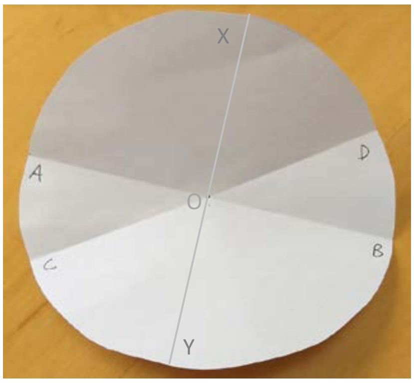
INVESTIGATING CIRCLE THEOREMS

Activity 1: Finding the centre of the circle



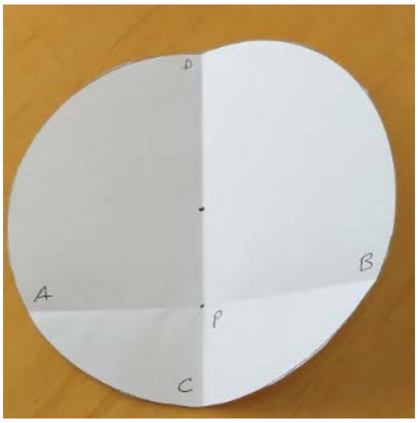
Draw round a large plate and cut out a paper circle.

(i)Use a paper circle that does not show the centre.

Fold the circle along a diameter. Mark ends A and B.

(ii) Fold the circle into quarters so that A and B come together. Open the circle out and mark the ends of the last fold line X and Y.

(iii) Now unfold your circle and then fold again in half along another diameter and mark the ends C and D. Also mark the point O where AB, XY and CD intersect.



(iv) Fold the circle exactly in half along other fold lines (diameters). What do you notice?

(v) All these folds go through the point O.
Why is this the centre of the circle?

Activity 2: Finding the perpendicular bisector of a chord.

Draw round a large plate, cut out a paper circle and mark the centre O.

(i)Fold over a segment of the circle to make a chord. Mark the ends A and B.

(ii) Fold the circle so that A and B come together. Unfold and mark the points where this line cuts the circle as C and D.

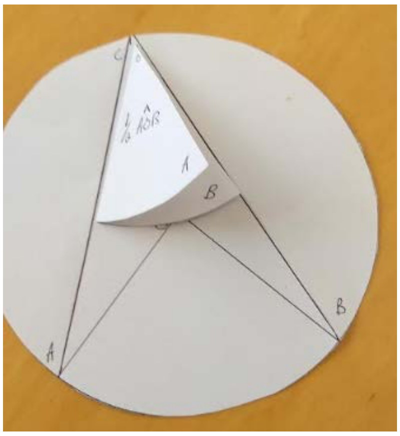
(iii) Draw the line segments (chords) AC, CB, BD and DA. Measure the angles and lengths of the quadrilateral ACBD? What do you notice?

(iv) If AB and CD intersect at P, what do you know about AP and PB? What about angle CPA?

(v) How is CD related to AB? Is CD a diameter of the circle? Explain.

Activity 3: Angles at the centre and circumference subtended by the same arc/chord.

Work in pairs if possible. Both need identical cut-out paper circles with centre O marked (draw round a large plate).



(i) Check the circles are identical by placing them on top of one another and mark corresponding points A and B one side of O on both circles, and a point C the other side of O as shown in the diagram.

(ii) One of you folds along the chords AC and BC. Then marks several other positions for C on the circumference labelling them C_1, C_2, C_3, …

(iii) The other person folds along AO and BO and then folds again so that AO and BO are aligned, making the angle ½ ∠AOB.

(iv) Compare this angle ½ ∠AOB with ∠ACB. It may be easier to cut out ∠AOB. What about $\angle AC_1B$, … ? Can you find any exceptional positions of C on the upper part of the circumference above A and B where ½∠AOB is not equal to ∠ACB?

HELP

Make some conjectures about what you think might be true. Gather evidence for your conjectures by measuring lengths and angles.

Observation of even a large number of cases supporting the conjecture does not ***prove*** it is true, and in all cases and a formal proof is needed.

To reinforce the mental pictures you have made from this folding exercise, and to help everyone remember the circle theorem, the class could make a large poster of the seven theorems for the classroom wall.

NEXT

Activity 4 The opposite angles of a cyclic quadrilateral are supplementary.

Work in pairs if possible. Both learners need identical cut-out paper circles with centre O marked (draw round a large plate).

(i) Both partners need to mark 4 points on the circumference so they have identical cyclic quadrilaterals, ABCD.

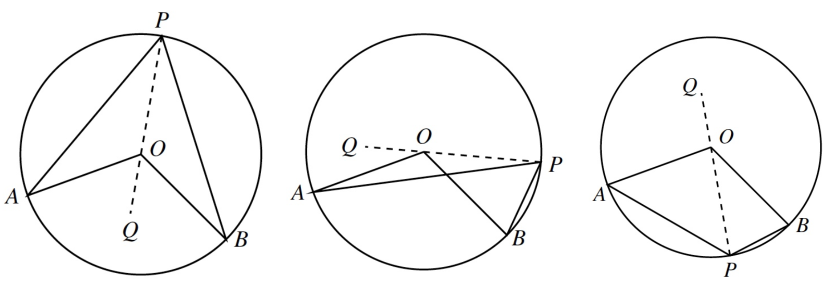
(ii) Fold along the chords AB, BC, CD, DA.

(iii) One learner should cut out the quadrilateral ABCD. Using one angle from each quadrilateral, place ∠A adjacent to ∠C. What do you notice?

(iv) Using one angle from each quadrilateral, place ∠B adjacent to ∠D. What do you notice?

Can you prove the results corresponding to activities 1 to 4?

In the same way as you did with Activity 4, using paper circles, **investigate ∠AOB and ∠APB as on the diagrams below**.



Can you prove the following theorem?

Theorem The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.

Three diagrams are necessary here to show the possible configurations but the proof is the same in each case.

Notice that there are two cases. In the first two diagrams we compare ∠APB with the obtuse angle ∠AOB but in the third diagram (where P is on the lower part of the circumference below A and B) we compare ∠APB with the reflex angle ∠AOB.