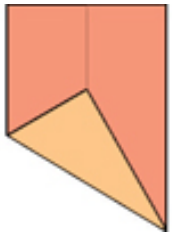
MAKING SIXTY



Why does this fold create an angle of sixty degrees?

Make a centre crease down the length of the paper then open it up.

Next fold one corner over and onto the centre crease so that the fold line passes through the corner next to it (on the short side of the paper).

You have created some angles of 60 degrees and of 30 degrees.

Can you prove this?

Now fold a square in half in different ways. What do you notice?

HELP

Draw and cut out an equilateral triangle.

Fold IT along a line of symmetry.

Work with a partner. Talk about how the line of symmetry splits the equilateral triangle into two congruent 30-60-90 degree triangles.

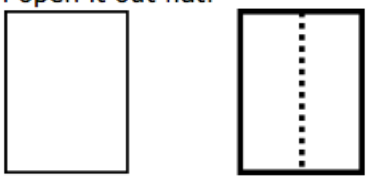
Look at your triangle in different orientations as you turn it around.

Does this help you to explain why the angle is 60^{o} in the Making Sixty paper folding?

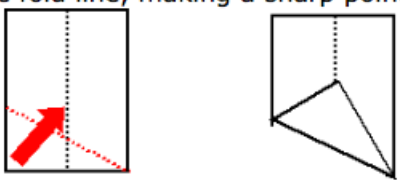
NEXT

Follow on by utilising this fold to make equilateral triangles.

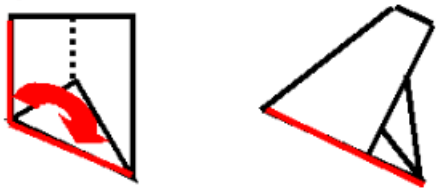
1. Fold the paper in half long-ways, then open it out flat.



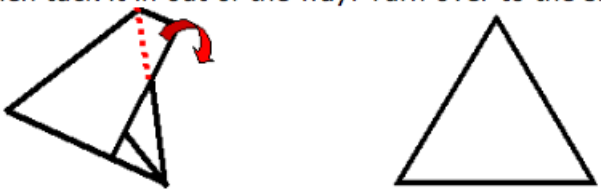
2. Fold a bottom corner up to touch the fold line, making a sharp point on the other corner.



3. Now fold the two RED edges together.

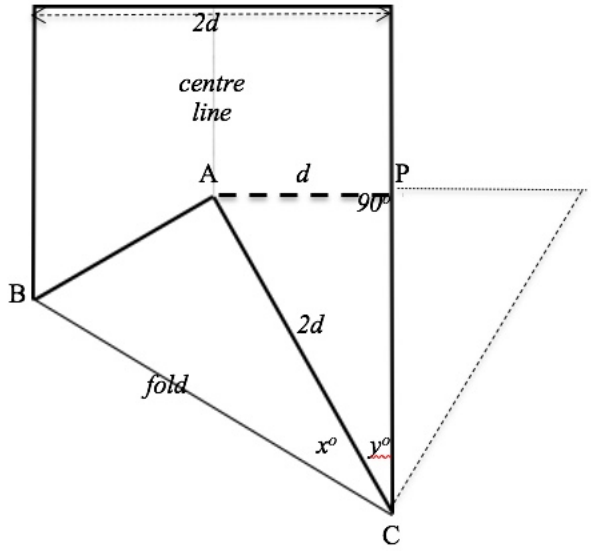


4. Fold under the corner and then tuck it in out of the way. Turn over to the smooth side.



NOTES FOR TEACHERS

SOLUTION



Draw AP so that angle APC is a right angle.

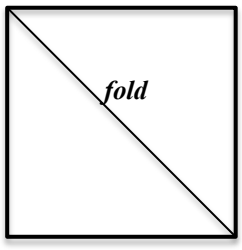
If the width of the paper is $2d$ (for any value of d) then AC = $2d$ and AP = d so triangle APC is half of an equilateral triangle and angle ACP = $y^o = 30^o$.

Because C is at a corner of the paper the fold CB makes the angle at C into 2 angles of x^o and an angle of y^o .

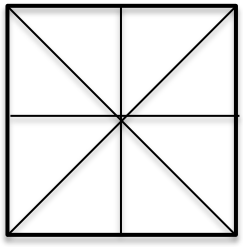
So $2x + y = 90$

and $x^o = 30^o$.

This fold gives angle BCP = 60^o and angle ACP = 30^o.



