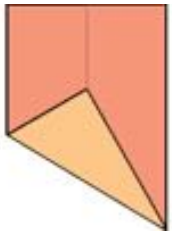
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18+ with
a common starter on the theme of EQUILATERAL TRIANGLES

Choose what seems suitable for the age or attainment level of your learners

The original MAKING SIXTY ACTIVITY was designed for Years 9 to 10

MAKING SIXTY



Why does this fold create an angle of sixty degrees?

Make a centre crease down the length of the paper then open it up.

Next fold one corner over and onto the centre crease so that the fold line passes through the corner next to it (on the short side of the paper).

You have created some angles of 60 degrees and of 30 degrees.

Can you prove this?

Now fold a square in half in different ways. What do you notice?

HELP

Draw and cut out an equilateral triangle.

Fold IT along a line of symmetry.

Work with a partner. Talk about how the line of symmetry splits the equilateral triangle into two congruent 30-60-90 degree triangles.

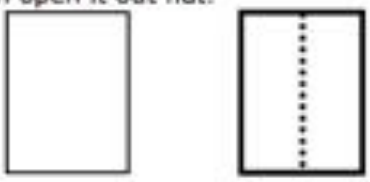
Look at your triangle in different orientations as you turn it around.

Does this help you to explain why the angle is 60^{o} in the Making Sixty paper folding?

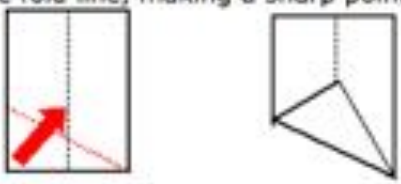
NEXT

Follow on by utilising this fold to make equilateral triangles.

