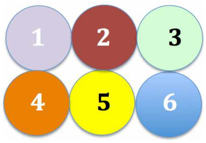
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

LUCKY NUMBERS



In the Lucky Numbers Game six balls are numbered 1 to 6.

Three balls are chosen at the same time, at random, from the six numbers, in no special order.

When you play this game you get a ticket with 3 numbers written on it.

You win a prize if your 3 numbers match the 3 numbers on the chosen balls.

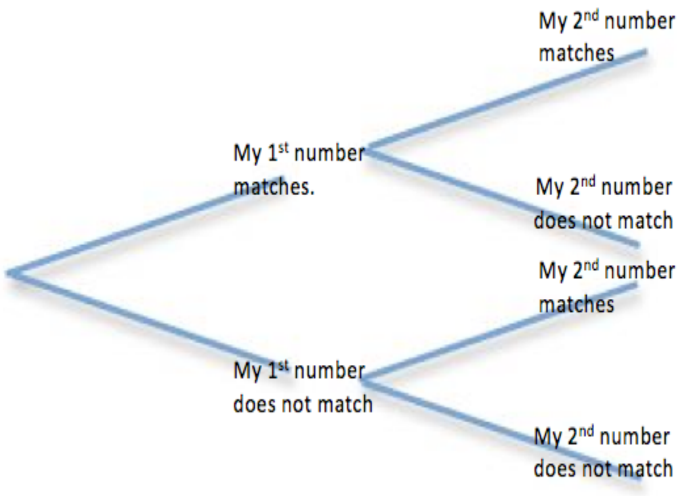
What is your chance of winning a prize?

If you find this problem difficult try the simpler case where 2 balls are chosen from 6 and you get a ticket with 2 numbers.

The probability of winning the 2 lucky number game is 1/15.

If 100 people pay R10 to play the 3-number game and the prize is R150 would the organisers of this game expect to make a profit? If so why?

HELP



Work with a partner if you can and use this tree diagram. You should be able to get the answer that the probability of winning is 1/15.

Add more branches for the 3rd number and calculate the probability of winning now.

NEXT When you have succeeded with the 2 ball and the 3 ball game, work out the chances of winning with four balls (from six).

MONTY HALL 1 IN 3 LUCKY NUMBERS

Each player has 3 cards numbered 1, 2 and 3. Either players take turns to be the dealer for a round or there can be one dealer for the game. The dealer chooses one of his cards for the lucky number placing it face down on the table keeping the choice a secret. The players also choose one card and place it face down on the table. The dealer then reveals one of the losing numbers and, with this extra information, the players can change their choices. Which is the better strategy, to change every time or never to change? Players score 1 point if they choose the lucky number. The player with the most points after 10 rounds is the winner.

NOTES FOR TEACHERS

SOLUTION

METHOD 1

Note that the order in which the numbers occur is not relevant so (1, 2) and (2, 1).

The possible choices of 2 numbers are:

(1 , 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 3) (2, 4) (2, 5) (2, 6)

(3, 4) (3, 5) (3, 6)

(4, 5) (4, 6)

(5, 6)

One of these pairs of numbers must be the winning combination so there is a 1 in 15 chance of winning.

All possible outcomes for choices of 3 numbers from 6 are given here as lists:

123, 124, 125, 126

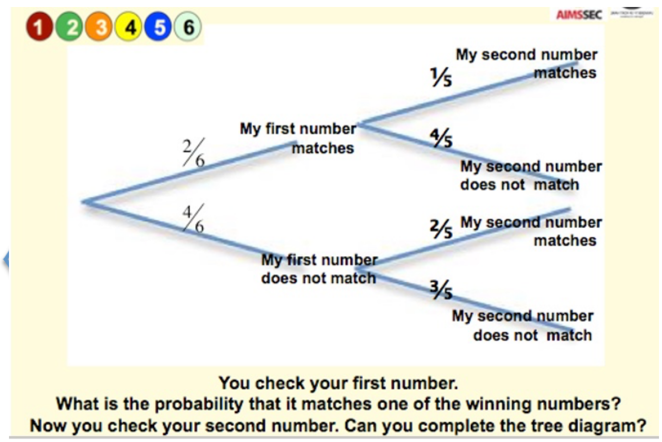
134, 135, 136, 234, 235, 236

145, 146, 245, 246, 345, 346

156, 256, 356, 456

One of these pairs of numbers must be the winning combination so there is a 1 in 20 chance of winning.