The diagonal fold line is an axis of reflection symmetry.
The two triangles are congruent right-angled triangles with angles $45^o – 45^o - 90^o$ and edge lengths in the ratio $1 : 1 : \sqrt{2}$



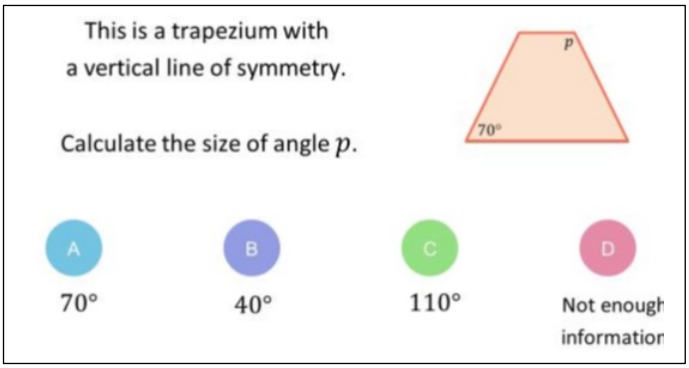
The square has 4 axes of reflective symmetry (mirror lines) as shown by these fold lines.

The square has four-fold rotational symmetry about its centre.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: C

Possible misconceptions:

A. The student appears to have taken a horizontal line of symmetry.
B. The student may not know that angles in a quadrilateral add to 360 degrees.
C. The vertical line of symmetry means that the bottom-right angle is also 70 degrees. Also, p has the same value as the top-left angle. A trapezium is a quadrilateral, and so all angles will add to 360 degrees. Hence, we can write: 70 + 70 + p + p = 360, and so 2p + 140 = 360. This means 2p = 220, giving p = 110 degrees.
D. The student appears not to realise the information given by the line of symmetry.

https://diagnosticquestions.com/

Why do this activity?

This activity involves a simple fold which gives a surprising and useful result. Having confirmed the sizes of angles this knowledge can be applied to making models of polyhedra. Different routes to the solution of the Making Sixty activity can also lead to useful discussions and give learners practice in explaining their reasoning.

The activity could lead to impressing on learners that they must learn to visualise the standard diagrams for the $45^o - 45^o - 90^o$ triangle (half a square so its edge lengths are in the ratio $1 : 1 : \sqrt{2}$) and the $30^o - 60^o - 90^o$ triangle (half an equilateral triangle so its edge lengths are in the ratio $1 : 2 : \sqrt{3}$). Visualising these two triangles enables a person to remember the exact values (in terms of surds) of the sine, cosine and tangent of 30^o, 45^o and 60^o.

Learning objectives

In doing this activity students will have an opportunity to:

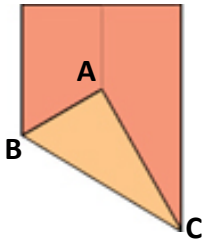
- deepen understanding of the properties of equilateral triangles and the special 30-60-60 degree triangle including the ratios of the lengths of its edges;
- deepen understanding of the properties of right-angled isosceles triangles, and the special 45-45-90 degree triangle including the ratios of the lengths of its edges.

Generic competences

In doing this activity students will have an opportunity to develop visualization skills.

Suggestions for teaching

Give everyone a piece of paper. It can be scrap paper and, as long as it is rectangular it can be any size. Demonstrate the first fold slowly and after the learners have made the fold, and then flattened out their sheets of paper, talk with the class about how this fold becomes the ***centre line*** and a line of symmetry for their piece of paper.



Draw this diagram on the board and mark the points A, B and C on the diagram.

Next demonstrate the second fold along BC emphasising that the corner A must go onto the centre line.

When the learners have made this fold ask them "what do you notice about the length of AC and the length of the top edge of the paper?" "Why does that happen?" If they are not sure they should measure these two lengths.

Then ask the learners to draw the line AP on their piece of paper so that angle APC is 90^o.

Then say "You have created some angles of 60 degrees and 30 degrees. Can you prove this?" Give learners time to work on this individually and in pairs so that they can discuss it and help each other. Give learners who finish the first activity well before the others the extension task of making an equilateral triangle.

Finally invite learners to explain to the class how they prove that the angles are 30^o and 60^o.

Then give learners scrap paper and show them how to make a square from a rectangle by making one fold. They should all do the and then cut out their squares. Ask the learners what properties of the square and right angled isosceles triangles they notice from this diagonal fold in the square.

Then ask the learners to fold their squares in half in different ways. Ask them what they notice about the symmetries of squares from this paper folding exercise.

Key questions

- What do you notice?
- Can you see and can you use any symmetry shown by that fold?
- If you fold an equilateral triangle in half what do you get?

Follow up

Tri-Fold https://aiminghigh.aimssec.ac.za/years-9-12-tri-fold/

Fold a Square https://aiminghigh.aimssec.ac.za/years-9-11-fold-a-square/

Investigating Circle Theorems
https://aiminghigh.aimssec.ac.za/years-10-11-investigating-circle-theorems/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13