1. Fold the paper in half long-ways, then open it out flat.

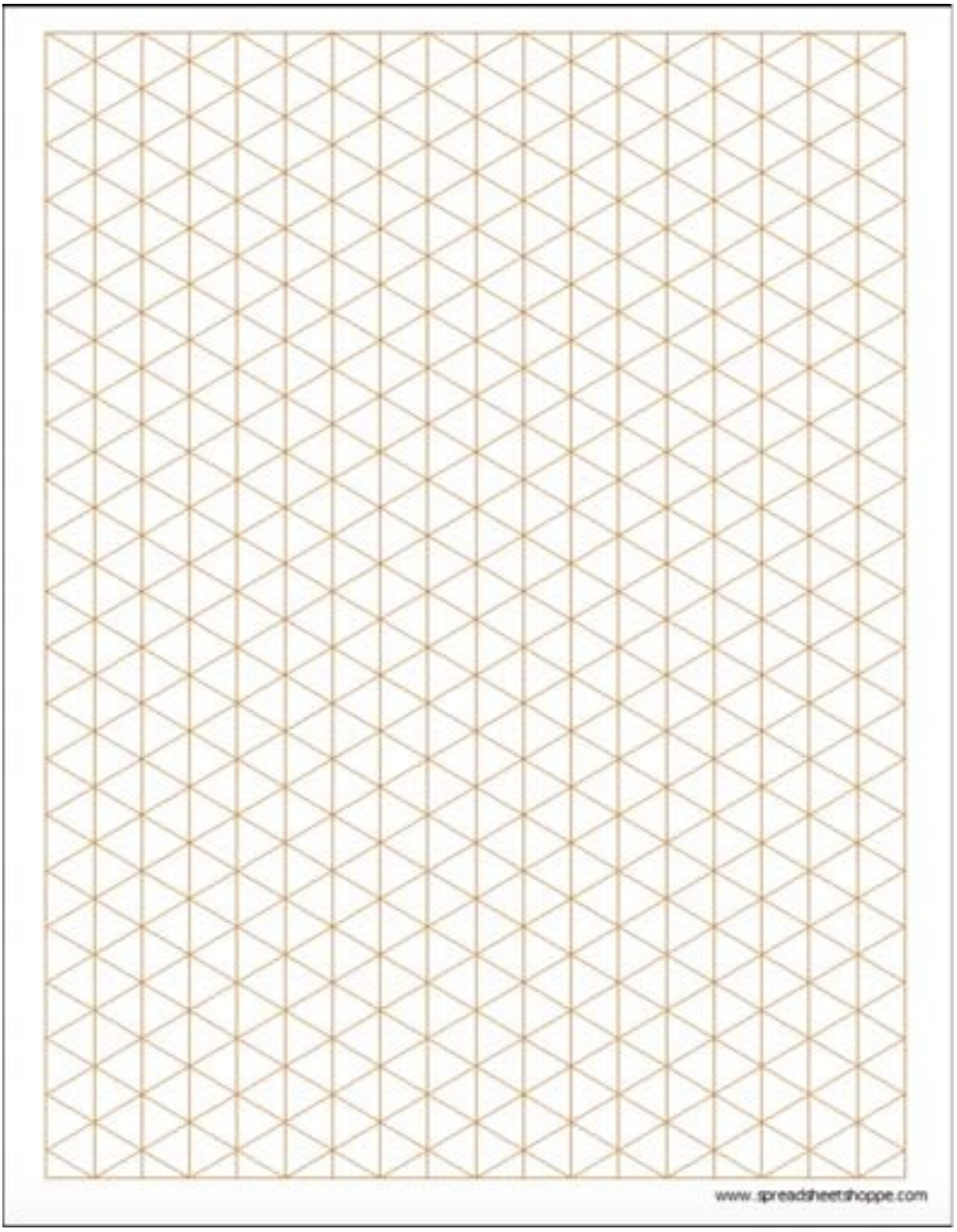


2. Fold a bottom corner up to touch the fold line, making a sharp point on the other corner.



3. Now fold the two RED edges together.

4. Fold under the corner and then tuck it in out of the way. Turn over to the smooth side.

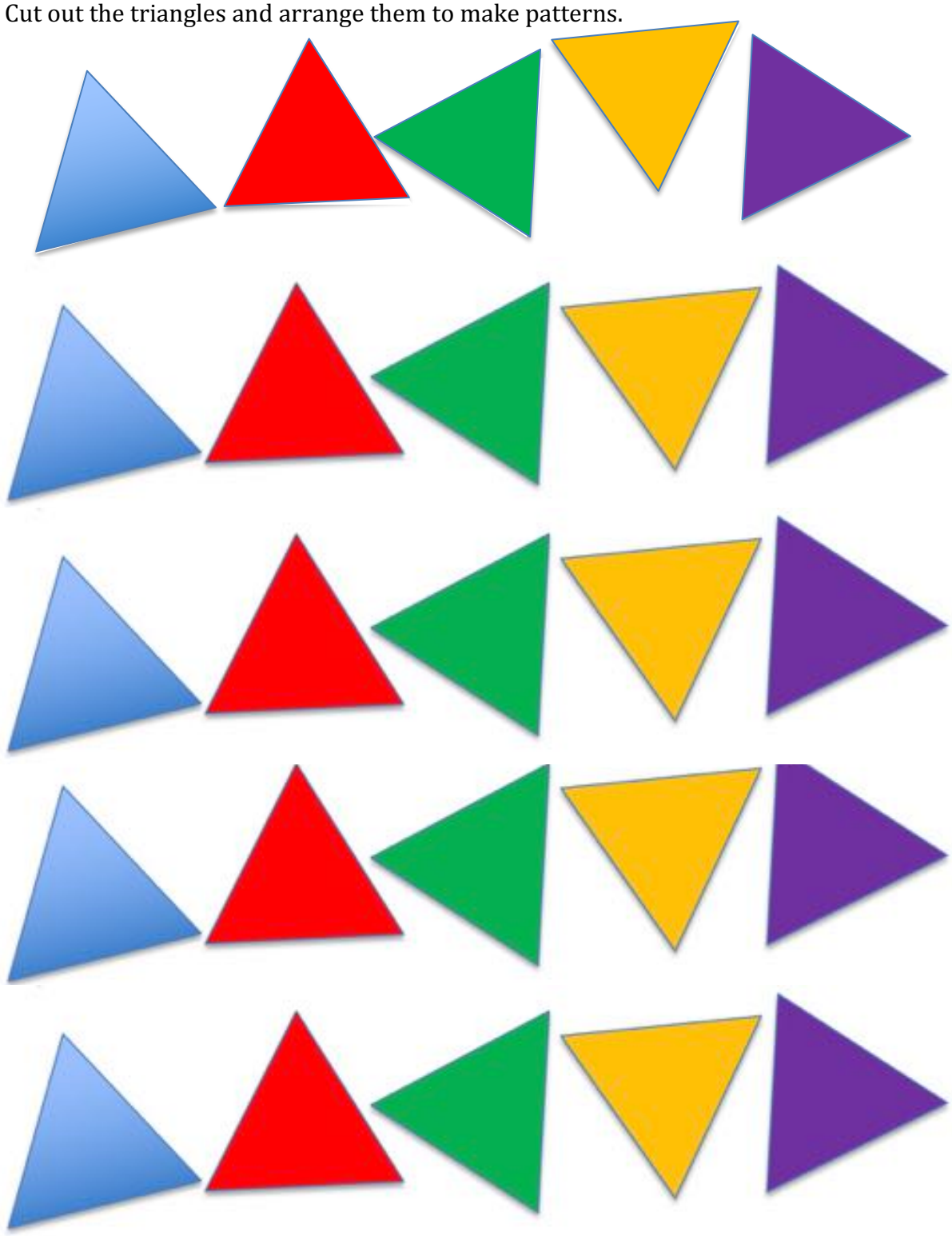

www.spreadsheetshoppe.com

INCLUSION AND HOME LEARNING GUIDE

THEME: EQUILATERAL TRIANGLES

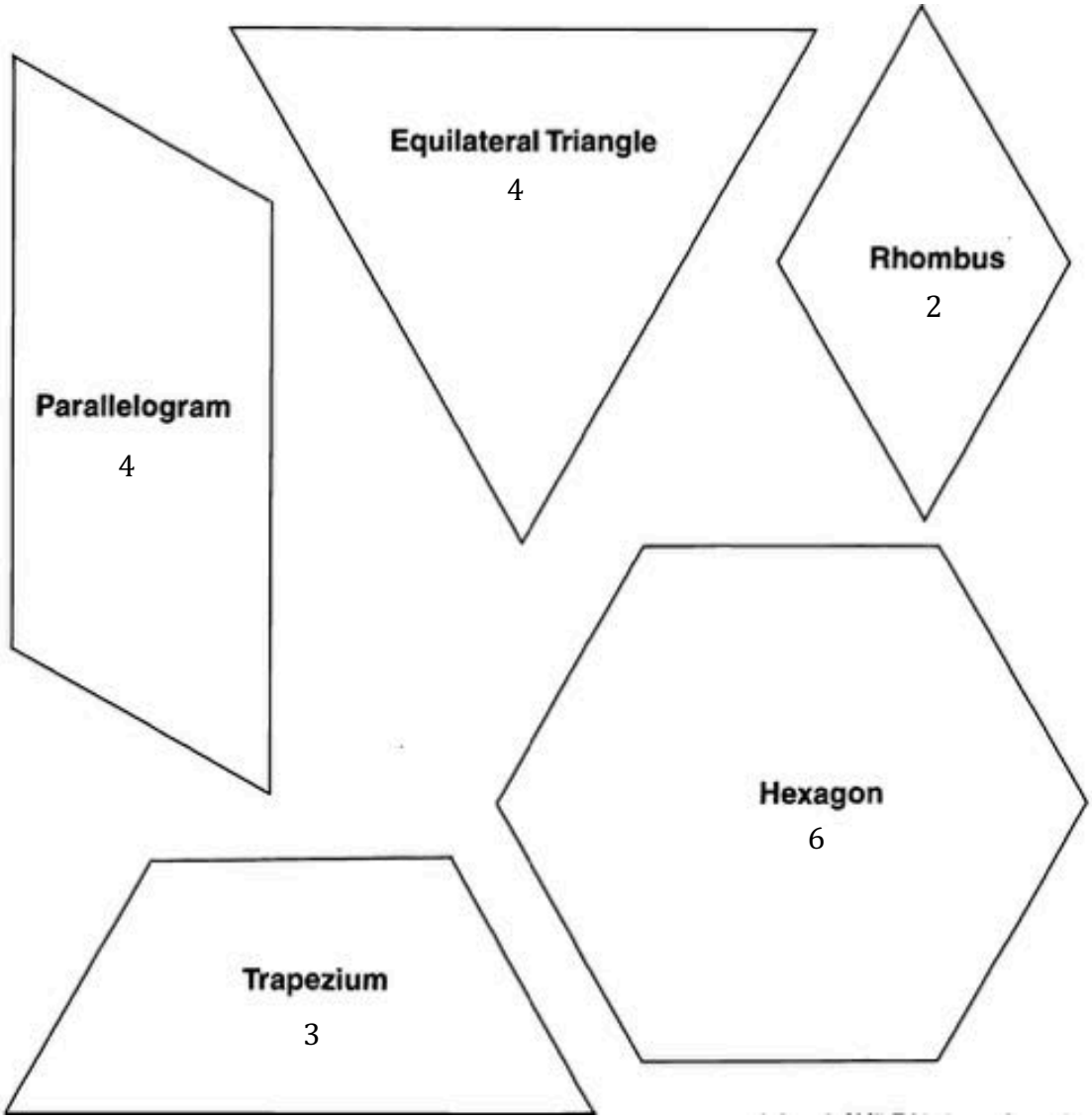
Early Years, Kindergarten and Year 1

Cut out the triangles and arrange them to make patterns.