METHOD 2



In the 2 numbers from 6 game, using the top branches of the tree diagram, the probability is 2/6 multiplied by 1/5 giving 1/15.

For the **3 Numbers from 6 Game** we don't need to draw the complete tree diagram but just use the branches at the top that represent matches for all three numbers picked to the 3 winning numbers. The probability of winning **The 3 Numbers from 6 Game** is:

3/6 x 2/5 x 1/4 = 1/20

If 100 people pay R10 to play the 3 balls game then the expectation is that there would be 1/20 x 100 = 5 winners so the pay-out would be R750. The organisers would collect R1000 and so make a profit of R250.

Monty Hall 1 in 3 Lucky Numbers

In counting all possible outcomes there are 3 totally symmetric results according to which of the 3 numbers is the lucky winning number and each set of results has a probability one third. The table shows the results when the lucky number is 1 and there are equivalent tables for it being 2or 3. The shaded region shows the total probability when a player changes his choice of winning number is 2/9 and when he keeps the same number it is 1/9.

Taking all events as described above the best strategy for players is to change the choice of card as the final probability of winning when the card is changed is:

$$\frac{2}{9} \times 3 = \frac{2}{3}$$

and the final probability of winning with no change is:

$$\frac{2}{18} \times 3 = \frac{1}{3}$$

	Card number choice by player	Lucky Number	Losing card number shown	Choice 2	Probability (Card ↓ Lucky Number ↓ Choice 2 ↓)	Win/Lose
Choice not changed	1	1	2	1	(1/3) × (1/3) × (1/2) = 1/18	W
	1	1	3	1	(1/3) × (1/3) × (1/2) = 1/18	W
	1	2	3	1	(1/3) × (1/3) = 1/9	L
	1	3	2	1	(1/3) × (1/3) = 1/9	L
Choice changed	1	1	2	3	(1/3) × (1/3) × (1/2) = 1/18	L
	1	1	3	2	(1/3) × (1/3) × (1/2) = 1/18	L
	1	2	3	2	(1/3) × (1/3) = 1/9	W
	1	3	2	3	(1/3) × (1/3) = 1/9	W

Learning objectives

In doing this activity students will have an opportunity to:

- engage with the problem of finding the probability of winning a game;
- experience making a decision to work on a simpler case first;
    - play the 2 from 6 game in order to estimate the experimental probability;
    - list all possible outcomes in a systematic way to ensure finding them all;
    - use a list of all possible outcomes to calculate the theoretical probability;
    - collaborate in a large group to pool results and observe that the experimental probability gets closer to the theoretical probability the more games are played;
- repeat the above for the 3 from 6 game.

Generic competences

In doing this activity students will have an opportunity to:

- develop the skill of planning and working systematically;
- develop the skill of working with a team all of whom contribute data and share results.

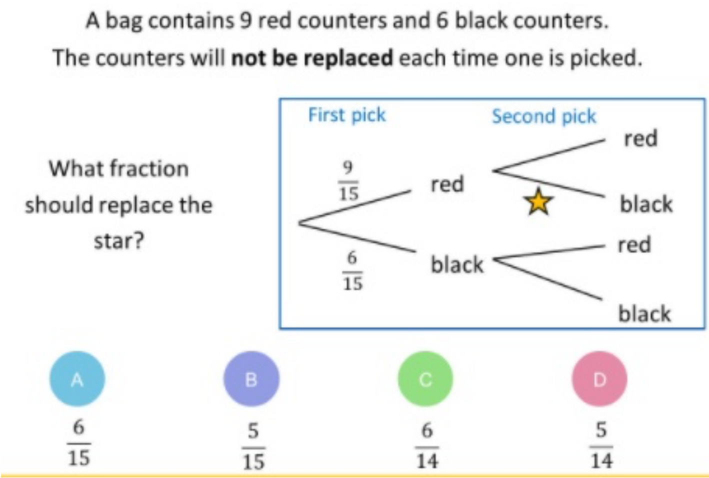
DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Use at the end of the lesson sequence for formative assessment.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.



