AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE

suggests related learning activities for all ages from 4 to 18

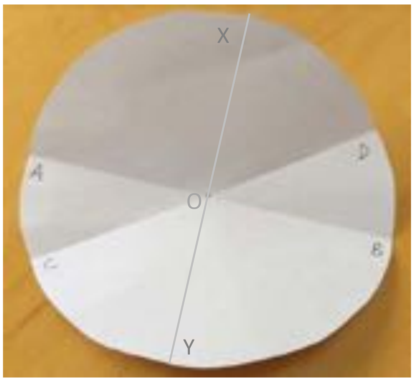
on the theme of CIRCLES

Just choose whatever seems suitable for your group of learners

The original INVESTIGATING CIRCLE THEOREMS activity was designed for Years 10 to 11

INVESTIGATING CIRCLE THEOREMS

Activity 1: Finding the centre of the circle



Draw round a large plate and cut out a paper circle.

(i)Use a paper circle that does not show the centre.

Fold the circle along a diameter. Mark ends A and B.

(ii) Fold the circle into quarters so that A and B come together. Open the circle out and mark the ends of the last fold line X and Y.

(iii) Now unfold your circle and then fold again in half along another diameter and mark the ends C and D. Also mark the point O where AB, XY and CD intersect.

(iv) Fold the circle exactly in half along other fold lines (diameters). What do you notice?

(v) All these folds go through the point O.
Why is this the centre of the circle?

Activity 2: Finding the perpendicular bisector of a chord.

Draw round a large plate, cut out a paper circle and mark the centre O.

(i)Fold over a segment of the circle to make a chord.
Mark the ends A and B.

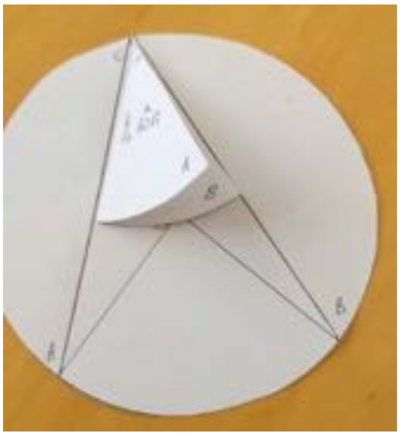
(ii) Fold the circle so that A and B come together. Unfold and mark the points where this line cuts the circle as C and D.

(iii) Draw the line segments (chords) AC, CB, BD and DA. Measure the angles and lengths of the quadrilateral ACBD? What do you notice?

(iv) If AB and CD intersect at P, what do you know about AP and PB? What about angle CPA?

(v) How is CD related to AB? Is CD a diameter of the circle? Explain.

Activity 3: Angles at the centre and circumference subtended by the same arc/chord.



Work in pairs if possible. Both need identical cut-out paper circles with centre O marked (draw round a large plate).

(i) Check the circles are identical by placing them on top of one another and mark corresponding points A and B one side of O on both circles, and a point C the other side of O as shown in the diagram.

(ii) One of you folds along the chords AC and BC. Then marks several other positions for C on the circumference labelling them C_1, C_2, C_3, …

(iii) The other person folds along AO and BO and then folds again so that AO and BO are aligned, making the angle ½ ∠AOB.

(iv) Compare this angle ½ ∠AOB with ∠ACB. It may be easier to cut out ∠AOB. What about ∠AC_1B, … ? Can you find any exceptional positions of C on the upper part of the circumference above A and B where ½∠AOB is not equal to ∠ACB?

HELP

Make some conjectures about what you think might be true. Gather evidence for your conjectures by measuring lengths and angles.

Observation of even a large number of cases supporting the conjecture does not ***prove*** it is true, and in all cases and a formal proof is needed.

To reinforce the mental pictures you have made from this folding exercise, and to help everyone remember the circle theorem, the class could make a large poster of the seven theorems for the classroom wall.

