.com/platonic_solids.html

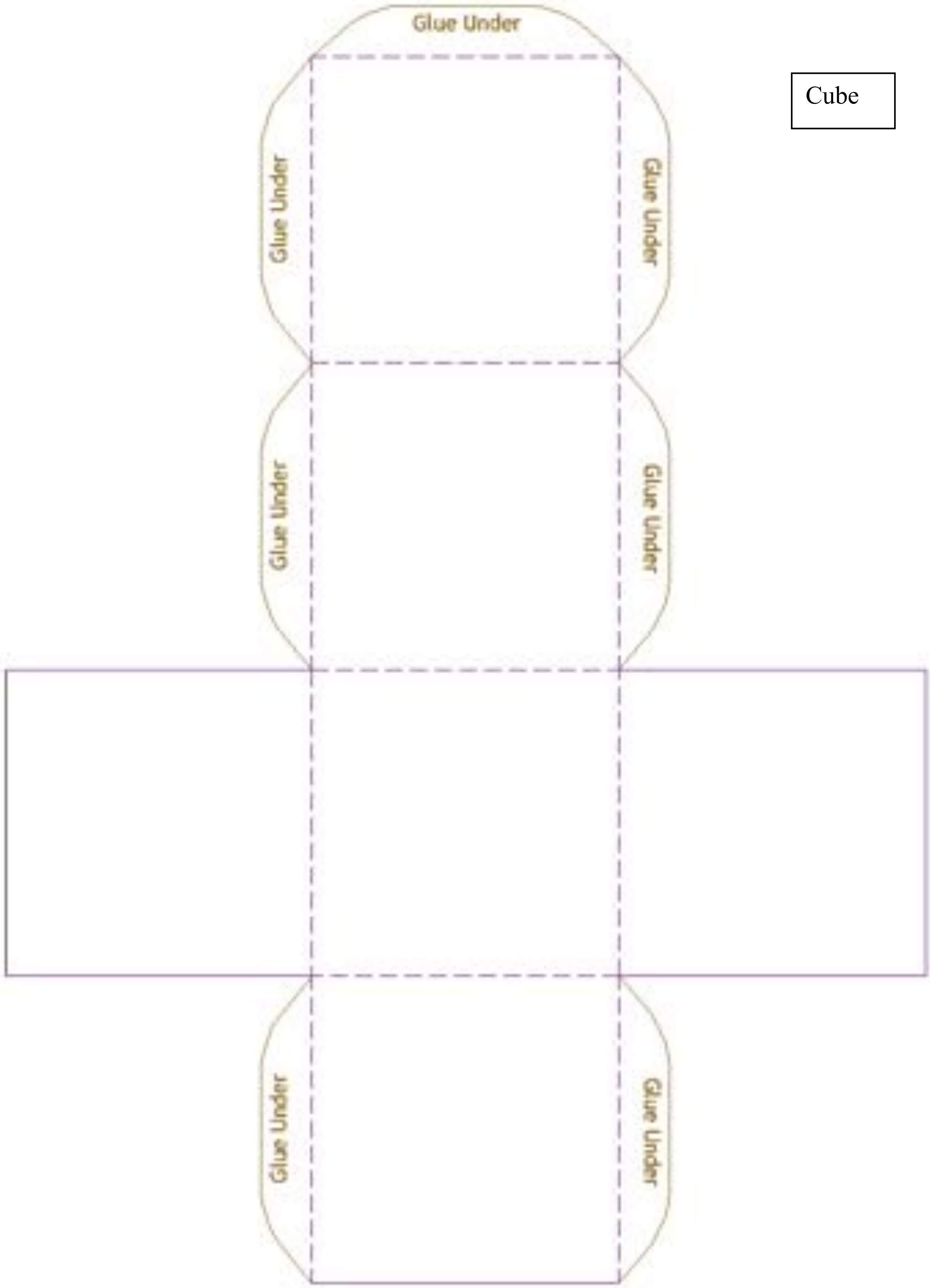
You can use scrap card (empty packets from your kitchen) to make the models using the nets below. Prick through the points on the nets and then copy the net onto your scrap card by joining the points you have drawn. For example you need to prick through 6 points for the 6 vertices of a tetrahedron.