4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: **D. There will be 14 counters in the bag of which 5 are white.**

Common Misconceptions

A. Learners have not understood that, after a counter has been removed, there are fewer in the bag.

B. Learners have realised there will be 5 black counters remaining but failed to use the fact that there will be 14 in all (not 15).

C. here reduced the total correctly but not the number of blacks.

https://diagnosticquestions.com

Why do this activity?

This activity can be used for two distinct learning objectives and so it can be used for different age groups. For older learners the teacher can combine both learning objectives. The class can play the game and then collect data from the whole class to find an experimental estimate of the probability of winning. Then they can work on calculating the true probability, either by systematically listing all the possibilities (learning objective 1), or by using a tree diagram (learning objective 2).

Teachers may require the probabilities to be given in the three forms : fractions, decimals and percentages, in order to give learners practice in converting from one to another and to emphasise that they are simply different ways of writing down the same number.

This problem offers a simple case of a lottery type game to help learners to develop an understanding that they can then build on to work out the probability of winning in the National Lottery. However it is a useful exercise in itself in experimental and theoretical probability. Only introduce learning objective 2 when learners already have some experience with tree diagrams for compound events in cases where the second event

does not depend on the outcome of the first event. The tree diagram for this problem starts with the branches: my first number matches or my first number does not match, then moves to the second branching and multiplying fractions based on conditional probabilities.

Suggestions for teaching

You could simulate the lottery in class by having 6 numbered cards in a bag. Discuss the problem. Ask "how could we make the game simpler?" As a simpler case you could start with the '2 numbers from 6' game. Each member of the class can choose 2 numbers for themselves and write them down making his or her own 'ticket'. Ask learners to predict the chance of winning. Then play the game. You draw 2 numbers from the bag. How many learners in the class have won? Work out an estimate of the chance of winning by dividing the total number of winners by the total number of tickets. Did we win as often as expected?

You could play the game as a class a few times and work out the estimated chance of winning when you put all the results together.

You might decide to move on to calculation of the probability or alternatively you could organize for learners to work in groups, give each group an envelope with 6 numbered cards in it and ask them to play the game, say 15 times, and record the number of winners. Then results should be collected for the whole class and discussed so that learners can see that results from a few experiments do not give a good estimate of the probability.

The class should try to calculate the probability without the teacher suggesting how to do it. Give learners some time to work on this.

For learning objective 1 and younger learners, teachers may ask the learners to list all the possible choices of 2 numbers. Bring the class together to share methods. Praise anyone who has listed systematically and discuss the importance of making sure every combination is considered. Several different systems can be used. How many **different systems** have the class found? It is important to help learners to develop the skill of working systematically. Take time to discuss the symmetry that emerges from starting from 1 or starting from 6, and ask students to consider why this happens.

For learning objective 2 or older learners, some students may list combinations (systematically or otherwise), whereas others may use tree diagrams. Ideally, they will work on this using both listing and tree diagram approaches. If necessary, move students towards a tree diagram approach.

The Monty Hall 1 in 3 game is very entertaining and older students can be asked to fill in the contingency table and work out the probabilities. High flyers might play the Monty Hall 2 in 5 game and work out the probabilities. See the Inclusion Guide for the solution in this case.

Key questions

- Did we win as often as expected?
- How could we calculate the probability of winning?
- Why is the probability of winning the two from six lottery the same as the probability of winning the four from six lottery?
- If this game had a lot more balls (say 49 balls like the National Lottery) would you want to list all the possibilities or would it be better to use a tree diagram?

Follow up

Mathopia Lottery https://aiminghigh.aimssec.ac.za/mathopia-lottery

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13