NEXT

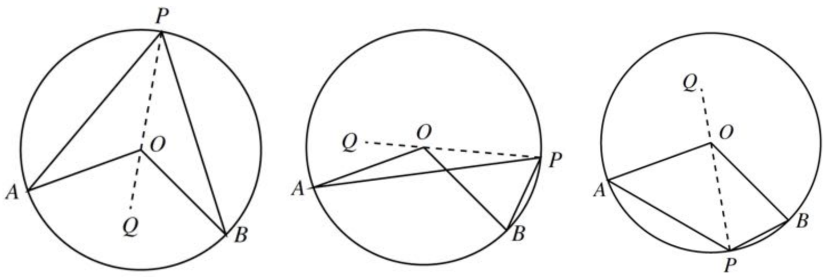
Activity 4 The opposite angles of a cyclic quadrilateral are supplementary.

Work in pairs if possible. Both learners need identical cut-out paper circles with centre O marked (draw round a large plate).

(i)	Both partners need to mark 4 points on the circumference so they have identical cyclic quadrilaterals, ABCD.
(ii)	Fold along the chords AB, BC, CD, DA.
(iii)	One learner should cut out the quadrilateral ABCD. Using one angle from each quadrilateral, place ∠A adjacent to ∠C. What do you notice?
(iv)	Using one angle from each quadrilateral, place ∠B adjacent to ∠D. What do you notice?

Can you prove the results corresponding to activities 1 to 4?

In the same way as you did with Activity 4, using paper circles, **investigate ∠AOB and ∠APB as on the diagrams below**.



Can you prove the following theorem?

Theorem The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.

Three diagrams are necessary here to show the possible configurations but the proof is the same in each case.

Notice that there are two cases. In the first two diagrams we compare ∠APB with the obtuse angle ∠AOB but in the third diagram (where P is on the lower part of the circumference below A and B) we compare ∠APB with the reflex angle ∠AOB.

INCLUSION AND HOME LEARNING GUIDE

THEME: CIRCLES

Early Years and Lower Primary


Use some plates and tins from your kitchen.

Draw round them to make circles.

Help the children to draw circles.

Make some patterns.

Can the children make circles with their arms? Or with their whole bodies?

Ask the children: "**What do you notice about these pictures**?"










Ask the children to find some more circles.

Drop a stone in water to see the circles formed.

Turn this into a game of 'Spot the circle'.

For more than 7000 years people across the world have built stone circles.

Sonehenge, UK

Tamera Stone Circle, Somalia view from satellite.

Cromeleque dos Almendres , Portugal

Gilgal Ra'im, Israel

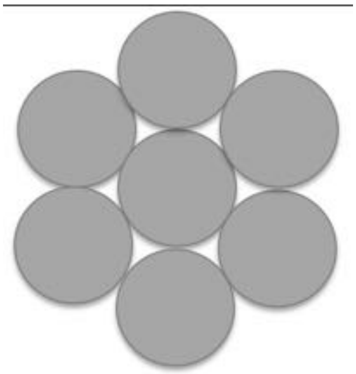
Investigate rolling

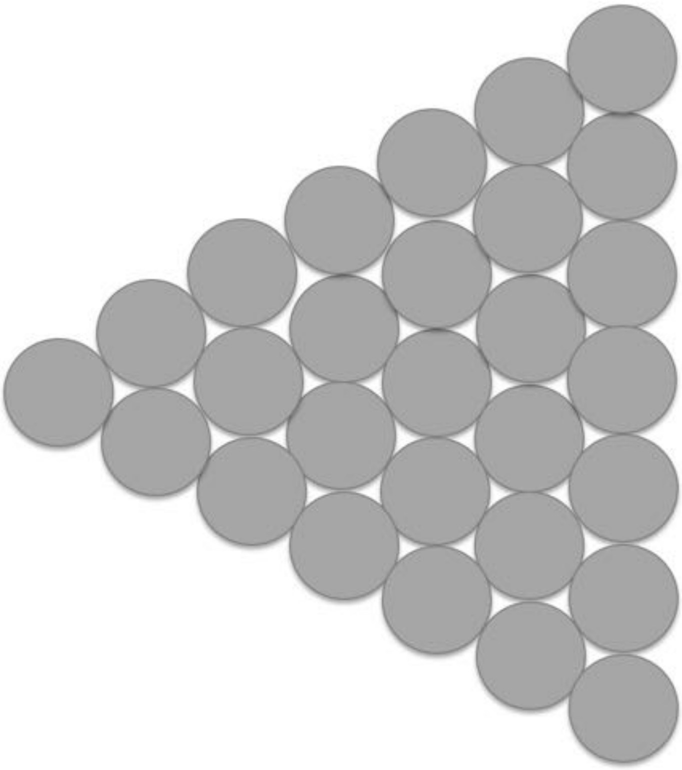
Ask the children about wheels and their shape and how they are used.

