AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18
on the theme of THE CHANCE OF WINNING

Just choose whatever section seems suitable for your group of learners

The original LUCKY NUMBERS learning activity was designed for Years 9 to 12

LUCKY NUMBERS

In the Lucky Numbers Game six balls are numbered 1 to 6. Three balls are chosen at the same time, at random, from the six numbers, in no special order.

When you play this game you get a ticket with 3 numbers written on it.

You win a prize if your 3 numbers match the 3 numbers on the chosen balls.

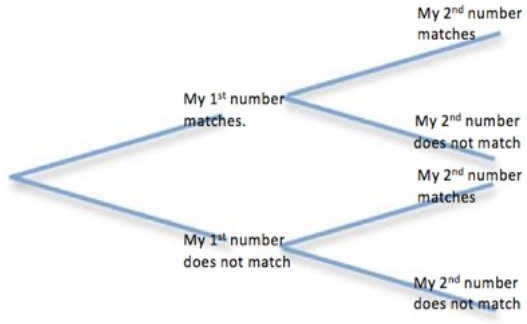
What is your chance of winning a prize?

If you find this problem difficult try the simpler case where 2 balls are chosen from 6 and you get a ticket with 2 numbers.

Show that the probability of winning the **2 in 6 Lucky Numbers Game** is 1/15.

If 100 people pay R10 to play the 3-number game and the prize is R150 would the organisers of this game expect to make a profit? If so why?

HELP



Work with a partner if you can and use this tree diagram. You should be able to get the answer that the probability of winning the 2 numbers in 6 game is 1/15.

Add more branches for the 3rd number and calculate the probability of winning the 3 numbers in 6 game.

NEXT MONTY HALL 1 IN 3 LUCKY NUMBERS

Each player has 3 cards numbered 1, 2 and 3. Either players take turns to be the dealer for a round or there can be one dealer for the game. The dealer chooses one of his cards for the lucky number placing it face down on the table keeping the choice a secret. The players also choose one card and place it face down on the table. The dealer then reveals one of the losing numbers and, with this extra information, the players can change their choices. Which is the better strategy, to change every time or never to change? Players score 1 point if they choose the lucky number. The player with the most points after 10 rounds is the winner.

INCLUSION AND HOME LEARNING GUIDE

THEME: THE CHANCE OF WINNING

Early Years ONE IN FOUR GAME FOR 2, 3 OR 4 PLAYERS.

Make 4 cards with the numbers 1, 2, 3 and 4 on them and put them in a bag.

Make a spinner from a square split into 4 with the numbers 1, 2, 3 and 4 as shown, a pin and a paperclip. Straighten one end of the paper clip and use the pin to hold it in place at the centre of the square.

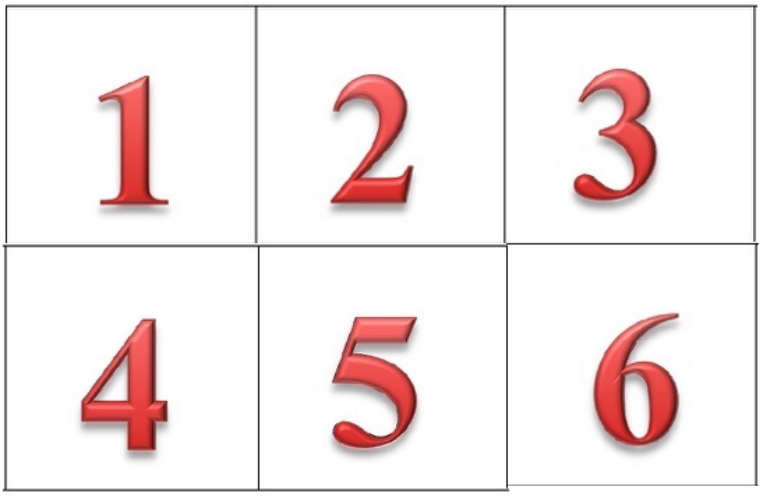
Now you are ready to play.

Each player takes a number card from the bag. Spin the spinner to choose the winning number. If it lands exactly on a line, spin again. Record 1 point for the winner. The match ends when someone reaches a total score of 10 points.