Then cut out your net, crease all the edges and stick the flaps down to hold the solid in shape. Young learners will need help with this.

Tetrahedron



Cube



Glue Under
Glue Under
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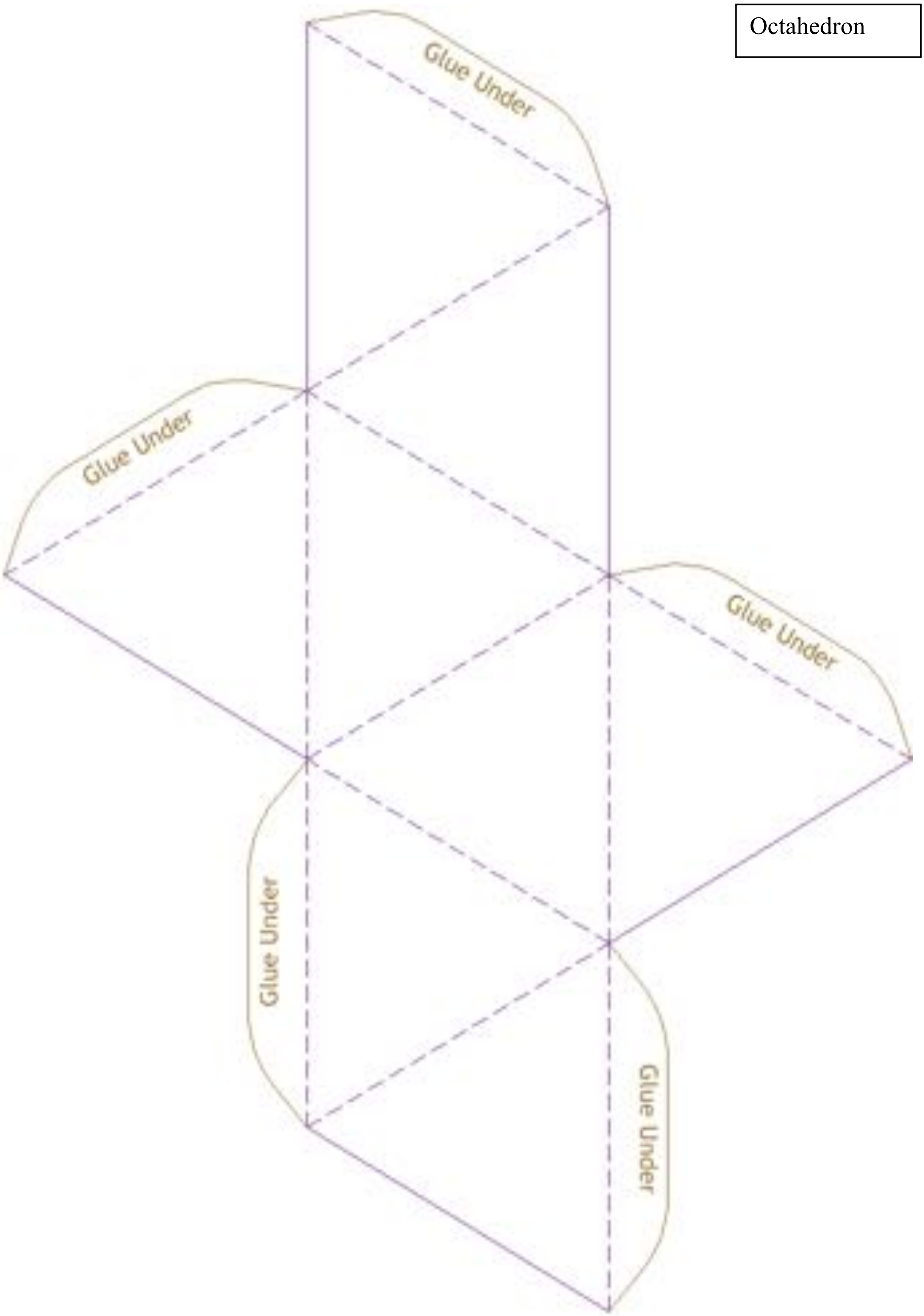
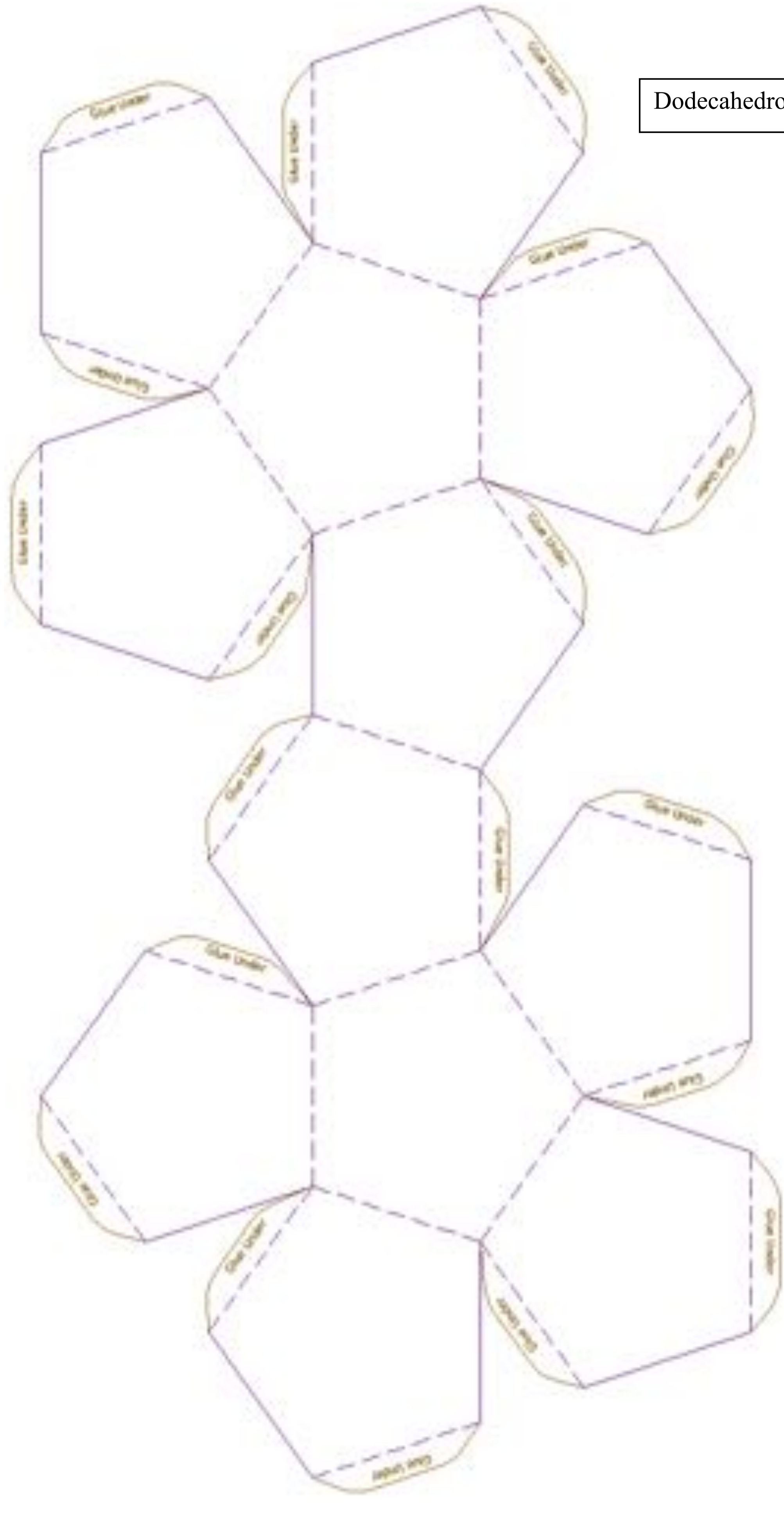
Glue Under