Try rolling cylindrical objects downhill.

The children might themselves roll downhill or do a somersault head over heels.

Make patterns with coins like these diagrams.





This is a wasps' nest. Bees make honeycombs like this.

Ask the children what they notice about it and compare it to the pattern of coins.

Upper Primary

Work through some or all of the activities for Early Years and Lower Primary.

What did people do before they invented wheels?

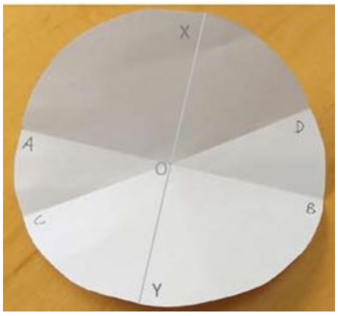
Many people had to work together to move heavy things like the big stones used to build the Pyramids of Giza in Egypt 4500 years ago and stone circles built 7000 years ago or more. People wet the sand, or spread animal fat on the ground, so the stones would slide when they were pushed along.

Rolling the stone on logs made it easier to move it forward.

As it came off each log at the back, the log was carried to put at the front.

Try this with pencils and a heavy weight.

Finding the centre of the circle



Draw round a large plate and cut out a paper circle.

(i)Use a paper circle that does not show the centre.

Fold the circle along a diameter. Mark ends A and B.

(ii) Fold the circle into quarters so that A and B come together. Open the circle out and mark the ends of the last fold line X and Y.

(iii) Now unfold your circle and then fold again in half along another diameter and mark the ends C and D. Also mark the point O where AB, XY and CD intersect.

(iv) Fold the circle exactly in half along other fold lines (diameters). What do you notice?

(v) All these folds go through the point O. Why is this the centre of the circle?

PAPERCLIP COMPASS

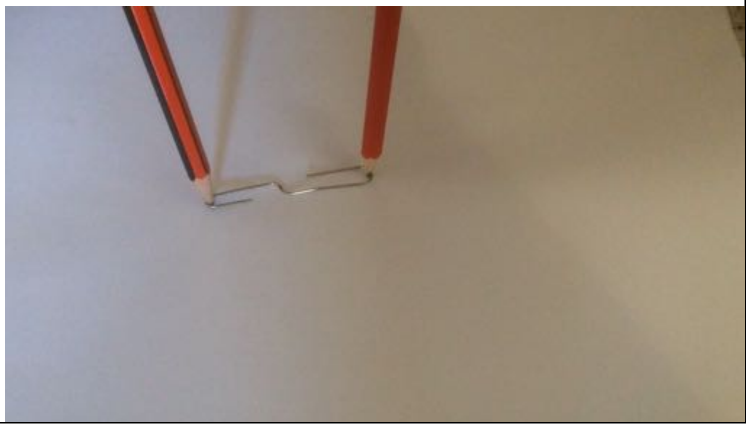
Double click on the picture to start the movie showing how to draw a circle using the paperclip compass.

Or go to:

https://youtu.be/iewxclQEAnk

Draw some circles with a paperclip compass.