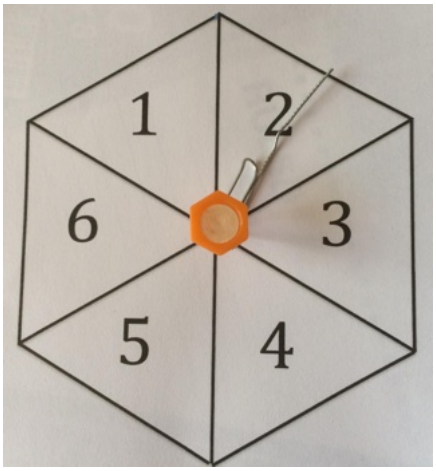
Ask the children what chance they have of winning when they pick a number. The chance of winning is one in four spins.

With 2 or 3 players nobody might get the winning number. Can they explain why that is?

Lower Primary **THE 2 IN 6 GAME** for 2 or 3 players.



Make 6 cards with the numbers 1, 2, 3, 4, 5 and 6 on them and put them in a bag or envelope.



Make a spinner using a pin, a paperclip and a hexagon split into 6 for the numbers 1, 2, 3, 4, 5 and 6 as shown. Straighten one end of the paper clip and use the pin to hold it in place at the centre of the hexagon.

Now you are ready to play.

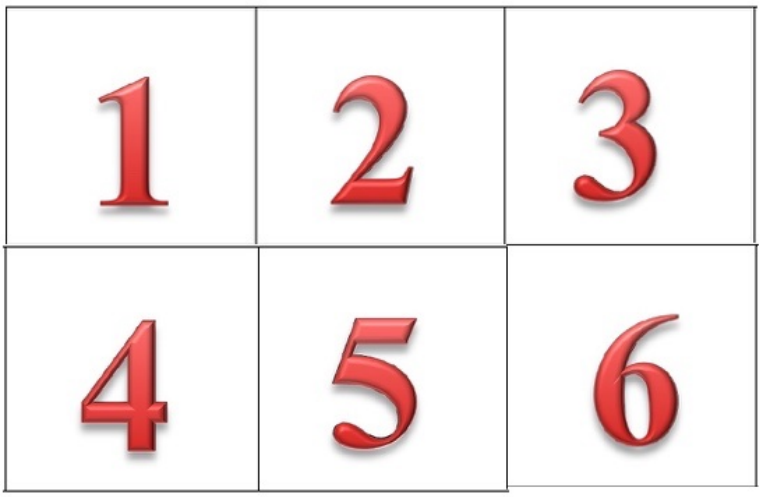
Each player takes 2 number cards from the bag. Spin the spinner twice to choose the 2 winning numbers. If it lands exactly on a line or on the first winning number a second time, then spin again.

Record 1 point for a player who has chosen both winning numbers. The match ends when someone reaches a total score of 10 points.

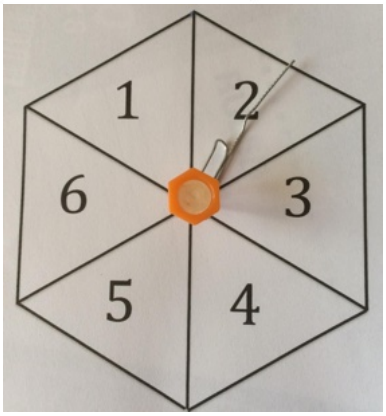
Why are there rounds when nobody is a winner?

Just play for fun. This will lay a foundation for future work.

Upper Primary '2 IN 6' and '3 IN 6' GAMES for 2 or 3 players.



Make 6 cards with the numbers 1, 2, 3, 4, 5 and 6 on them and put them in a bag.



Make a spinner using a pin, a paperclip and a hexagon split into 6 for the numbers 1, 2, 3, 4, 5 and 6 as shown. Straighten one end of the paper clip and use the pin to hold it in place at the centre of the hexagon.

Now you are ready to play **The 2 in 6 Game**.
Each player takes 2 number cards from the bag.
Spin the spinner twice to choose the 2 winning numbers. If it lands exactly on a line or on the first winning number a second time, then spin again.

Record 1 point for a player who has chosen both winning numbers. The match ends when someone reaches a total score of 10 points. Why are there rounds when nobody is a winner?

