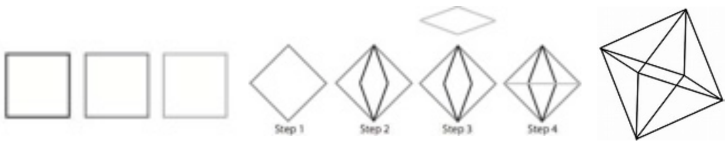
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

TETS AND OCTS PUZZLES

First make 6 paper sticks, all the same length, by rolling paper tightly round pieces of string and securing with sticky tape.
Cut a piece of string long enough to overlap the paper at both sides.
Stick string to the shorter edge of the paper to avoid it slipping out of the roll.
Roll up paper very tightly keeping the edges straight.
Secure the rolled paper with more sticky tape.
Then make a rhombus by tying together 5 sticks to make 2 equilateral triangles.

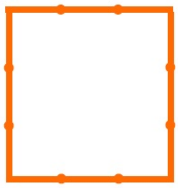
TET PUZZLE: Use the rhombus and the one remaining stick to make a shape with 4 triangles.
What do you notice about the shape that you have made?

OCT PUZZLE 1: For the second puzzle make 12 paper sticks and tie them together



making 3 squares, preferably in 3 colours. Make the 3 squares into the skeleton of a solid shape that has 8 triangular faces.

What do you notice about this solid shape? Make a list.



OCT PUZZLE 2:
For the third puzzle, make an octahedron from a square made from 12 sticks, or starting from a continuous line of 12 sticks.

HELP

Make the two models. All the sticks used should be the same length.

When you make the TET there should be 3 sticks at each vertex.

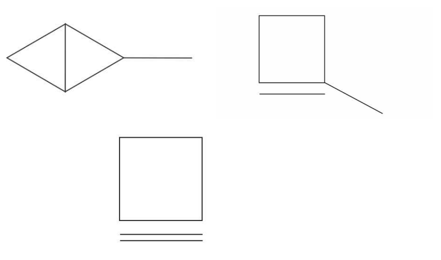
What do you notice about the faces?

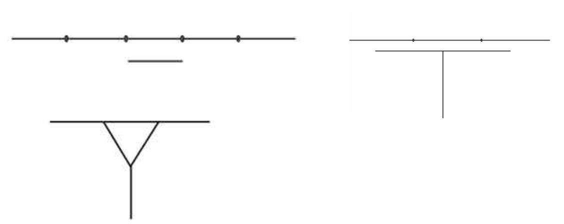
Why are these solids described as **regular solids.**

Count the faces, edges and vertices.

NEXT

Here are 6 more puzzles. Make each of the 6 patterns as shown into a regular tetrahedron by tying the sticks together.





NOTES FOR TEACHERS

SOLUTION

The diagram shows the solution to the first puzzle, a tetrahedron with 4 triangular faces.

The tetrahedron **has 4 faces, 6 edges and 4 vertices**.

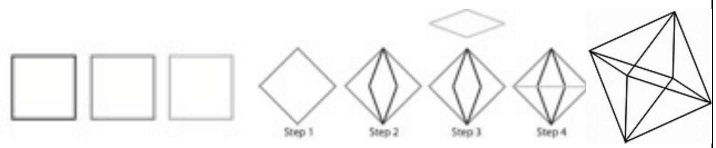
If all the edges are the same length making 4 equilateral triangles then the tetrahedron is **regular** so it is **one of the Platonic solids.** The other Platonic solids are the cube, octahedron, icosahedron and dodecahedron.

One half of the tetrahedron is **reflected in the plane cutting it through an edge and the midpoint of the opposite edge**. There are 4 planes of reflection (mirror planes)

The tetrahedron has rotational symmetry of order 3 about axes of rotation through a vertex and the centre of the opposite face and **rotational symmetry of order 2** about axes of rotation through the midpoints of opposite edges.

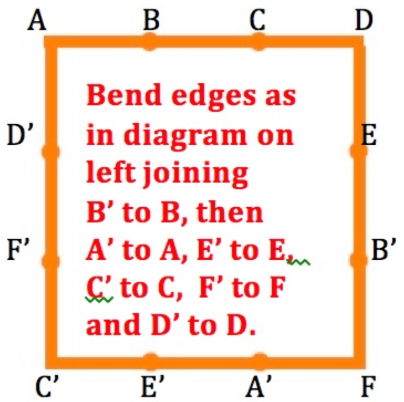
OCT PUZZLE 1.