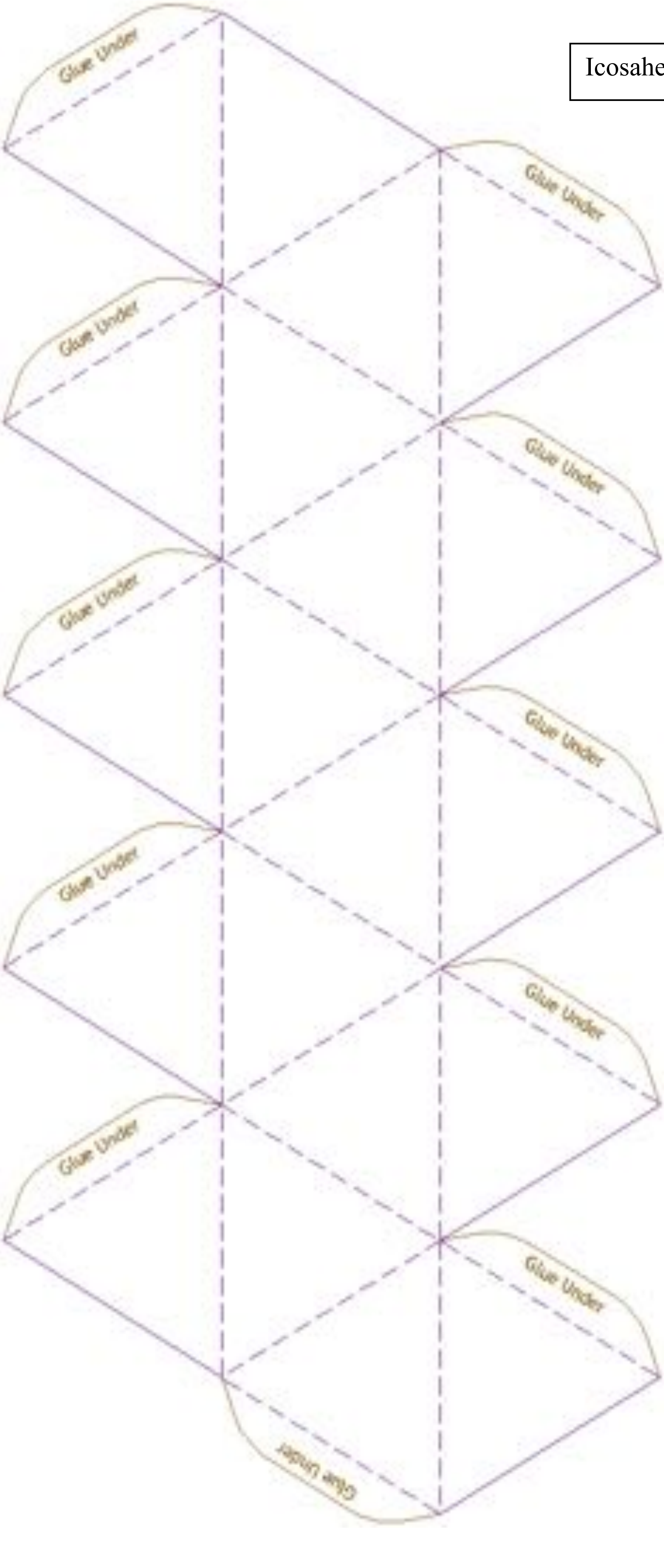
Octahedron



Dodecahedron



Icosahedron



INCLUSION AND HOME LEARNING GUIDE

THEME: PROJECTION OF IMAGES

Early Years and Lower Primary

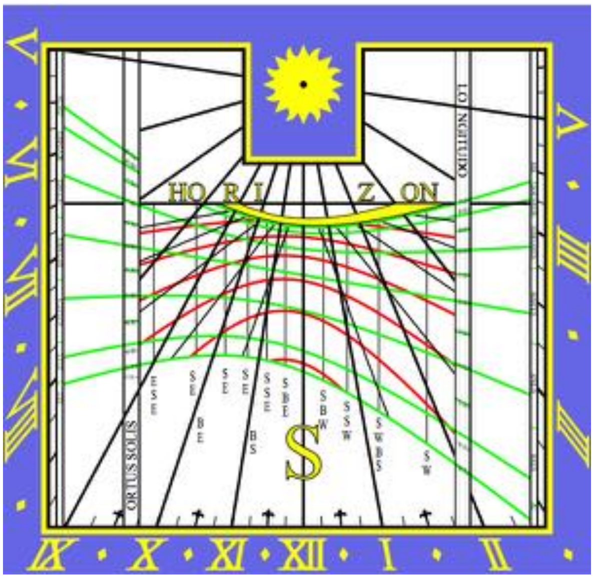
Encourage children to observe their own shadows on the ground. Talk about why the length of a shadow changes during the day and also moves around.

