TIMES NINE

Complete these calculations.

What do you notice?

To explain how the patterns of numbers arise:

Either change from multiplying by 9 to multiplying by (10 – 1)

or use the table below first completing the calculations on each line.

The picture shows the pyramids in the background because this number pattern, and others like, it were known to the ancient Egyptians.

100 000 000 × 9 =	
20 000 000 × 9 =	
3 000 000 × 9 =	
400 000 × 9 =	
50 000 × 9 =	
6 000 × 9 =	
700 × 9 =	
80 × 9 =	
9 × 9 =	
123 456 789 × 9 =	

HELP

The Game is to help you to understand how and why the pattern works – there is no competition here. Play with a friend if you can.

PLAY THE GAME: Cut out the strips on page 3.

Write down the multiples of 9 from 9 to 99 before you start.

Mix up the strips.
Fill in the answer in the last box on the right hand end of each strip.
Arrange the strips in order.

Talk about how the patterns come about and why they occur.

NEXT

What do you notice about this pattern of numbers?

Can you explain why the pattern occurs?

Work out the numbers to replace the ? marks.

? × 9 + 7 = 88
? × 9 + 6 = 888
? × 9 + 5 = 8888
? × 9 + 4 = 88888
? × 9 + 3 = 888888
? × 9 + 2 = 8888888
? × 9 + 1 = 88888888
? × 9 + 0 = 888888888

COPY AND CUT OUT THESE CARDS FOR THE CLASS GAME SET 1

$0 \times 9 + 1 =$	
$1 \times 9 + 2 =$	
$12 \times 9 + 3 =$	
$123 \times 9 + 4 =$	
$123\ 4 \times 9 + 5 =$	
$123\ 45 \times 9 + 6 =$	
$123\ 456 \times 9 + 7 =$	
$1\ 234\ 567 \times 9 + 8 =$	
$12\ 345\ 678 \times 9 + 9 =$	

123 456 789 × 9 + 10 =	

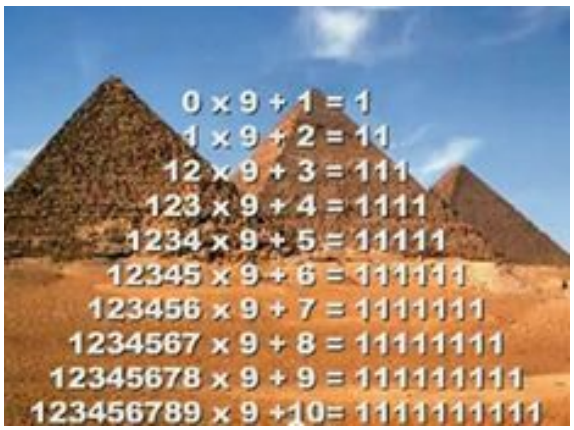
COPY AND CUT OUT THESE CARDS FOR THE CLASS GAME SET 2

0 × 9 + ? =	1	? =
1 × 9 + ? =	11	? =
12 × 9 + ? =	111	? =
123 × 9 + ? =	1 111	? =
123 4 × 9 + ? =	11 111	? =
123 45 × 9 + ? =	111 111	? =
123 456 × 9 + ? =	1 111 111	? =
1 234 567 × 9 + ? =	11 111 111	? =

12 345 678 × 9 + ? =	111 111 111	? =
123 456 789 × 9 + ? =	1 111 111 111	? =

NOTES FOR TEACHERS

SOLUTION



You could explain why this pattern appears in different ways.

If we think of multiplying by 9 as multiplying by 10-1 then, for example

12 345 678 × (10 – 1) + 9

= 123 456 789 – 12 345 678

= 111 111 111

Looking at another calculation we see the pattern of 1's appearing again:

12 345 × (10 – 1) + 6

= 123 456 – 12 345

= 111 111

The table below shows part of the last calculation

100 000 000 × 9 =	900 000 000
20 000 000 × 9 =	180 000 000
3 000 000 × 9 =	27 000 000
400 000 × 9 =	3 600 000
50 000 × 9 =	450 000
6 000 × 9 =	54 000
700 × 9 =	6 300
80 × 9 =	720
9 × 9 =	81
123 456 789 × 9 =	**1 111 111 101**

Another way to explain the patterns uses the fact that the patterns come from the 9 times table, that is multiples of 9:

9, 18, 27, 36, 45, 54, 63, 72, 81

We see the bracketed pairs of numbers in the same list all add up to 10:

(9, 1)(8, 2)(7, 3)(6, 4)(5, 5)(4, 6)(3, 7)(2, 8)1

or, re-writing these pairs:

9
18
27
36
45
54
63
72
81

So, as in the table, we have these pairs of numbers occurring in the units, tens, hundreds, thousands, ten thousands … places where each column adds up to 10 with 1 carried over from the column on the right of it.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Which of the follow is equivalent to $201 \times 9 - 9 \times 102$?