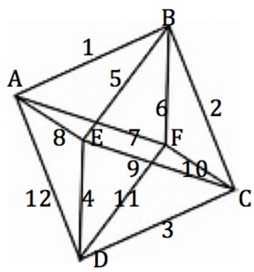
By making the octahedron from 3 squares we see that these squares act as mirrors (or planes of reflection).



Each half of the octahedron is a square based pyramid and it is **reflected in one of these planes.**

Solution to **OCT PUZZLE 2.**

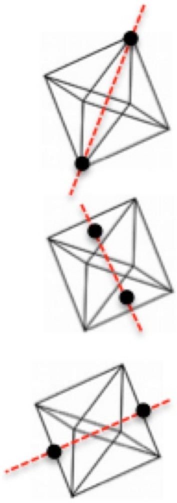




This diagram shows the order in which the edges are formed to make the octahedron.

The octahedron **has 8 faces, 12 edges and 6 vertices**.

If all the edges are the same length making 8 equilateral triangles then the octahedron is **regular.**



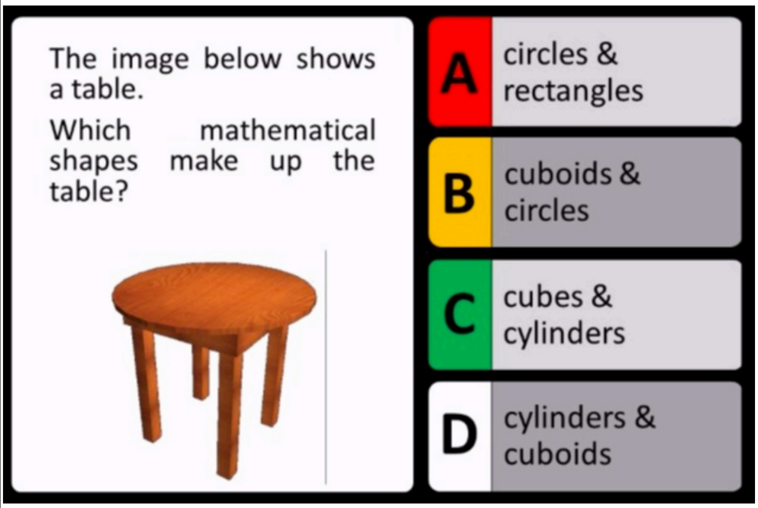
The octahedron has rotational symmetry of order 4 about lines through each pair of opposite vertices.

It has **rotational symmetry of order 3** about lines through each pair of centres of opposite faces.

It has **rotational symmetry of order 2** about lines through each pair of midpoints of opposite edges.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. It is important for learners to explain the reason for their answer. Putting thoughts into words and help to clarify understanding and develop communication skills.
4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
5. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
6. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer D
A. Students only recognise 2D shapes and not 3D shapes.
B. Students can see that the legs are cuboids. For the top they can't tell the difference between a circle and a cylinder or disc.
C. Students are muddling the names of cubes and cuboids. https://diagnosticquestions.com

Why do this activity?

Learners enjoy this practical activity which helps them to visualise the 3D shapes and to understand and remember their properties. Some learners who often struggle with mathematics may be able to do better with this activity because it is concrete and practical. The activity builds on knowledge and understanding of properties of 2D shapes and also connects to work on transformations (reflections and rotations).

Learning objectives

In doing this activity students will have an opportunity to:

- deepen their understanding of the properties of 3D shapes;
- develop problem solving skills;
- (for older learners) deepen their understanding of symmetry.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts;
- **persevere and work systematically** to **solve problems**;
- **communicate** in writing, speaking and listening, present information and ideas to others and record ideas effectively.

Suggestions for teaching

Start with the Diagnostic Quiz to introduce the idea of 3D objects. Instead of the picture of a table you could substitute other solid objects that you have available.

The activity has 3 parts: the tetrahedron and the two puzzles involving the octahedron. You may choose to use them in the same lesson or in different lessons. With younger learners, just make the models and describe them but leave out the ideas about symmetry.