Talk about sundials. Make your own sundial with a stick. Go and see a sundial if there is one near where you live.

Upper Primary and Lower Secondary

Shadows is an ideal activity for a group of children of different ages and there is sufficient for more than a week. It can be used as a revision activity for learners of any age. You can select some tasks from this Guide and return at a later date to work on other tasks specified here.

A considerable range of mathematics is involved apart from properties of 2D and 3D shapes and development of visualisation skills. The theme of **projection of an image** leads to similarity, enlargement and scale factors. This in turn is connected to applications to constructing and interpreting accurate scale plans such as building plans, maps or engineering designs.



Everyone will enjoy making shadows and working out what made those shadows. Learners can observe their own shadows on the ground and **make sundials.**

Look at the picture. This sundial, first painted in 1642, is on a wall in Queens' College, Cambridge. It has a stick (called a gnomon) projecting out at the centre of the sun near the top. Time is determined where the shadow of the stick falls along the hour-lines.

Ask 'What do you notice?' Encourage the learners to talk about what they notice and to ask themselves: 'What do these things mean?'

This could involve talking about the word HORIZON, the Roman numerals, the compass directions and the red and green curves, 12 and 24 hour cycles and more.

Learners should do the activity as described in the worksheet on page1. Encourage discussion and invite suggestions but do not say whether the suggestions are right or wrong. Later the group can work together to make a clear and accurate summary of the facts observed.

Older children can help younger ones to make the models of the regular solids (Platonic Polyhedra) from the templates given on pages 2 to 6. Skeletons of the polyhedra can be made from paper sticks. See https://bit.ly/MakePaperStickPolyhedra

Invite the learners to say what might cast each shadow. From the collection of models that they have made and put on display, let them choose and test the solids that they think will make that shadow They can check if their choice was correct by using sunlight or a torch to cast the shadow. Make lists of the different objects that could have made each shadow.

Older learners can be expected to know the names and the properties of the shapes. Many older learners have missed out on developmental experiences like this when they were younger, so this activity will also help them to make connections between concrete experience and abstract ideas.

Key questions

1. Can you think of any 2D objects that would cast a shadow like this?
2. Can you think of any 3D objects that would cast a shadow like this?
3. Can you see any objects on display that would cast this shadow?
4. Do you know the name of this object?
5. Are there any other objects that would cast this shadow?
6. Is the shadow the same size as the object?
7. Why does it make a difference if we hold the object nearer to the torch?
8. Does it make a difference if we turn the object around, or if we hold the object at different angles to the wall?

Why do this activity?

This activity helps learners to make connections between abstract ideas about shapes and concrete objects that they can handle and look at from different directions. The activity builds on the learners' own experience of actually handling objects and experimenting. Young children need a lot of experiences of working with concrete materials for their brains to develop powers of visualization and abstract thinking.

Learning objectives

In doing this activity students will have an opportunity to:

- learn the properties of 3D solids through handling the solids and discussing their properties;
- learn the properties of Platonic Polyhedra by making models and experimenting with making shadows;
- learn about transformations, similarity and scale factors by making shadows.

Generic competences

In doing this activity students will have an opportunity to

- **develop visualization** and skill to interpret or create images to represent concepts;
- apply their knowledge to drawing accurate scale drawings;
- apply their knowledge to interpreting scale drawings, maps, building plans etc

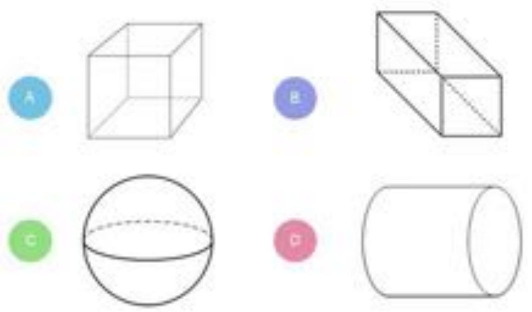
Diagnostic Assessment This should take about 5–10 minutes.

