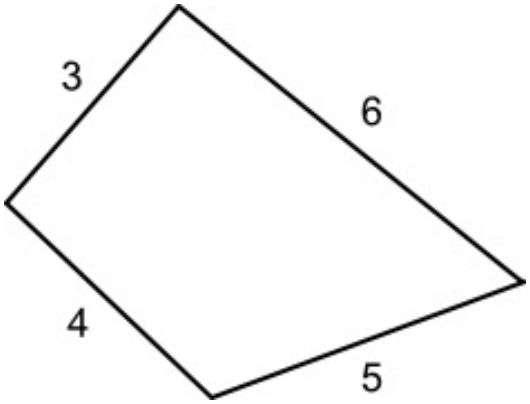
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18
on the theme of QUADRILATERALS

The original BENDY QUAD ACTIVITY was designed for Years 11 to 12
but this document has versions for all ages.
Choose what seems suitable for the age or attainment level of your learners.

BENDY QUADS

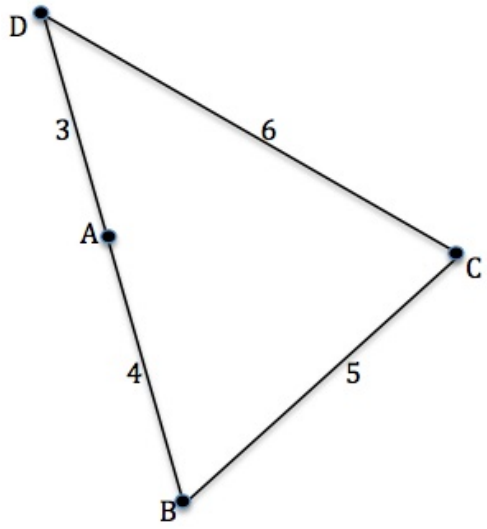
See the Bendy Quads video **https://bit.ly/BendyQuadsVideo**



Four rods are hinged at their ends to form a convex quadrilateral with sides of length 3, 4, 5 and 6 (in that order). Investigate the different shapes that the quadrilateral can take if the polygon is always convex.

How do the angles change as the bendy quad changes shape?

Can any of the angles reduce to zero degrees?

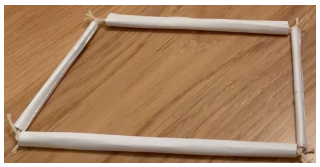


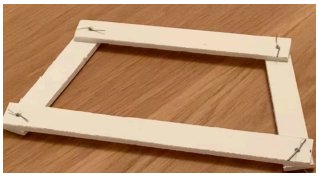
Can any of the angles increase to 180 degrees?

Calculate the size of angle C when the rods form a triangle as shown. If the polygon remains convex, can angle C get any smaller than shown in this diagram? What is the smallest size of angle C and what is the largest?

Find the smallest and largest values that the other angles can take in a similar way.

HELP





You might make a model that you can manipulate and experiment with, changing the angles. You could use 4 paper sticks of lengths 3, 4, 5 and 6 units choosing your own scale. For example, your sticks could be 6 cm, 8 cm, 10 cm and 14 cm (linear scale factor 2).

The special quads in the two pictures, with edge lengths 2, 3, 2 and 5, can both form a symmetric trapezium.

For the stiff quad model cut 4 strips of card and join them to form a quadrilateral of the given dimensions using split pins to link the strips of card.

The final calculations only require the use of cosine and sine rules.

NEXT

You could investigate non-convex quadrilaterals.

You could investigate the area of the quadrilateral and how this changes.

Can you make all the types of quadrilateral with 4 rods, for example a trapezium or a cyclic quadrilateral?

Try a quadrilateral with edges of lengths: 3, 5, 8 and 6. What is special about this quadrilateral?

INCLUSION AND HOME LEARNING GUIDE
THEME: QUADRILATERALS

Early Years

Make a large supply of paper sticks of different lengths.

See the video http://bit.ly/HowToMakePaperSticksVideo

Make triangles by tying the ends of 3 sticks together. Make a game of trying to find who can be the first to make a triangle that is different from any you have found before.

Talk about what is the same and what is different.

Make quadrilaterals by tying the ends of 4 sticks together. Make a game of trying to find who can be the first to make a quadrilateral that is different from any you have found before.

Talk about what is the same and what is different.

Lower Primary

The activity is similar to that described for early years but now introduce some of the names of the different shapes.

Make a large supply of paper sticks of different lengths.

