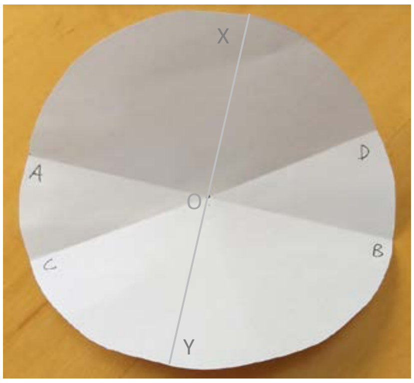
INVESTIGATING CIRCLE THEOREMS

Activity 1: Finding the centre of the circle



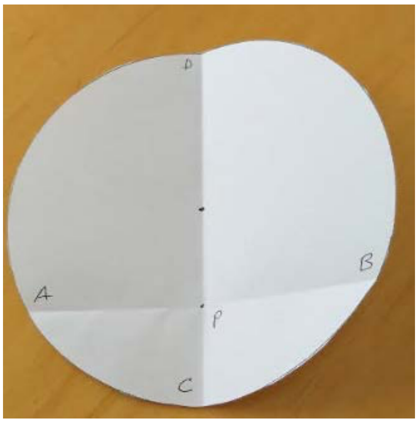
Draw round a large plate and cut out a paper circle.

(i)Use a paper circle that does not show the centre.

Fold the circle along a diameter. Mark ends A and B.

(ii) Fold the circle into quarters so that A and B come together. Open the circle out and mark the ends of the last fold line X and Y.

(iii) Now unfold your circle and then fold again in half along another diameter and mark the ends C and D. Also mark the point O where AB, XY and CD intersect.



(iv) Fold the circle exactly in half along other fold lines (diameters). What do you notice?

(v) All these folds go through the point O.
Why is this the centre of the circle?

Activity 2: Finding the perpendicular bisector of a chord.

Draw round a large plate, cut out a paper circle and mark the centre O.

(i)Fold over a segment of the circle to make a chord. Mark the ends A and B.

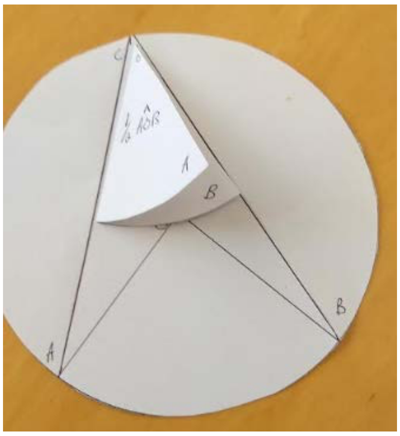
(ii) Fold the circle so that A and B come together. Unfold and mark the points where this line cuts the circle as C and D.

(iii) Draw the line segments (chords) AC, CB, BD and DA. Measure the angles and lengths of the quadrilateral ACBD? What do you notice?

(iv) If AB and CD intersect at P, what do you know about AP and PB? What about angle CPA?

(v) How is CD related to AB? Is CD a diameter of the circle? Explain.

Activity 3: Angles at the centre and circumference subtended by the same arc/chord.



Work in pairs if possible. Both need identical cut-out paper circles with centre O marked (draw round a large plate).

(i) Check the circles are identical by placing them on top of one another and mark corresponding points A and B one side of O on both circles, and a point C the other side of O as shown in the diagram.

(ii) One of you folds along the chords AC and BC. Then marks several other positions for C on the circumference labelling them C_1, C_2, C_3, …

(iii) The other person folds along AO and BO and then folds again so that AO and BO are aligned, making the angle ½ ∠AOB.

(iv) Compare this angle ½ ∠AOB with ∠ACB. It may be easier to cut out ∠AOB. What about $\angle AC_1B$, … ? Can you find any exceptional positions of C on the upper part of the circumference above A and B where ½∠AOB is not equal to ∠ACB?

HELP

Make some conjectures about what you think might be true. Gather evidence for your conjectures by measuring lengths and angles.

Observation of even a large number of cases supporting the conjecture does not ***prove*** it is true, and in all cases and a formal proof is needed.

To reinforce the mental pictures you have made from this folding exercise, and to help everyone remember the circle theorem, the class could make a large poster of the seven theorems for the classroom wall.

NEXT

Activity 4 The opposite angles of a cyclic quadrilateral are supplementary.

Work in pairs if possible. Both learners need identical cut-out paper circles with centre O marked (draw round a large plate).