A. 99×9
B. 101×9
C. 0
D. 9

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: A "I chose A because I subtracted 201 and 102 and got 99"
Possible misconceptions – all quotes are reasons for these answers given by learners.
B. "its almost halving each other"
C. "you take away what you have calculated therefore it equally 0"
D. "201 times 9 is 1809 -1818 is 9"

Why do this activity?

Learners get practice in doing calculations and the activity re-enforces their understanding of the concept of place value. The pattern is pleasing. Explaining why the pattern occurs develops learners' logical reasoning and number sense. The calculations can be done by multiplying by 9 for each line first, then doing the additions, or doing the whole calculation line by line.

This can be game for a whole class as described below. Or all the calculations can be done as a class effort where a learner gives an answer and then names the learner to give the next answer.

The activity can be structured to cater for everyone in a class; simpler calculations and more challenging tasks like giving explanations and doing longer calculations and can be distributed by the teacher according to the abilities of the learners to cater for all attainment levels.

Learning objectives

In doing this activity students will have an opportunity to:

- practise multiplication, calculation and solving equations;
- develop visualisation, logical reasoning and number sense;
- re-enforce understanding of the concept of place value.

Generic competences

In doing this activity students will have an opportunity to practise pattern spotting and explaining why the patterns occur, which has applications to solving problems in a range of situations.

Suggestions for teaching

Start the lesson with discussion about this 16th Century Nursery Rhyme. Explain that this is about counting in fives and the lesson is about counting in nines.

Fingers and Toes

16th Century Nursery Rhyme

Every Lady in this Land

Hath 20 nails on *each* Hand;

Five & twenty on Hands and *Feet*;

And this is true, without deceit.

Different punctuation changes the meaning:

Every lady in this land
hath 20 nails.
On each hand five; and
twenty on hands and feet.

LEARNING ACTIVITY FOR INDIVIDUAL LEARNERS OR PAIRS

The question can be set to learners as on page 1.

When most of the learners have completed the calculations the teacher can ask learners to come and complete the calculations on the board, one line at a time and then lead a whole class discussion of the patterns in these calculations and why these patterns occur.

GAME FOR THE WHOLE CLASS

Copy the sets of cards from pages 4 and 5 that you see illustrated below. Cut out the strips (rows in the table) and give them out to the class so that each individual or pair of learners gets one strip. Then ask the learners to complete the calculations on the strip that they have been given. The first few calculations are much easier and will be done more quickly, and SET 1 cards are easier than SET 2, so you can give these cards to the learners who are likely to be slower. Also you could give out the first 3 or 4 lines together depending on the number in your class. For example, for a class of 28 learners you could cut out the first 4 rows (strips) together as shown giving 14 cards in all. Give these out to 14 pairs of learners.

SET 1

0 × 9 + 1 =	
1 × 9 + 2 =	
12 × 9 + 3 =	
123 × 9 + 4 =	
123 4 × 9 + 5 =	
123 45 × 9 + 6 =	
123 456 × 9 + 7 =	
1 234 567 × 9 + 8 =	
12 345 678 × 9 + 9 =	
123 456 789 × 9 + 10 =	

SET 2

0 × 9 + ? =	1	? =
1 × 9 + ? =	11	? =
12 × 9 + ? =	111	? =
123 × 9 + ? =	1 111	? =
123 4 × 9 + ? =	11 111	? =
123 45 × 9 + ? =	111 111	? =
123 456 × 9 + ? =	1 111 111	? =
1 234 567 × 9 + ? =	11 111 111	? =
12 345 678 × 9 + ? =	111 111 111	? =
123 456 789 × 9 + ? =	1 111 111 111	? =

While the learners are busy draw up a table on the board filling in each line up to and including the + sign but leaving the rest of the calculation to be filled in.
When most of the learners have completed the calculations tell them to get up from their places and find the pair of learners who have a matching calculation. This will give the learners a chance to check with their new partner to see if their calculations were

correct. Then ask learners to come and complete the calculations on the board, one line at a time. The learners can remain standing in small groups or, if convenient, the learners could sit down with their new partners who have the matching calculation.

Then lead a whole class discussion of the patterns in these calculations and why these patterns occur.

Key questions

- How do the multiples of 9 appear in the calculations?
- How do the multiples of 9 appear in the pattern?
- What do you notice?
- Can you explain why the pattern comes out like that?

Follow up

Two by Two Puzzle https://aiminghigh.aimssec.ac.za/years-4-7-two-by-two-puzzle/
Multiplication Squares
https://aiminghigh.aimssec.ac.za/years-4-7-multiplication-squares/
Magic 1387 https://aiminghigh.aimssec.ac.za/years-6-10-magic-13837/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13