See the video http://bit.ly/HowToMakePaperSticksVideo

Make triangles by tying the ends of 3 sticks together. Make a game of trying to find who can be the first to make a triangle that is different from any you have found before.

Talk about what is the same and what is different.

Make quadrilaterals by tying the ends of 4 sticks together. Make a game of trying to find who can be the first to make a quadrilateral that is different from any you have found before.

Talk about what is the same and what is different.

Upper Primary

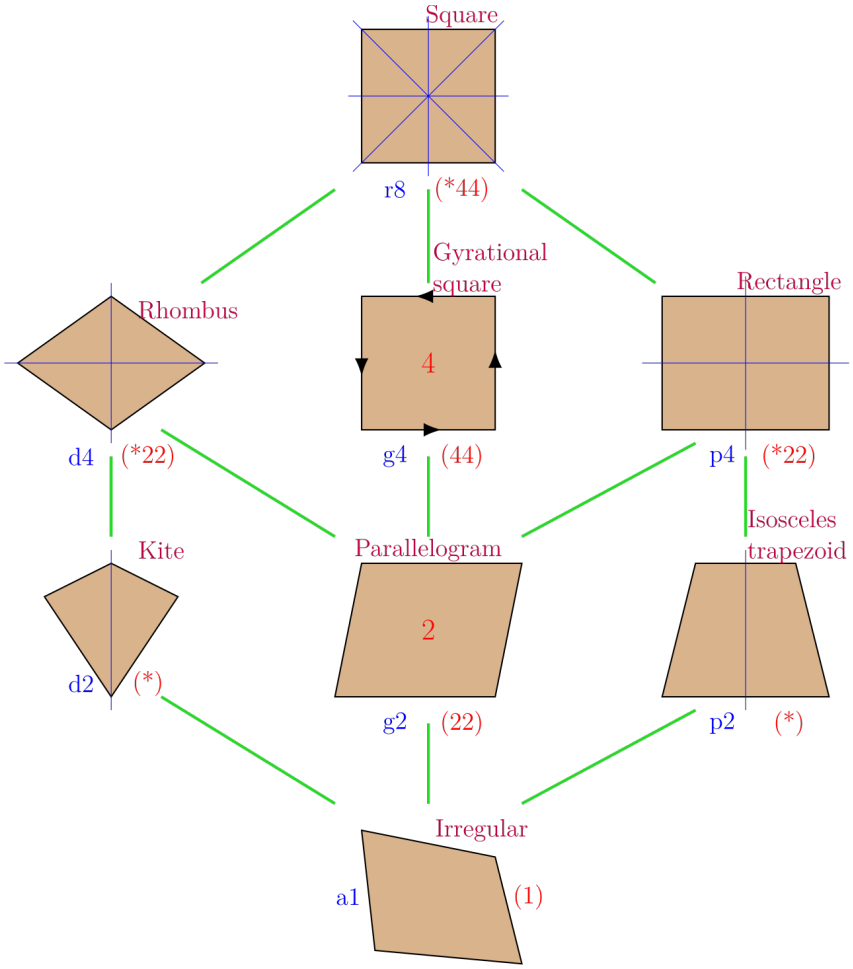
The activity is similar to that described for early years and Lower Primary but now make triangles and quadrilaterals of all the different shapes possible and introduce the names of the different shapes.

Make a large supply of paper sticks of different lengths.

See the video http://bit.ly/HowToMakePaperSticksVideo

Make triangles by tying the ends of 3 sticks together. Make a game of trying to find who can be the first to make a triangle that is different from any you have found before.

Talk about what is the same and what is different. Make a poster of triangles in which you stick the paper stick edges onto a backing sheet and write the names and properties beside the models.



Make quadrilaterals by tying the ends of 4 sticks together.

Make a game of trying to find who can be the first to make a quadrilateral that is different from any you have found before.

Talk about what is the same and what is different.

Make a quadrilaterals poster in which you stick the paper stick edges onto a backing sheet and write all the names and properties beside the models.