Count the possible pairs of numbers

Challenge the group to find all possible pairs. Make a game of 'who can find another new pair' until the group, working together, has found all 15 pairs. Remind the learners that numbers are not repeated and the order does not make any difference: 2 & 1 is the same as 1 & 2.

Then show the learners this pattern and ask them what they notice about it. The aim should be to introduce the idea of making a plan to record results systematically.

(1 , 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 3) (2, 4) (2, 5) (2, 6)
(3, 4) (3, 5) (3, 6)
(4, 5) (4, 6)
(5, 6)

The 3 in 6 Game for 2 players

The game is the same as the 2 in 6 Game except that players take it in turns to pick 3 numbers this time, and the other player gets the remaining 3 numbers. Then you spin the spinner 3 times and see if there is a winner. The contest ends when a player gets 3 points.

Count the possible triples of numbers

Challenge the group to find all possible triples. Make a game of 'who can find another new triple' until the group, working together, has found all 20 triples. Remind the learners that numbers are not repeated and the order does not make any difference. All six combinations 123 ; 132 ; 213 ; 231 ; 312 and 321 are the same.

Then show the learners this pattern and ask them what they notice about it. The aim should be to introduce the idea of making a plan to record results systematically so that you can check that you have found all possible results.

123, 124, 125, 126

134, 135, 136, 234, 235, 236

145, 146, 245, 246, 345, 346

156, 256, 356, 456

Key Questions

- Did we win as often as expected?
- How could we calculate the probability of winning?
- Why is the probability of winning the two from six lottery the same as the probability of winning the four from six lottery?

Lower Secondary

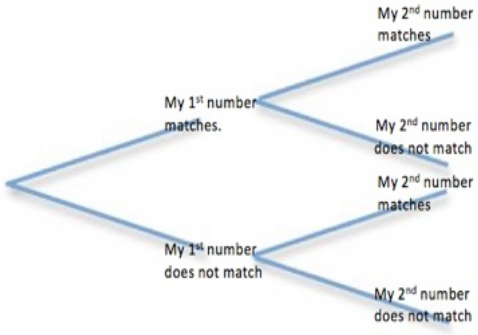
Do the same activities as for Upper Primary.

(1) Suggest that the learners write down all possible pairs of numbers chosen from 1, 2, 3, 4, 5 and 6. Bring the class together to share methods. Discuss the importance of making sure every possible pair of numbers is considered. It is important to help learners to develop the skill of working systematically so praise anyone who has worked systematically. How many **different systems** have the class found? Take time to discuss the symmetry that emerges from starting from 1 or starting from 6.

(2) Simulate the lottery by playing the **2 Numbers from 6 Game** as a group or class. Each member of the class can make his or her own ticket choosing 2 numbers for themselves and writing them down. Ask learners to predict the chance of winning. Use 6 numbered cards and draw 2 cards from a bag or envelope. How many learners in the class or group have won? Ask the learners to use the list of possible pairs to work out the probability of winning when you buy a ticket. Divide the total number of winners in the class by 15 (the total number of different tickets). Did we win as often as expected? You could play the game as a class a few times, put all the results together, and work out the estimated statistical probability of winning.

(3) Then gently introduce the idea of using a **tree diagram for the 2 in 6 Game**:

- How many numbers are there in the bag at the start?
- How many will be winning numbers.
- What is my chance of picking a winning number (answer 2 in 6 or 1/3)
- How many numbers are there in the bag when I pick my 2nd number?
- If I picked one winning number already, how many numbers in the bag will be winning numbers now?
- What is my chance of picking a second winning number (answer 1 in 5 or 1/5)
- What is my chance of winning by picking both winning numbers?



Key Questions

- Did we win as often as expected?
- How could we calculate the probability of winning?
- Why is the probability of winning the two from six lottery the same as the probability of winning the four from six lottery?
- If this game had a lot more numbers and the top prize is for 6 winning numbers (say 6 numbers from 49 balls like the National Lottery) would you want to list all the possibilities or would it be better to use a tree diagram?

For Year 9 - A tree diagram for The 3 in 6 Game.

Do all the activities from Page 1.

If your learners have not met the **Lucky Numbers Game** before then first play **the 2 in 6 Game** as described for Upper Primary, make a list of all possible pairs of numbers and introduce the tree diagram for this game.

