AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

FIBONACCI'S RABBITS

Fibonacci (in the year 1202) investigated a problem about how fast a population of rabbits would grow in the following circumstances, starting with just one pair of rabbits:

Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits.

Suppose that our rabbits **never die** and that the female **always** produces one new pair (one male, one female) **every month** from the second month on.

Use the diagram to find out how many pairs of rabbits there will be in one year and so to solve the problem?

The diagram shows the population of rabbits at times t = 0, 1, 2, 3, 4 and 5 months.

At time t = 0 the original pair of new-born rabbits are shown by a yellow disc.

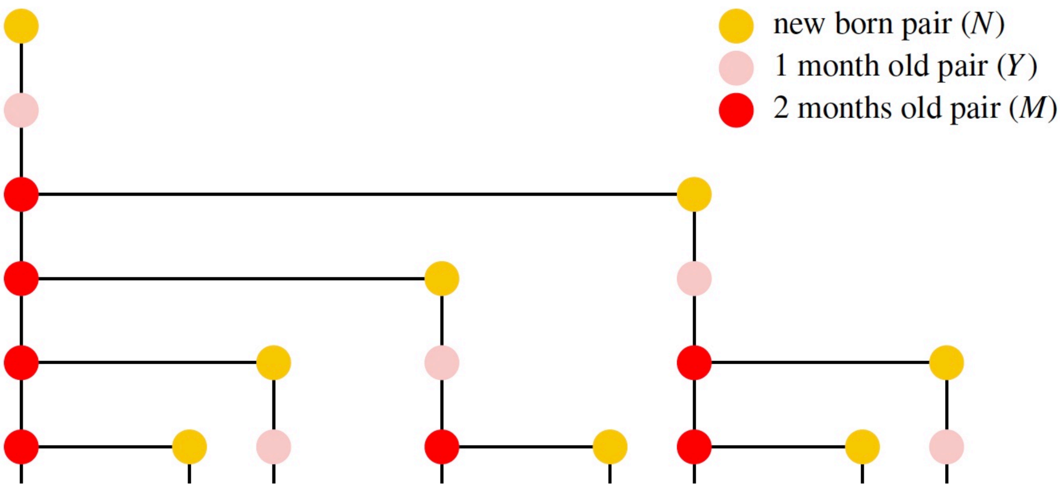
At time t = 1 month the same pair are shown in pink, as young rabbits.

At time t = 2 months the diagram shows the same rabbits (now mature) in red together with a pair of new-borns (in yellow).

At time t = 3 months the original pair have produced a 2nd pair of new-borns and their 1st pair are now young and shown in pink.

The population is also shown at times t = 4 and 5 months.

Extend the tree diagram to show the population at t = 6 months.



Fill in the table for times t = 0 to t = 6 months.

Write down formulae for N_t, Y_t, M_t and F_t and explain why the pattern will continue month after month.

Time in months	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of new-born pairs N_t													
Number of young pairs Y_t													
Number of mature pairs M_t													
Total number of pairs F_t													

In 1202 Leonardo of Pisa, who is now known as Fibonacci, published a ground-breaking book called 'The Book of Calculation' (Liber Abaci in Latin) which included this problem. Fibonacci had travelled in Asia as a merchant and he brought from India to Europe a new way of writing numbers – the Hindu-Arabic Numeral system. His methods influenced the development of mathematics in Europe and provided merchants with efficient ways to record commercial transactions.

Henry E Dudeney (1857 – 1930), an Englishman famous for his puzzles, adapted Fibonacci's Rabbits problem. He changed months into years, and rabbits into bulls (male) and cows (females) and stated the problem as follows:

If cows produce their first she-calf at age 2 years, and after that another she-calf every year, how many she-calves are there after 12 years, assuming that none die?

Explain the solution to this problem.

HELP

You need to read the information, and interpret the diagram, in order to fill in the table and to solve the problem. It may help you to work with another student so that you can talk about the information given and help each other.

NEXT

Consider times t = 0, 1, 2, … and use the notation N_t for the number of new-born pairs, Y_t for the number of young pairs and M_t for the number of mature pairs at time t.

Then $N_{t+1} = M_t + Y_t$

Complete the following:

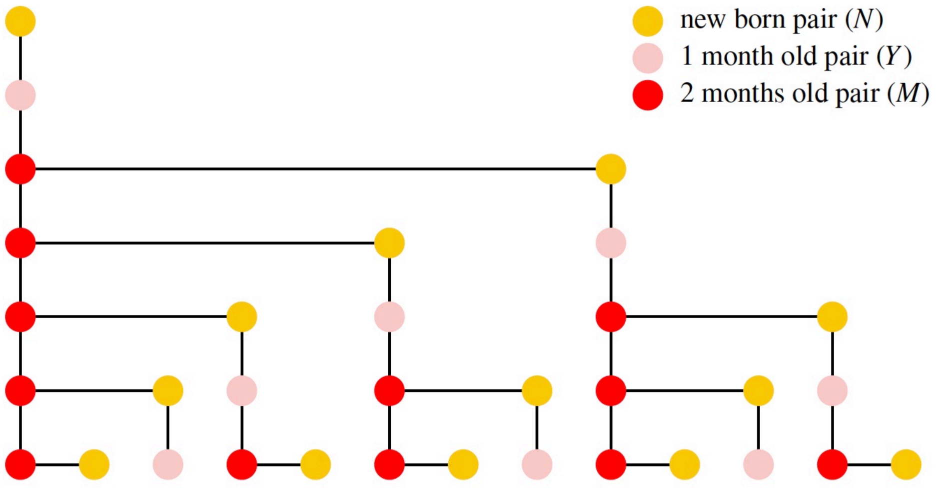
Y_{t+1} = …

M_{t+1} = …

and prove that the sequences of the number of pairs of rabbits are Fibonacci sequences.