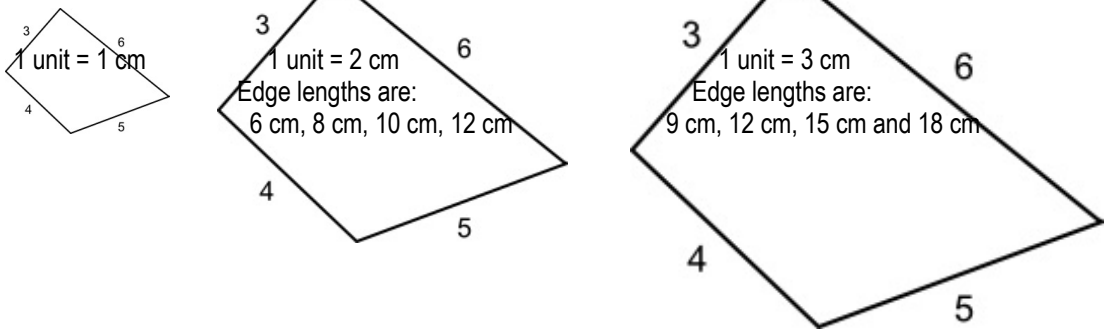
Lower Secondary

Solution by modelling, scale drawing and measurement

You will need a large supply of scrap paper, string and some scissors, or a supply of ready-made paper sticks of lengths 12 cm, 16 cm, 20 cm and 24 cm.

Organise your class so that learners work in pairs or groups of four. Start your lesson by reviewing what the leaners know about scale and enlargement. In this learning activity the lengths of the edges of the quadrilateral are 3, 4, 5 and 6 units. Sticks of lengths 3 cm, 4 cm, 5 cm and 6 cm sticks are too small to make accurately and even to handle.

The percentage error in measurement of angles will be less for similar quadrilaterals made with longer sticks. Remember that angles remain the same when objects are enlarged.



If you don't have a ready-made supply of paper sticks then give out scrap paper and string to the learners, demonstrate how to make a paper stick and the learners should all make a stick. See the video http://bit.ly/HowToMakePaperSticksVideo

Use a scale of 1 unit = 4 cm so the sticks in your models have lengths 12 cm, 16 cm, 20 cm and 24 cm. If learners work in groups of 4 each learner can make a stick of one of the four lengths so they are ready to start the investigation with their 4 sticks.

Using your model, explain to the class briefly that they must explore all the differently shaped quadrilaterals that can be made by 'bending' the quadrilateral.

Discuss what it means for a quadrilateral to be convex so it has interior angles all less than 180°. Explain that, for simplicity, they are going to work with convex quadrilaterals (and not arrow or dart shaped quadrilaterals).

Ask them to explore how the angles change. Give some time for the learners to think about the range of possibilities and to draw sketches.

You might make a demonstration model by cutting 4 strips of card and joining them to form a quadrilateral of the given dimensions using split pins to link the strips of card.

Allow time for learners to explore the different shapes that the quadrilateral can take, using a model as described or dynamic geometry software such as Geogebra. This will

help them to identify what can be varied and how much variation is possible. Ask them to answer the questions (as on page 1). They should first decideon the shapes that give the greatest and smallest angles possible, then draw accurate scale drawings and measure the angles.

You can make it more challenging by simply showing the first diagram on page 1 and leaving it to the learners to discover how some angles can reduce to zero or increase to 180^{o}, and how the triangle forms the limiting shape if the quadrilateral remains convex (so that the quadrilateral is **not** an arrowhead).

Guide the work by asking Key Questions.

You might also make it easier for the learners by suggesting that they consider the configurations where ABC and ADC become straight lines or where DAB becomes a straight line.

When learners have had time to do all this have a general discussion in which the learners share their discoveries. For each angle they have found, write the learners' answers on the board and find the average to get an approximation answer.

Key questions

- If you flex the quadrilateral can the angles be any size?
- Can any of the angles reduce to 0^{o}?
- Can any of the angles increase to 180^{o}?
- Can the rods form a triangle?

Upper Secondary

Solution by modelling and calculation using trigonometry

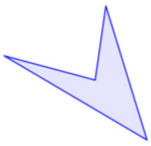
You will need a supply of scrap paper, string and some scissors, or a supply of ready-made paper sticks of lengths 12 cm, 16 cm, 20 cm and 24 cm.

Following the instructions in the video http://bit.ly/HowToMakePaperSticksVideo if you have not made them before, make paper sticks of lengths 12 cm, 16 cm, 20 cm and 24 cm. Make a scale model of the 3-4-5-6 quadrilateral using a scale of 1 unit = 4 cm, because 3cm, 4 cm, 5 cm and 6 cm sticks are too small to handle, and the percentage error in measurement of angles will be less for similar quadrilaterals made with longer sticks.

Start bending your quadrilateral to discover how the angles change and to find the smallest and largest possible angles. Explore the different shapes that the quadrilateral can take using a model or dynamic geometry software such as Geogebra. This will help you to identify what can be varied and how much variation is possible. Then measure the smallest and largest possible angles. Think about the range of possibilities and to make some notes.