For **the 3 in 6 Game** we don't need to draw the complete tree diagram but just use the branches at the top that represent matches to winning numbers for **all three numbers picked**.

Ask the learners questions like those above for the 2 in 6 game to find the probability of winning in the 3 in 6 Game.

- Investigate what happens if you collect more data from games and use the data to estimate the probabilities. How close will the estimates be? Learners could play the game in pairs or groups of three, say 20 times, and record the number of times that the there is a winner who has chosen all 3 winning numbers. Each group will need an envelope with 6 numbered cards in it. Collect and merge the data from all the participants. By collecting and discussing the data for the whole class learners can see results from a small number of experiments do not give a good estimate of the probability. When more rounds are included, the statistical estimate of the probability get closer to the theoretical probability?

(See page 9 for solutions)

Upper Secondary

Work through the learning activities on page 1. Either the teacher will follow the lesson described for Lower Secondary (or a modified version of it) or you can work, by yourself, or with a partner, on the following challenges.

The challenge is for you to

(1) write down all possible pairs of numbers chosen from 6 numbers in a systematic way and use this to work out the probability of winning in **The 2 in 6 Game;**

(2) use the tree diagram method to find the probability of winning;

(3) write down all possible triples of numbers chosen from 6 numbers in a systematic way and use this to work out the probability of winning in **The 3 in 6 Game;**

(4) Use the tree diagram method to find the probability as of winning.

(5) Try the Mathopia Lottery and investigate how the National Lottery works in your country and the method of calculating the chance of winning with one ticket. https://aiminghigh.aimssec.ac.za/mathopia-lottery/ Reflect on and discuss the reasons why people buy tickets and the dangers of gambling.

(6) Try the Monty Hall version of the Lucky Numbers Game in the NEXT section on page 1. Is it a good strategy to change your numbers when you have the opportunity to do so?

(7) **MONTY HALL 2 IN 5 LUCKY NUMBERS**
(easy to play but finding the probabilities is more challenging)
Each player has 5 cards numbered 1, 2 , 3, 4 and 5. Players either take turns to be the dealer for a round or there can be one dealer for the game. The dealer chooses two of his or her cards for the lucky numbers placing them face down on the table keeping the choice a secret. The players also choose two cards and place them face down on the table. The dealer then reveals one of the losing numbers and, with this extra information, the players can change their choices. Which is the better strategy, to change every time or never to change? Players score 5 points for having chosen 2 lucky numbers and 1 point for 1 lucky number. The player with the most points after 10 rounds is the winner.

SOLUTION

METHOD 1

Note that the order in which the numbers occur is not relevant so (1, 2) and (2, 1) are the same.

The possible choices of 2 numbers are:

(1 , 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 3) (2, 4) (2, 5) (2, 6)
(3, 4) (3, 5) (3, 6)
(4, 5) (4, 6)
(5, 6)

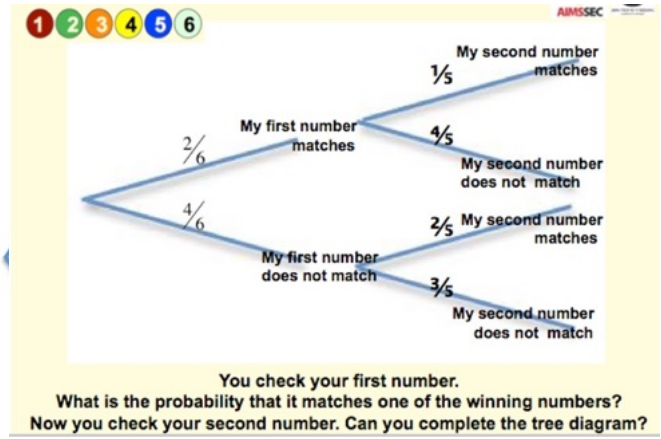
One of these pairs of numbers must be the winning combination so there is a 1 in 15 chance of winning.

All possible outcomes for choices of 3 numbers from 6 are given here as lists:

123, 124, 125, 126
134, 135, 136, 234, 235, 236
145, 146, 245, 246, 345, 346
156, 256, 356, 456

One of these pairs of numbers must be the winning combination so there is a 1 in 20 chance of winning.

METHOD 2



In the 2 numbers in 6 game, using the top branches of the tree diagram, the probability is 2/6 multiplied by 1/5 giving 1/15.