(i) Both partners need to mark 4 points on the circumference so they have identical cyclic quadrilaterals, ABCD.

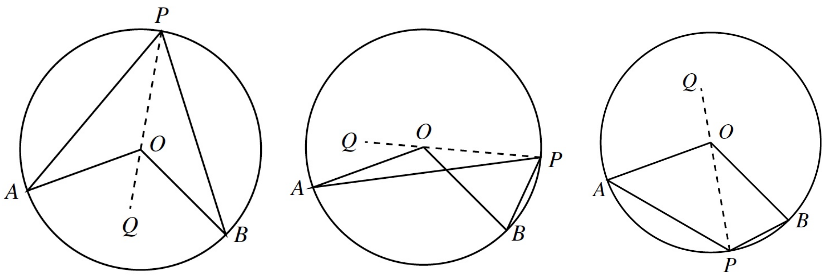
(ii) Fold along the chords AB, BC, CD, DA.

(iii) One learner should cut out the quadrilateral ABCD. Using one angle from each quadrilateral, place ∠A adjacent to ∠C. What do you notice?

(iv) Using one angle from each quadrilateral, place ∠B adjacent to ∠D. What do you notice?

Can you prove the results corresponding to activities 1 to 4?

In the same way as you did with Activity 4, using paper circles, **investigate ∠AOB and ∠APB as on the diagrams below**.



Can you prove the following theorem?

Theorem The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.

Three diagrams are necessary here to show the possible configurations but the proof is the same in each case.

Notice that there are two cases. In the first two diagrams we compare ∠APB with the obtuse angle ∠AOB but in the third diagram (where P is on the lower part of the circumference below A and B) we compare ∠APB with the reflex angle ∠AOB.

NOTES FOR TEACHERS

SOLUTION

Activity 1: O is the centre because the intersection point of the two diameters bisect each diameter therefore OA, OB, OC and OD are all radii.

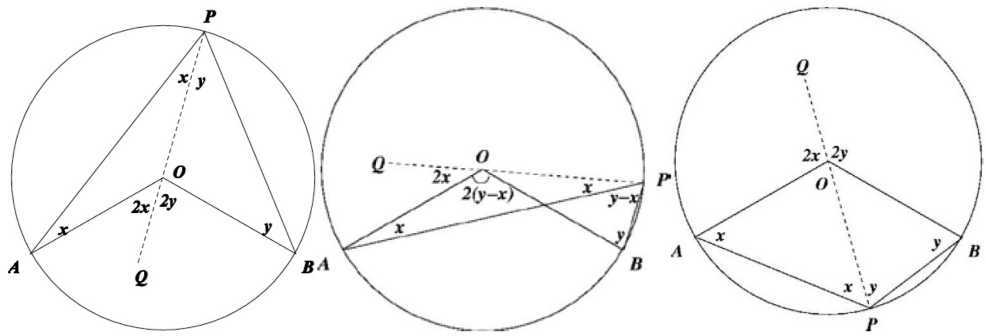
Another way is to fold a semi-circle again into a quarter circle to get the centre.

Activity 2: CD is the perpendicular bisector of a chord AB

CD is the diameter as it goes through the centre.

Activity 3: The angle at the centre is twice the size of the angle at the circumference - if they are both subtended by the same arc or chord.

NEXT Theorem The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.



Proof Three diagrams are necessary here to show the possible configurations but the proof is the same in each case.

As the radii are equal and the base angles of an isosceles triangle are equal:

∠OAP =∠OPA = x and ∠OBP =∠OPB = y.

As the exterior angle of a triangle equals the sum of the two interior opposite angles:

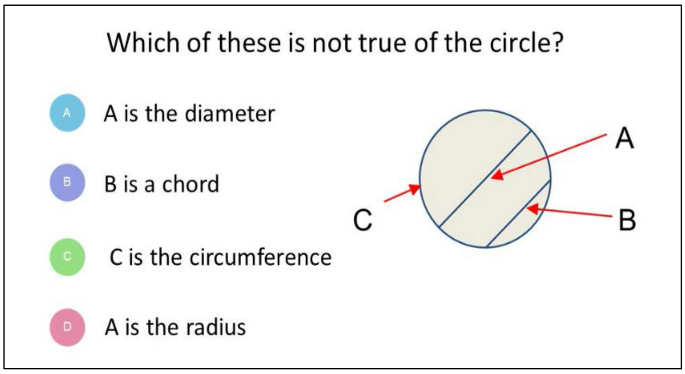
∠AOQ = 2x and ∠BOQ = 2y. Hence ∠AOB = 2∠APB.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.