If you want to be able to measure the angles more accurately then make a model by cutting 4 strips of card and joining them to form a quadrilateral of the given dimensions using split pins to link the strips of card.

A convex quadrilateral has interior angles all less than 180° (or equal to 180° on the extreme case when two edges form a straight line).



Non-convex quadrilaterals like the one shown here are called arrowheads or darts. For this investigation only work on convex quadrilaterals at first. If you want to extend your investigation to non-convex quadrilaterals, then do that as a 'follow-up'.

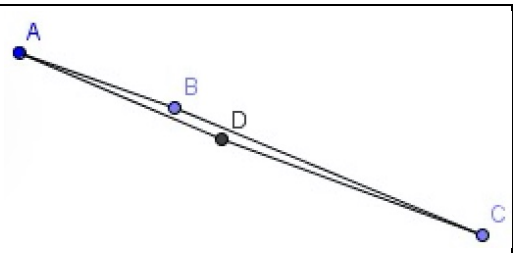
To get accurate answers you need to calculate the angles. There are different methods for doing this. You might work out your own solution, and then discuss what you have done with other students and explain your methods to each other. Write a summary of your work and explain how both methods apply.

Key questions

- If you flex the quadrilateral can the angles be any size?
- Can any of the angles reduce to 0°?
- Can any of the angles increase to 180°?
- Can the rods form a triangle?

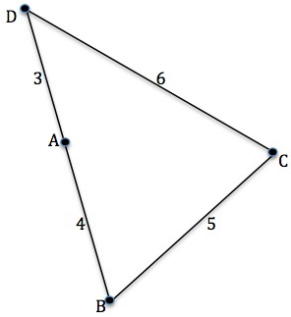
SOLUTION

As AB +BC = CD + DA = 9



we see that angles A and C can reduce to 0^o.

So angle A can change in size from 0^o to 180^o



However, if we just consider convex polygons, then angle C cannot get smaller than shown in this diagram where the rods form a triangle.

The smallest possible sizes of angles C, B and D are found from this diagram.

By the cosine rule: $7^2 = 6^2 + 5^2 - 60\ \cos C$

So $\cos C = 12/60 = 1/5$ and angle C = 78.5^o to the nearest tenth of a degree.

Angle C can change from 0^o to 78.5^o.

Using the sine rule:

$\sin B = 6/7\ (\sin C) = 5/7\ (\sin D)$ so angle B = 57.1^o and angle D = 44.4^o

Angle B changes from 57.1^o to 180^o.

Angle D changes from 44.4^o to 180^0.

Why do this activity?

This activity involves the interpretation of a very simple concrete structure, a linkage of 4 rods with the joints between the rods at the vertices totally flexible. Experiment and investigation lead to ideas about the angles that can be formed in these bendy quadrilaterals. Different cases can be considered, including convex and non-convex bendy quads in 2D and even in 3D. The conjectures need justification and proof by forming convincing arguments.

To find the constraints on the angles in the general case requires an argument using inequalities.

Solutions can be found by mathematical thinking and scale drawing. Accurate values of the angles can be calculated using the cosine and sine rules.

Learning objectives

In doing this activity students will have an opportunity to:

- investigate a range of geometrical possibilities for a quadrilateral;
- find solutions by scale drawing;
- practise applying the sine and cosine rules.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

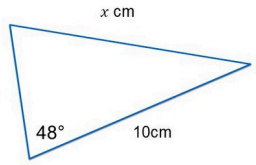
Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer so that they develop their communication skills and deepen their understanding by putting their thoughts into words.

Which of the following would correctly calculate the value of x?



A $\sqrt{10^2 + 6^2 - 2(10)(6)\cos(48)}$

B $\frac{10}{\sin(48)} \times 6$

C $\frac{10}{6} \times \sin(48)$

D $\sqrt{10^2 - 6^2 + 2(10)(6)\cos(48)}$

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
5. The concept is needed for the lesson to follow, so explain the right answer or give a remedial task.

The correct answer is A using the cosine rule.
Students giving answers B and C are incorrectly trying to use the sine rule.
Students giving answer D are misusing the cosine rule getting that signs wrong.
https://diagnosticquestions.com

Follow up

A challenging question that requires the setting up and solution of a quadratic equation:
https://aiminghigh.aimssec.ac.za/years-11-12-solve-the-triangle/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
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