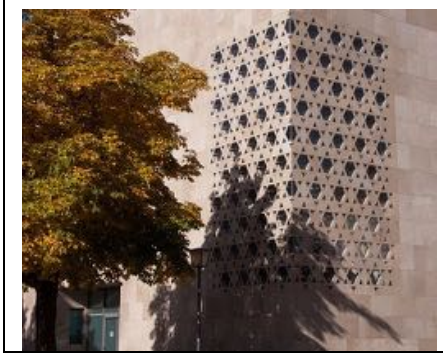


Years 2 to 5

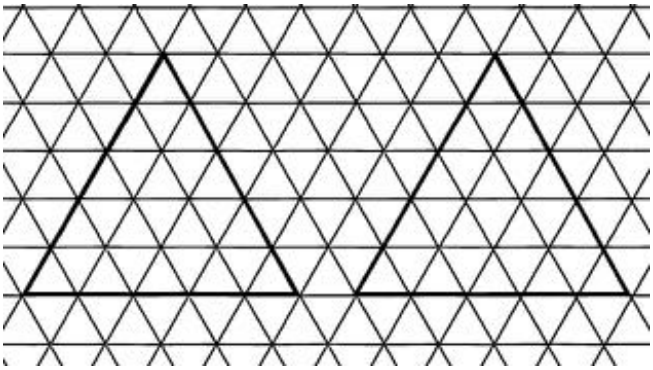
Make lots of equilateral triangles, use the isometric paper on page 3, or cut out the triangles on page 4, or draw your own triangles. Ask the children to make the shapes below using equilateral triangles. The number to use is shown on each diagram. They could stick the triangles into their workbooks to make these shapes.



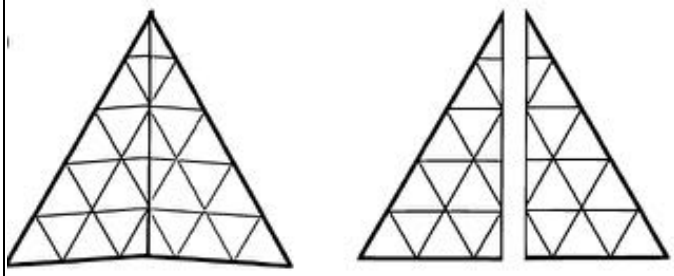
Describe what you see in these pictures?



Years 6 to 8



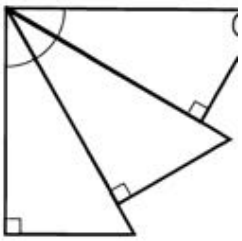
Give each learner some isometric paper and ask them to draw and cut out 2 equilateral triangles.



Next the learners must fold the triangles in half and cut out 4 triangles as in the diagram.

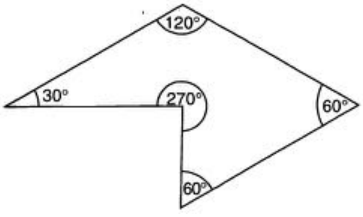
Give the following instructions:

1. On each small triangle mark the right angle on both sides of the paper.
   How many degrees are there in a right angle?
2. On each small triangle mark the smallest angle on both sides of the paper.
3. Place 3 of the smallest angles together. What size is the angle they make?
   How many degrees is this angle?
4. Mark the third angle.