For the **3 Numbers in 6 Game** we don't need to draw the complete tree diagram but just use the branches at the top that represent matches for all three numbers picked to the 3 winning numbers.

The probability of winning **The 3 Numbers in 6 Game** is:

3/6 x 2/5 x 1/4 = 1/20

If 100 people pay R10 to play the 3 balls game then the expectation is that there would be 1/20 x 100 = 5 winners so the pay-out would be R750. The organisers would collect R1000 and so make a profit of R250.

Monty Hall 1 in 3 Lucky Numbers

In counting all possible outcomes there are 3 totally symmetric results according to which of the 3 numbers is the lucky winning number and each set of results has a probability one third. The table shows the results when the lucky number is 1 and there are equivalent tables for it being 2or 3. The shaded region shows the total probability when a player changes his choice of winning number is 2/9 and when he keeps the same number it is 1/9.

Taking all events as described above the best strategy for players is to change the choice of card as the final probability of winning when a card is changed is:

$$\frac{2}{9} \times 3 = \frac{2}{3}$$

and the final probability of winning when no change is:

$$\frac{2}{18} \times 3 = \frac{1}{3}$$

	Card number choice by player	Lucky Number	Losing card number shown	Choice 2	Probability (Card ↓ Lucky Number ↓ Choice 2 ↓)	Win/Lose
Choice not changed	1	1	2	1	(1/3) × (1/3) × (1/2) = 1/18	W
	1	1	3	1	(1/3) × (1/3) × (1/2) = 1/18	W
	1	2	3	1	(1/3) × (1/3) = 1/9	L
	1	3	2	1	(1/3) × (1/3) = 1/9	L
Choice changed	1	1	2	3	(1/3) × (1/3) × (1/2) = 1/18	L
	1	1	3	2	(1/3) × (1/3) × (1/2) = 1/18	L
	1	2	3	2	(1/3) × (1/3) = 1/9	W
	1	3	2	3	(1/3) × (1/3) = 1/9	W

Monty Hall 2 in 5 Lucky Numbers

Each player has 5 cards numbered 1, 2 , 3, 4 and 5. Players either take turns to be the dealer for a round or there can be one dealer for the game. The dealer chooses two of his or her cards for the lucky numbers placing them face down on the table keeping the choice a secret. The players also choose two cards and place them face down on the table. The dealer then reveals one of the losing numbers and, with this extra information, the players can change their choices. Players score 5 points if they have chosen 2 lucky numbers and 1 point for 1 lucky number. The player with the most points after 10 rounds is the winner.

Which is the better strategy, to change every time or never to change?

The probabilities can be read from the contingency table:

P(winning)	= 1/100 × 10 = 1/10
P(changing and winning)	= 9/1000 × 10 = 9/100
P(keeping first choice and winning)	=1/1000 × 10 = 1/100
P(losing)	= 9/100 × 10 = 9/10
P(changing and losing)	= 81/1000 × 10 = 81/100
P(keeping first choice and losing)	= 9/1000 × 10 = 9/100

The best strategy is to change every time (as shown by the shading). By changing cards every time players are 9 times more likely to win than if they keep the same cards.

Player 1st choice of 2 cards	Lucky numbers	1 from 3 losing cards as shown	All possible players' 2nd choices that don't include the losing card shown	Probability Lucky Cards ↓ Numbers ↓ Choice 2 ↓	Win /Lose	1000ths ×1/1000
1,2	1,2	3,4,5	1,2	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	1,2	3,4,5	9 choices	1/10 × 1/10 × 9/10 = 9/1000	Lose	9
1,2	1,3	2,4,5	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	1,3	2,4,5	1,3	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	1,3	2,4,5	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	1,4	2,3,5	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	1,4	2,3,5	1,4	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	1,4	2,3,5	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	1,5	2,3,4	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	1,5	2,3,4	1,5	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	1,5	2,3,4	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	2,3	1,4,5	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	2,3	1,4,5	2,3	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	2,3	1,4,5	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	2,4	1,3,5	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	2,4	1,3,5	2,4	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	2,4	1,3,5	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	2,5	1,3,4	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	2,5	1,3,4	2,5	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	2,5	1,3,4	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	3,4	1,2,5	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	3,4	1,2,5	3,4	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	3,4	1,2,5	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	3,5	1,2,4	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	3,5	1,2,4	3,5	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	3,5	1,2,4	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8
1,2	4,5	1,2,3	1,2	1/10 × 1/10 × 1/10 = 1/1000	Lose	1
1,2	4,5	1,2,3	4,5	1/10 × 1/10 × 1/10 = 1/1000	Win	1
1,2	4,5	1,2,3	8 choices	1/10 × 1/10 × 8/10 = 8/1000	Lose	8

Why do this activity?