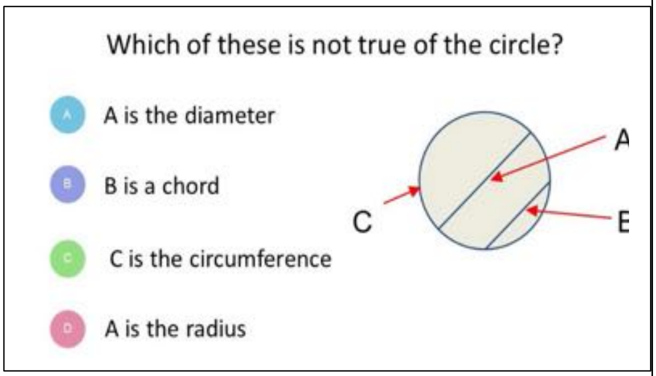
Make some circle patterns.



DIAGNOSTIC ASSESSMENT SUITABLE FOR YEARS 6 – 9

This can be done before or after the lesson, and as a group as described below, or the question can be answered individually. Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: D https://diagnosticquestions.com

Lower Secondary **Finding the perpendicular bisector of a chord.**

Draw round a large plate, cut out a paper circle and mark the centre O.

(i)Fold over a segment of the circle to make a chord. Mark the ends A and B.

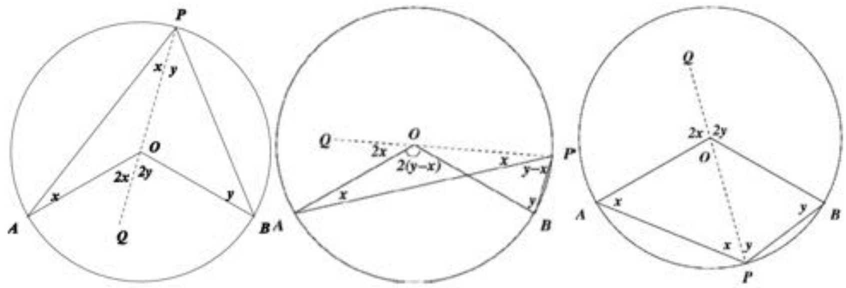
(ii) Fold the circle so that A and B come together. Unfold and mark the points where this line cuts the circle as C and D.

(iii) Draw the line segments (chords) AC, CB, BD and DA. Measure the angles and lengths of the quadrilateral ACBD? What do you notice?

(iv) If AB and CD intersect at P, what do you know about AP and PB? What about angle CPA?

(v) How is CD related to AB? Is CD a diameter of the circle? Explain.

Theorem The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.



Three diagrams are necessary here to show the possible configurations but the proof of the theorem is the same in each case.

Explain why the angles marked x are all equal.

Explain why the angles marked y are all equal.

Explain why ∠AOQ = 2*x* and ∠BOQ = 2*y.*

Key questions

- Can you explain what you have done so far?
- Do you think that this would work with other sketches or drawings?
- Did you use any new words today? What do they mean? How do you spell them?

SOLUTION

Activity 1: O is the centre because the intersection point of the two diameters bisect each diameter therefore OA, OB, OC and OD are all radii.

Activity 2: CD is the perpendicular bisector of a chord AB
CD is the diameter as it goes through the centre.

Activity 3: The angle at the centre is twice the size of the angle at the circumference if they are both subtended by the ***same*** arc or chord. Notice that this is true both for the obtuse angle at O and also for the reflex angle. These angles add up to 360^{o} and proves that the opposite angels of a cyclic quadrilateral add up to 180^{o} (supplementary).