1). Write the question on the board, say to the class:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D”.

Either ask: Which 3D shape is different from the others? Why?

or Which 3D shape is not a prism?



2). Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

3). Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4.Ask the class **again** to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

C. is the correct answer. Learners may say that the sphere is different because it has no flat faces, which is a good reason. Alternatively they might say that the other 3 shapes are all prisms. They are ‘the same all the way through’ or ‘have the same cross section’, that is if you cut them parallel to the end face the cut will always be the same shape.

Common Misconceptions

Many learners will just guess so it is important that they give reasons for their answers.

If the class has not met the word ‘prism’ they can still talk about the properties of the shapes and what happens if you cut through the shape.

https://diagnosticquestions.com

SOLUTION

There are lots of solutions here.

Disc	Square	Triangle	Rectangle	
	could be the shadow of:			
a circular plate (2D) a sphere a cylinder a cone. It could even be the shadow of a rugby ball if the longest axis of the ball is at right angles to the wall.	a flat (2D) square a cube a square prism (which is a special cuboid).	a flat (2D) triangle a tetrahedron (triangular pyramid) other pyramids a triangular prism a cone	a flat (2D) rectangle a cuboid a cylinder or any prism	Have you got any other ideas?

Upper Secondary

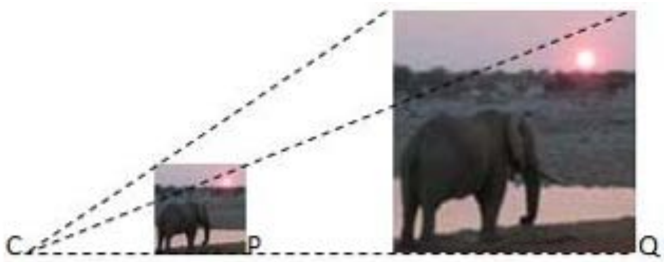
1. SIMILARITY AND ENLARGEMENT GROUP ACTIVITY

Experiment by making shadows to investigate similarity and enlargement.

You will need to work with a partner, or better still a group of four, one person to hold the torch, another to hold the object, a third person to make measurements and a fourth to draw a diagram and record all the findings. Measure distances and lengths for your shadows and check all the facts about enlargements that are summarised below. Then swop roles and repeat by making shadows of another object.

Enlargement is a transformation that maps shapes to **similar** shapes, keeping angles the same. All the lengths are changed by the same **scale factor (also called the linear scale factor)**. To describe an enlargement you have to give the centre of enlargement and the scale factor.

If points on the object are joined to the corresponding image points all these lines go through the centre of enlargement.



In the diagram the point C is the centre of enlargement.

The light source is the centre of enlargement for the projection of an image on a screen, or a shadow cast on a wall by a torch.

The scale factor of an enlargement is the ratio of the lengths in the image to the lengths in the original object. In the diagram above the scale factor is approximately 2.7which means that all the length measurements in the bigger picture are 2.7 times the lengths in the smaller picture.

To construct the image Q of a point P, join the centre of enlargement C to P and extend this line. Then multiply the distance CP by the scale factor to get the distance CQ and mark Q on the line through C and P. The linear scale factor of the enlargement is the ratio CQ : CP.

Find a map or a scale drawing such as a design or construction plan, or a building plan. All members of the group should, independently and in turn, study the map or scale drawing and make a list of 3 questions about it, and the answers to their questions. Then the group should have a discussion in which each member in turn asks his or her questions and all the group members have to come to an agreement about the correct answers.