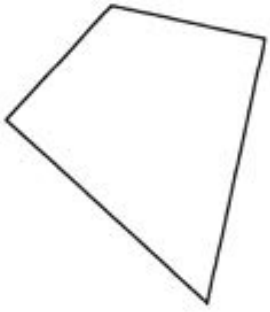


5. Make this shape with two small triangles.
   What is the name of this shape?
   What do you know about the angles in the corners of this shape?
   How many degrees is the third angle of the small triangle?

6. Make this shape with 3 small triangles



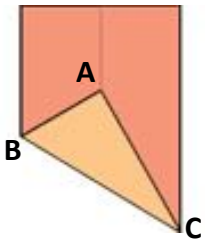
7. Make a kite with small triangles and work out the angles.
8. Make a parallelogram with 2 small triangles and work out the angles.



Inspired by SMILE

Years 9 and 10

Give everyone a piece of paper. It can be scrap paper and, as long as it is rectangular it can be any size. Slowly demonstrate making a centre crease down the length of the paper then open it up. After the learners have made the fold, and then flattened out their sheets of paper, talk with the group about how this fold becomes the ***centre line*** and a line of symmetry for their piece of paper.



Draw this diagram and mark the points A, B and C on the diagram.

Next demonstrate the second fold along BC emphasising that the corner A must go onto the centre line.

When the learners have made this fold ask them "what do you notice about the length of AC and the length of the top edge of the paper?" "Why does that happen?" If they are not sure they should measure these two lengths.

Then ask the learners to draw the line AP on their piece of paper so that angle APC is 90^o.

Then say "You have created some angles of 60 degrees and 30 degrees. Can you prove this?" Give learners time to work on this individually and in pairs so that they can discuss it and help each other. Give learners who finish the first activity before the others the extension task of making an equilateral triangle.

Finally invite learners to explain how they prove that the angles are 30^o and 60^o.

Then give learners scrap paper and show them how to make a square from a rectangle by making one fold. They should all do this and then cut out their squares. Ask the learners what properties of the square and right angled isosceles triangles they notice from the diagonal fold in the square.

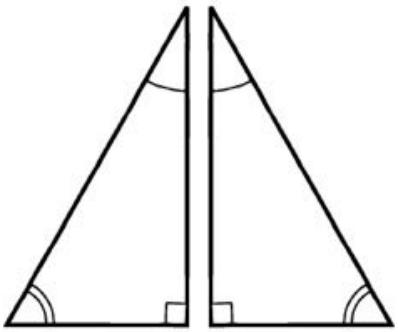
Then ask the learners to fold their squares in half in different ways. Ask them what they notice about the symmetries of squares from this paper folding exercise.

Key questions

- What do you notice?
- Can you see and can you use any symmetry shown by that fold?
- If you fold an equilateral triangle in half what do you get?

Years 11 to 13 – FOR INDEPENDENT STUDY

Do the learning activities on pages 1 and 2.



Fold an equilateral triangle of edge length 1 unit in half along a line of symmetry,

Cut it into two right angled triangles.

Work out:

a) the sizes of the angles;

b) the edge lengths;

c) the sine, cosine and tangent of 30^o, 60^o and 90^o.

Why do this activity?

This activity involves a simple fold which gives a surprising and useful result. Different routes to the solution of the Making Sixty activity can also lead to useful discussions and give learners practice in explaining their reasoning.

The activity should lead to impressing on learners that they must learn to visualise the standard diagrams for the $45^o - 45^o - 90^o$ triangle (half a square so its edge lengths are in the ratio $1 : 1 : \sqrt{2}$) and the $30^o - 60^o - 90^o$ triangle (half an equilateral triangle so its edge lengths are in the ratio $1 : 2 : \sqrt{3}$). Visualising these two triangles enables a person to remember the exact values (in terms of surds) of the sine, cosine and tangent of 30^o, 45^o and 60^o.

Having confirmed the sizes of angles this knowledge can be applied to making models of polyhedra.