Upper Secondary
Accept the following as axioms: (i) results established in earlier grades, (ii) a tangent is perpendicular to the radius drawn at its point of contact with the circle.
Similar paper folding activities can easily be devised that illustrate Theorem 6 and 7 below. **Investigate these theorems of the geometry of circles by working through the paper folding exercises on pages 1 and 2. Then prove the theorems formally:** 1. *the line drawn from the centre of a circle, perpendicular to a chord, bisects the chord, and its converse theorem 2;* 2. *the perpendicular bisector of a chord passes through the centre of the circle;* 3. *the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle;* 4. *angles subtended by a chord of the circle on the same side of the chord are equal and its converse;* 5. *the opposite angles of a cyclic quadrilateral are supplementary and its converse;* 6. *two tangents drawn to a circle from the same point outside the circle are equal in length;* 7. *the angle between a tangent, and a chord drawn to the point of contact of the tangent, is equal to the angle that the chord subtends in the alternate segment of the circle, and conversely.*

Why do this activity?

These paper folding activities provide experiences of discovering the properties of circles theorems. Folding cut-out paper circles does not prove the theorems, but ideas for the formal proofs are suggested in the diagrams themselves.

Learning objectives

In doing this activity students will have an opportunity to:

- gain a deeper understanding of the geometrical properties of circles;
- build their vocabulary about circle theorems;
- visualise the positions and relationships between the lines and angles in the circle, and the triangles they make, and so gain insights that will help them to understand the proofs of the theorems.

Generic competences

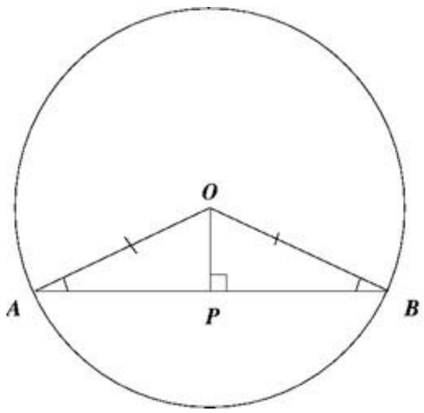
In doing this activity students will have an opportunity to develop problem solving skills and to:

- see relationships and connections that help in solving the problem.
- develop logical/critical thinking;
- develop visualization skills that enable them to form mental images and to think in pictures relating to a problem.

SOLUTION

Theorem 1

The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.



Proof

In ΔOAP and ΔOBP

OA=OB (radii)

∠OPA=∠OPB=90^0 (as OP is perpendicular to chord AB)

OP is a common side

ΔOAP≡ΔOBP RHS

So AP=PB

Converse If AP=PB then OP⊥AB

Proof ΔOAP≡ΔOBP (SSS)

So ∠OPA=∠OPB=90^0

Theorem 2

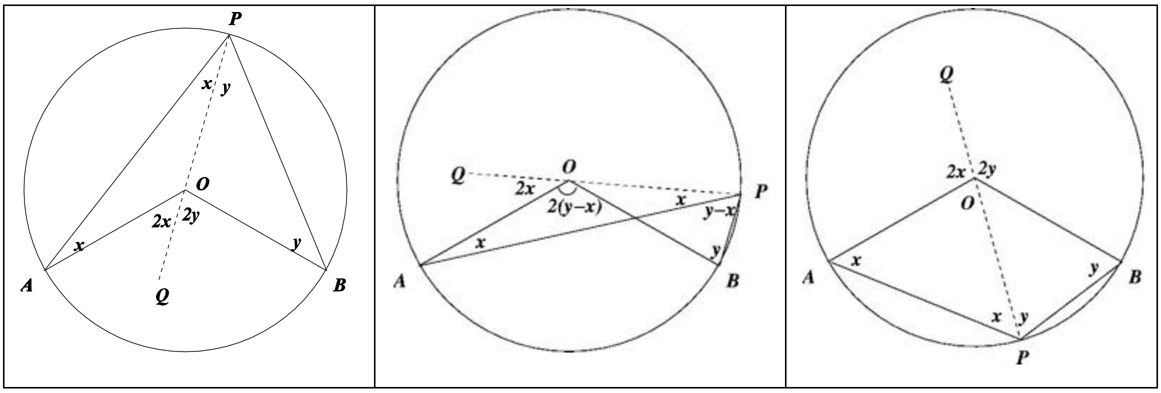
The perpendicular bisector of a chord passes through the centre of the circle.