3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: **D**

Why do this activity?

These paper folding activities provide experiences of discovering the properties of circles theorems. Folding cut-out paper circles does not prove the theorems, but ideas for the formal proofs are suggested in the diagrams themselves.

Learning objectives

In doing this activity students will have an opportunity to:

- gain a deeper understanding of the geometrical properties of circles;
- build their vocabulary about circle theorems;
- visualise the positions and relationships between the lines and angles in the circle, and the triangles they make, and so gain insights that will help them to understand the proofs of the theorems.

Generic competences

In doing this activity students will have an opportunity to develop problem solving skills.

- develop logical/critical thinking;
- develop visualization skills that enable them to form mental images relating to a problem, and to see relationships that help in solving the problem.

Suggestions for teaching

Similar paper folding activities can easily be devised that illustrate Theorem 6: the tangents drawn to a circle from a point outside the circle are equal in length and Theorem 7: the angle between a chord and the tangent to the circle at the endpoint of the cord is equal to the angle subtended by that chord in the alternate segment.

Key questions

- Can you explain what you have done so far?
- Do you think that this would work with other sketches or drawings?
- Did you use any new words today? What do they mean? How would you spell them?

Follow up

Surprise Ratios https://aiminghigh.aimssec.ac.za/years-9-12-surprise-ratios/

Kissing Circles https://aiminghigh.aimssec.ac.za/years-11-12-kissing-circles/

Construct with Circles
https://aiminghigh.aimssec.ac.za/years-7-9-construct-with-circles/

Circle Inscribed in a Quadrilateral
https://aiminghigh.aimssec.ac.za/years-10-12-circle-inscribed-in-quadrilateral/

Cyclic https://aiminghigh.aimssec.ac.za/years-11-12-cyclic/

Salinon https://aiminghigh.aimssec.ac.za/years-9-11-salinon/

Polycircles https://aiminghigh.aimssec.ac.za/years-11-12-polycircles/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

MATHS TOYS

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

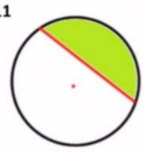
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13

DIAGNOSTIC ASSESSMENT https://diagnosticquestions.com

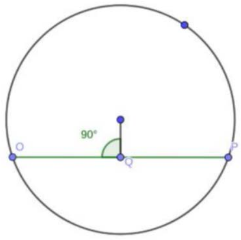
Question 11



What is the name of the circle part in green?

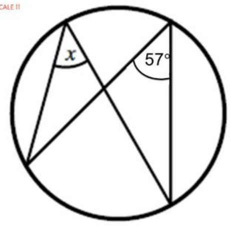
A) Tangent
B) Sector
C) Segment
D) Chord

The line segment QP is 7cm. What is the length of the chord OP?



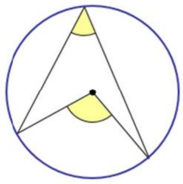
A 7cm
B 14cm
C 49cm
D Don't know

!! NOT TO SCALE !!



What is the size of angle x?

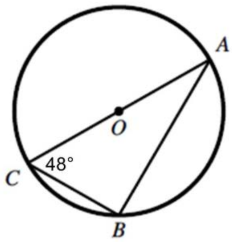
A 125°
B 57°
C 26.5°
D Not enough information



What is the correct name of this circle theorem?

A 'Opposite angles in a cyclic quadrilateral sum to 180°'
B 'The angle at the centre is twice the angle at the circumference'
C 'Angles in the same segment are equal'
D 'Alternate segment theorem'

!! NOT TO SCALE !!

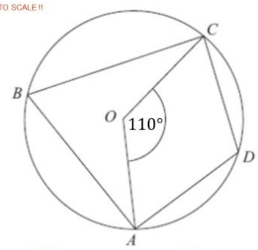


Angle ACB = 48°

What is the size of angle CAB?

A 52°
B 90°
C 48°
D 42°

NOT TO SCALE !!

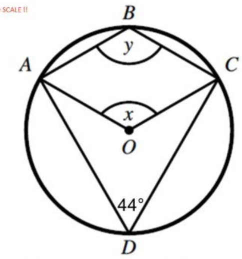


Angle AOC = 110°

What is the size of angle CDA?

A 125°
B 110°
C 55°
D 70°

!! NOT TO SCALE !!



Angle ADC = 44°

What is the size of angle x?

A 136°
B 22°
C 88°
D 44°