NOTES FOR TEACHERS

SOLUTION



Time in months	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of new-born pairs N_t	1	0	1	1	2	3	5	8	13	21	34	55	89
Number of young pairs Y_t	0	1	0	1	1	2	3	5	8	13	21	34	55
Number of mature pairs M_t	0	0	1	1	2	3	5	8	13	21	34	55	89
Total number of pairs F_t	1	1	2	3	5	8	13	21	34	55	89	144	233

Consider times t = 0, 1, 2, ...
Then

$N_{t+1} = M_t + Y_t$ (1) because each mature pair and each young pair give birth to a pair of new-borns at intervals of 1 month.

$Y_{t+1} = N_t$ (2) because after one month a new-born rabbit becomes a young rabbit.

$M_{t+1} = M_t + Y_t$ (3) because none of the rabbits die so the mature rabbits live on month after month, and the young rabbits become mature after one month.

The total number of pairs of rabbits is:

$F_t = N_t + Y_t + M_t$ (4)

Now

$F_{t+2} - F_{t+1} = (N_{t+2} + Y_{t+2} + M_{t+2}) - (N_{t+1} + Y_{t+1} + M_{t+1})$ substituting from (4)

$= (M_{t+1}+Y_{t+1}+N_{t+1}+M_{t+1}+Y_{t+1}) - (N_{t+1}+Y_{t+1}+M_{t+1})$ from (1), (2) and (3)

$= \quad M_{t+1}+Y_{t+1}$ simplifying the previous expression

$= \quad M_t + Y_t + N_t$ from (2) and (3)

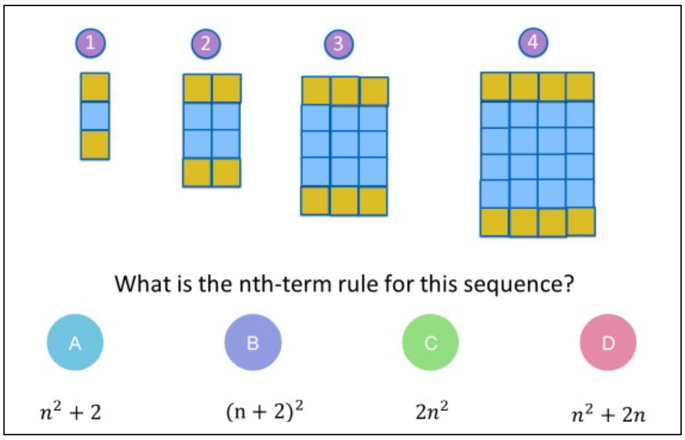
$= \quad F_t$ from (4).

This proves that the sequence month by month of the number of pairs of rabbits is a Fibonacci sequence.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: D

Why do this activity?

This is a classic and very old problem and it has been scaffolded so that most upper secondary students should be able to use the tree diagram to read off the numbers of pairs of rabbits in successive generations and extrapolate this to give the solution in terms of the Fibonacci sequence. The problem provides a challenge for older or more able students to give a proof of the result. Students also meet a small and important piece of the history of mathematics and are made aware that, in Europe, we learned valuable mathematical concepts from India and from the Arabs.

Learning objectives

In doing this activity students will have an opportunity to:

- interpret complex information and analyse it in order to solve a problem
- recognize a number pattern and express the nth term of a sequence as a formula.

Generic competences

In doing this activity students will have an opportunity to develop problem solving skills.

Suggestions for teaching

This problem provides an excellent opportunity for students to develop their problem solving skills.

Use the **1, 2, 4 more** teaching strategy. Give the students 10 minutes to work on the problem on their own, then tell them to work in pairs. After about 30 minutes tell the pairs to team up with another pair to compare their discoveries. Then have a class discussion inviting representatives from different groups of 4 to explain their solutions and their reasoning to the class.

Key questions

- Can you pick out the original pair of rabbits in the diagram at every stage?
- Can you pick out a new-born pair of rabbits for every mature pair and also for every young pair?
- What do you notice about the number patters in the sequences for the numbers of pairs of rabbits at each stage?

Follow up

One Step Two Steps https://aiminghigh.aimssec.ac.za/years-7-10-one-step-two-steps/
Elephant Dreaming https://aiminghigh.aimssec.ac.za/years-8-12-elephant-dreaming/
Sheep Talk https://aiminghigh.aimssec.ac.za/years-7-12-sheep-talk/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13