Proof

The perpendicular bisector of the chord AB is the locus of points equidistant from A and B and so it contains the centre of the circle.

Theorem 3 (The theorem in the NEXT section on page 3.)

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle.



Proof

Three diagrams are necessary here to show the possible configurations but the proof is the same in each case.

As the radii are equal and the base angles of an isosceles triangle are equal:

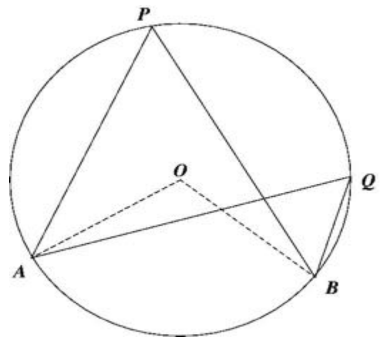
∠OAP=∠OPA=x and ∠OBP=∠OPB=y.

As the exterior angle of a triangle equals the sum of the two interior opposite angles:

∠AOQ=2x and ∠BOQ = 2y.

Hence ∠AOB = 2∠APB.

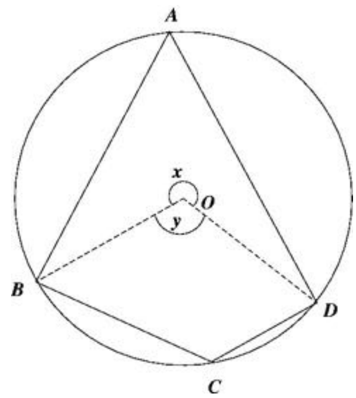
Theorem 4 The angles subtended by a chord at the circle on the same side of the chord are equal.



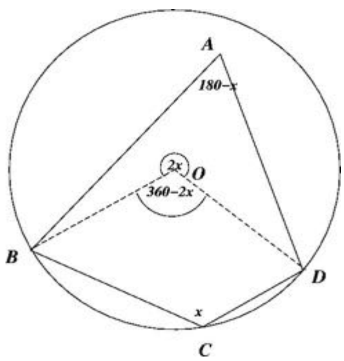
Proof
Using Theorem 3
∠APB = ∠AQB = ½ ∠AOB

Converse Given a line segment AB if the point P moves such that ∠APB is constant, and P stays on the same side of AB, then P lies on the arc of a circle.

Theorem 5
The opposite angles of a cyclic quadrilateral are supplementary (that is they add up to 180°).



Proof
By Theorem 3
∠BAD = ½ y
∠BCD = ½ x
x + y = 360°
Hence ∠BAD + ∠BCD = 180°.

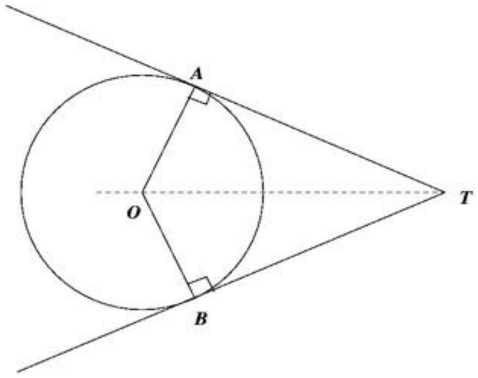


Converse If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.

Proof
Let the opposite angles be x and 180-x.
Draw a circle through BCD (not assuming it goes through A). Join OB and OD.
By Theorem 3 ∠BOD = 2x .
Hence the complementary angle at O, on the opposite side to A, is ∠BOD = 360-2x

By the converse of Theorem 4, the circle subtended by chord BD is the locus of points P such that
∠BPD = 180- x.
Hence A must lie on this circle so the quadrilateral ABCD is cyclic.