Learning objectives

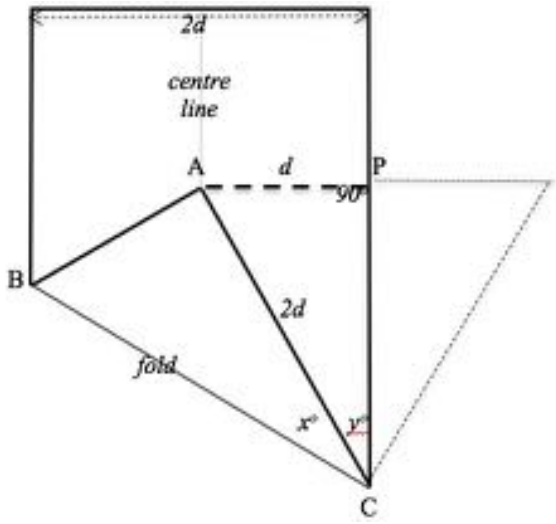
In doing this activity students will have an opportunity to:

- deepen understanding of the properties of equilateral triangles and the special 30-60-60 degrees triangle including the ratios of the lengths of its edges;
- deepen understanding of the properties of right-angled isosceles triangles, and the special 45-45-90 degrees triangle including the ratios of the lengths of its edges.

Generic competences

Students will have an opportunity to develop visualization skills.

SOLUTION



Draw AP so that angle APC is a right angle.

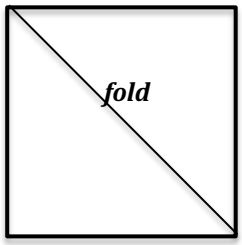
If the width of the paper is $2d$ (for any value of d) then AC = $2d$ and AP = d so triangle APC is half of an equilateral triangle and angle ACP = $y^o = 30^o$.

Because C is at a corner of the paper the fold CB makes the angle at C into 2 angles of x^o and an angle of y^o .

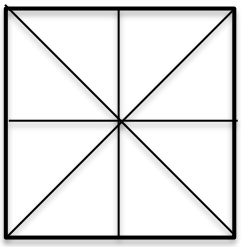
So $2x + y = 90$

and $x^o = 30^o$.

This fold gives angle BCP = 60^o and angle ACP = 30^o.



The diagonal fold line is an axis of reflection symmetry.
The two triangles are congruent right-angled triangles with angles 45^o – 45^o- 90^o and edge lengths in the ratio $1 : 1 : \sqrt{2}$



The square has 4 axes of reflective symmetry (mirror lines) as shown by these fold lines.

The square has four-fold rotational symmetry about its centre.

a) The angles are 30^o, 60^o and 90^o.

b) The edge lengths are 1, $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$

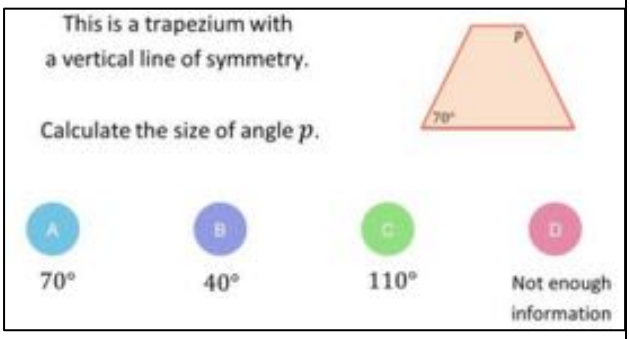
c) $\sin 30^o = \cos 60^o = \frac{1}{2}$

$\cos 30^o = \sin 60^o = \frac{\sqrt{3}}{2}$

$\tan 30^o = \cot 60^o = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot 30^o = \tan 30^o = \sqrt{3}$

DIAGNOSTIC ASSESSMENT This can be done before or after the lesson and as a group as described below, or the question can be answered individually. Show this question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: C https://diagnosticquestions.com/

Possible misconceptions:

A. The student appears to have taken a horizontal line of symmetry.
B. The student may not know that angles in a quadrilateral add to 360 degrees.
C. The vertical line of symmetry means that the bottom-right angle is also 70 degrees. Also, p has the same value as the top-left angle. A trapezium is a quadrilateral, and so all angles will add to 360 degrees. Hence, we can write: 70 + 70 + p + p = 360, and so 2p + 140 = 360. This means 2p = 220, giving p = 110 degrees.
D. The student appears not to realise the information given by the line of symmetry.

Follow up

Tri-Fold https://aiminghigh.aimssec.ac.za/years-9-12-tri-fold/

Fold a Square https://aiminghigh.aimssec.ac.za/years-9-11-fold-a-square/

Investigating Circle Theorems
https://aiminghigh.aimssec.ac.za/years-10-11-investigating-circle-theorems/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
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Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13