Upper Secondary

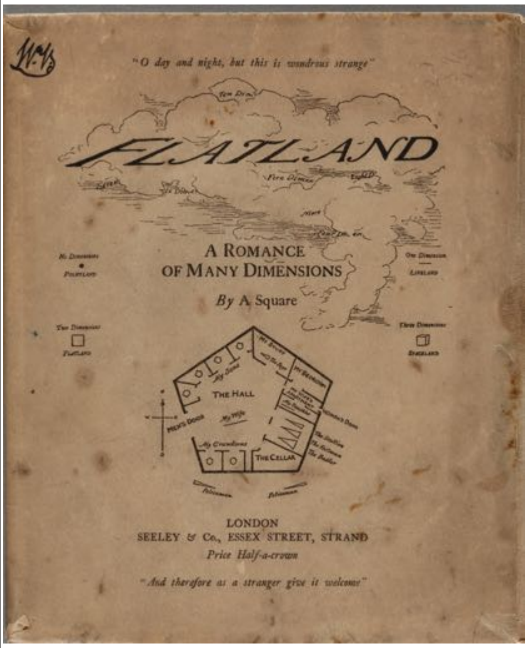
2. LIVING IN FLATLAND – A WORLD IN 2 DIMENSIONS

Imagine that you live in Flatland, a 2-dimensional world peopled by polygons, lines and points, where you can move forward and to one side and turn around, but have no knowledge or experience of up and down, those directions don't exist for you.

Flatlanders often introduce themselves by turning round full circle so that others can deduce their shapes. Houses and streets are always brightly lit with parallel beams of light so flatlanders can see the shadows of people they meet on the wall behind them.

(a) As a flatlander yourself, how would you identify other flatlanders?
(b) Could you tell a triangle from a square, and if so how?
(c) Could you identify the different sorts of triangle: equilateral, isosceles and scalene?
(d) Circles are many sided polygons with so many edges that they appear circular. How would you know a circle when you saw one?
(e) Could you tell a rectangle from a square, and if so how?
(f) Could you calculate the length of the shortest edge of a rectangle? What about the length of its diameter?

This is an investigation into the concept of dimension. It will give you a better understanding of the 4th dimension and curved space. You will get a glimpse of four dimensions and **the mathematics and physics of relativity introduced by Albert Einstein**. When he won the Nobel Prize for Physics in 1922 the world was introduced to the fourth dimension. If there is motion of our three-dimensional space relative to the fourth dimension, all the changes we experience and assign to the flow of time will be due simply to this movement, the whole of the future as well as the past always existing in the fourth dimension.



At the time of its publication in 1884, the novel, **Flatland: A Romance of Many Dimensions by Edwin A. Abbott** did not attract much attention, but it became popular in the 1920's, and has remained so since, because it enables us to imagine living in a flat world in 2D, and moving between worlds in 2D and 3D, and so to begin to understand movement into 4D beyond our own 3D world.

To understand some of these ideas and see a demonstration of the curvature of space watch the video **Flatland & the 4th Dimension – by Carl Sagan** https://youtu.be/iiWKq57uAlk

Edwin Abbot's book was also a political satire set in a society rigidly divided into classes, with a class-based hierarchy of men and the Victorian concept of women's roles in the society. Men are portrayed as polygons whose social status is determined by their regularity and the number of their edges, with a Circle considered the "perfect" shape. On the other hand, women consist only of lines and are required by law to sound a "peace-cry" as they walk, lest they pierce another flatlander or be mistaken for a point

Social ascent is, in theory, granted to everyone but strictly controlled by the top of the hierarchy. Freedom is despised and the laws are cruel. Innovators are imprisoned or suppressed. Members of lower classes who could be intellectually valuable, or potential leaders of riots, are promoted to the higher classes or killed. Every attempt for change is considered dangerous and harmful. This world is not prepared to receive "revelations from another world".

Watch **Flatland – The Film** https://youtu.be/Mfglluny8Z0 set in the year 3000 and enjoy the maths and the social satire. Do you recognise features of your world?

Follow up

Viewing Cubes https://aiminghigh.aimssec.ac.za/viewing-cubes/

Viewing Cubes Again https://aiminghigh.aimssec.ac.za/viewing-cubes-again/

How do you see it? https://aiminghigh.aimssec.ac.za/how-do-you-see-it/

Collapsible cube https://aiminghigh.aimssec.ac.za/collapsible-cube/

Scale Paper Airplanes https://aiminghigh.aimssec.ac.za/scale-paper-airplanes/
Metre and centimetre cubes
https://aiminghigh.aimssec.ac.za/metre-and-centimetre-cubes/

Enlargement https://aiminghigh.aimssec.ac.za/enlargement/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

MATHS TOYS

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Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13