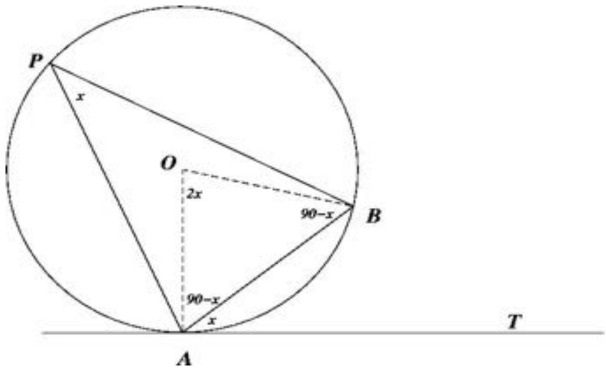
Theorem 6
Two tangents drawn to a circle from the same point outside the circle are equal in length.



Proof
In ΔOAT and ΔOBT
$OA = OB$
$\angle OAT = \angle OBT = 90^o$
OT is common to both triangles
$\Delta OAT \equiv \Delta OBT$ *(RHS)*

Hence TA = TB

Theorem 7
The angle between a tangent, and a chord drawn to the point of contact of the tangent, is equal to the angle that the chord subtends in the alternate segment of the circle.



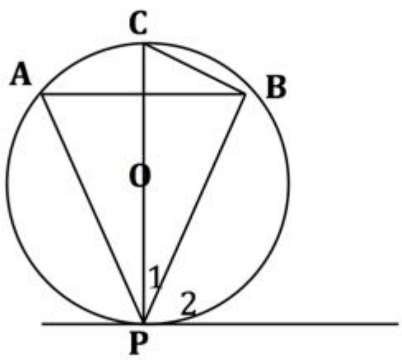
Proof
Let $\angle BAT = x$, then $\angle OAB = 90 - x$ because the radius is perpendicular to the tangent.

Triangle OAB is isosceles and so $\angle OBA = 90 - x$ and $\angle AOB = 2x$.

By Theorem 3, $\angle APB = x$.
Hence $\angle BAT = \angle APB = x$.

Converse If $\angle BAT = \angle APB$ then AT is a tangent to the circle at A.
Proof
If $\angle APB = x$ then $\angle AOB = 2x$ by Theorem 3.
Triangle OAB is isosceles and so $\angle OAB = \angle OBA = 90 - x$
As we have $\angle BAT = x$ this gives $\angle OAT = 90^o$ so $OA \perp AT$. Hence AT is a tangent to the circle at A.

An Alternative Proof of this Theorem.
Circle with centre O; PT a tangent at P; PB a chord and A is a point on the circumference.
Required to prove: $\angle BPT = \angle \mathbf{B}\hat{A}P$



Construction Draw diameter PC and join CB.
Proof: $\angle P_1 + \angle P_2 = 90^\circ$ (Tan $\perp$ diameter)
$\angle CBP = 90^\circ$ ($\angle$ in semi – circle)
$\therefore \angle P_1 + \angle C = 90^o$ ($\angle$ s of Δ CBP)
$\therefore \angle P_2 = \angle C$
$\angle C = \angle A$ (Subtended by PB)
$\therefore \angle P_2 = \angle A$

Follow up

Surprise Ratios https://aiminghigh.aimssec.ac.za/years-9-12-surprise-ratios/

Kissing Circles https://aiminghigh.aimssec.ac.za/years-11-12-kissing-circles/

Construct with Circles
https://aiminghigh.aimssec.ac.za/years-7-9-construct-with-circles/

Circle Inscribed in a Quadrilateral
https://aiminghigh.aimssec.ac.za/years-10-12-circle-inscribed-in-quadrilateral/

Cyclic https://aiminghigh.aimssec.ac.za/years-11-12-cyclic/

Salinon https://aiminghigh.aimssec.ac.za/years-9-11-salinon/

Polycircles https://aiminghigh.aimssec.ac.za/years-11-12-polycircles/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13