This activity can be used for distinct learning objectives where learners make a systematically ordered list of all possible sets of numbers that can be chosen, or use tree diagrams in working out the probability of winning or use the symmetry in the set of results and a contingency table in working out probabilities.

The activity can be used for different age groups. Older learners (Years 11, 12, and 13 can independently work through their section learning to work independently or in a pair or small group. In guiding them, the teacher can combine all the learning objectives. For younger learners, the class can play the games and then collect data from the whole class to find an experimental estimate of the probability of winning.

This problem offers a simple case of a lottery type game to help learners to develop an understanding that they can then build on to work out the probability of winning in the National Lottery.

The tree diagram for this problem starts with the two branches: my first number is a winning number or my first number is not a winning number. It then moves to the second branching and multiplying fractions based on conditional probabilities. It is a useful exercise in itself in experimental and theoretical probability, but it's best to introduce learning objective 2 when learners have already had some experience with tree diagrams for compound events when the events are independent of each other.

Learning objectives

In doing this activity students will have an opportunity to engage with the problem of finding the probability of winning a game, and to make a decision to start with a simpler case.

Objective 1:

- play **The 2 in 6 Game** in order to estimate the experimental probability;
- collaborate in a large group to pool results and observe that the experimental probability gets closer to the theoretical probability the more games are played;
- list all possible outcomes in a systematic way to ensure finding them all;
- use a list of all possible outcomes to calculate the theoretical probability;
- repeat the above for **The 3 in 6 Game**.

Objective 2: Use tree diagrams to find the probability of winning.

Objective 3: Use the symmetry in the set of results and a contingency table in working out probabilities, for example in the Monty Hall versions of the games.

Generic competences

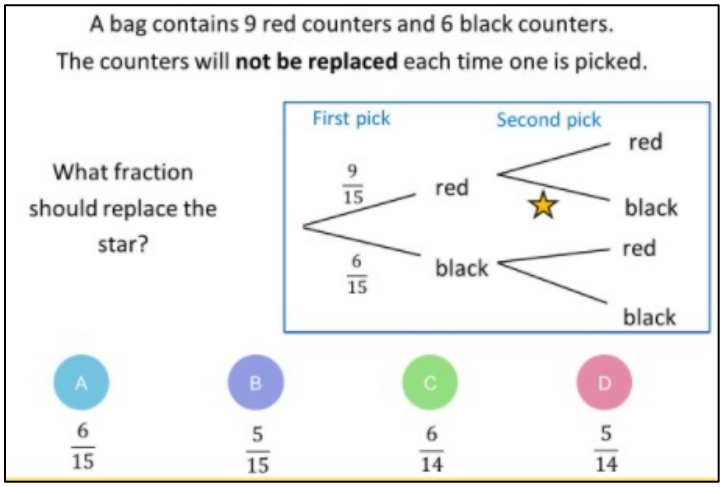
In doing this activity students will have an opportunity to:

- develop the skill of planning and working systematically;
- develop the skill of working with a team all of whom contribute data and share results.

DIAGNOSTIC ASSESSMENT This can be done before or after the lesson and as a group as described below, or the question can be answered individually.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Tell the learners to talk about the tree diagram, to ask questions about and to try to answer those questions.
5. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: D. There will be 14 counters in the bag of which 5 are white.

Common Misconceptions

A. Learners have not understood that, after a counter has been removed, there are fewer in the bag.

B. Learners have realised there will be 5 black counters remaining but failed to use the fact that there will be 14 in all (not 15).

C. here reduced the total correctly but not the number of blacks.

https://diagnosticquestions.com

Follow up

Mathopia Lottery https://aiminghigh.aimssec.ac.za/mathopia-lottery/

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