Be sure to tell your learners to tie the sticks together in a bow so that they can be undone (like shoelaces). This is important as the learners may want to correct their mistakes.

Once they have their 3D polyhedra to look at and turn around in their hands, ask the learners to make a list of all the properties that they can think of. Then, in a class discussion, get the learners to say what they have noticed and ask the key questions below to prompt them to think of all the properties. With older learners you may choose to ask about the symmetries.

Making the octahedron from a chain of 12 sticks is quite challenging and could be given as an extension exercise for the quickest learners. You could have several of these chains available for learners to play with from time to time. Some learners will take up the challenge and even repeat the performance over and over to become more proficient.

Key questions

- How many faces are there altogether?
- How many edges are there altogether?
- How many vertices are there altogether?
- Do you remember the name for this? What is it called?

Follow up for older learners

About reflections and reflective symmetries of the tetrahedron:

Imagine cutting your tetrahedron in half. How would you do that? Show me.

Imagine one half of your tetrahedron is a reflection of the other half, where would the mirror be?

The mirror is called a plane of symmetry. How many planes of symmetry can you find? Can you show me? Describe them.

What happens if you cut through an edge and the midpoint of the opposite edge?

About rotations and rotational symmetries of the tetrahedron:

Turn the tetrahedron around.
Can you find any rotational symmetry?
Show me a vertex. Now show me the centre of the opposite face.
Hold those 2 points.
Now turn the tetrahedron round slowly. Is there any rotational symmetry?
How many times do you turn it? Are you turning it through the same angle each time?
What is the angle?
What is the order of that rotational symmetry? **(Answer 120^{o}, order 3)**

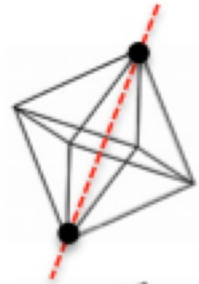
Show me the midpoint of an edge. Now the midpoint of the opposite edge.
Hold those 2 points.
Now turn the tetrahedron round slowly. Is there any rotational symmetry?
How many times do you turn it? Are you turning it through the same angle each time?

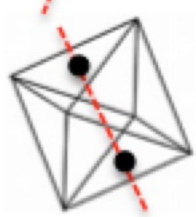
While asking these questions you, or a learner, can pick up the model and show where the cuts are made for planes of symmetry

or

rotate the model to show rotational symmetry.

What is the angle?
What is the order of that rotational symmetry?
(Answer 180°, order 2)

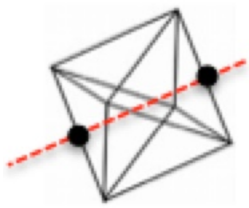




About reflections and reflective symmetries of the octahedron

Imagine cutting your octahedron in half. How would you do that? Show me.
Imagine one half of your octahedron is a reflection of the other half, where would the mirror be?
The mirror is called a plane of symmetry. How many planes of symmetry can you find?
Can you show me? Describe them.

About rotations and rotational symmetries of the octahedron:



Hold your model at two opposite vertices, turn it round slowly and stop when it gets to fill the same position in space.
How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
What is the order of that rotational symmetry?
(Answer 90°, order 4)

Hold your model at the centres of two opposite faces. turn it round slowly and stop when it gets to fill the same position in space.
How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
What is the order of that rotational symmetry? **(Answer 120°, order 3)**

Hold your model at the midpoints of two opposite edges, turn it round slowly and stop when it gets to fill the same position in space.
How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
What is the order of that rotational symmetry? (**Answer 180°, order 2)**

More Follow up

Two simple starters for younger learners:
https://aiminghigh.aimssec.ac.za/years-4-8-shadows-activity/
https://aiminghigh.aimssec.ac.za/years-4-7-paper-stick-tetrahedron/

Building with tetrahedra:
https://aiminghigh.aimssec.ac.za/years-6-12-sierpinski-number-and-shape-patterns/

A more challenging puzzle but similar to the Tets and Octs Puzzle:
https://aiminghigh.aimssec.ac.za/years-7-10-icosahedron-puzzle/

Making models using nets: https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
More about nets: https://aiminghigh.aimssec.ac.za/years-6-10-cut-nets/

Two puzzles about tetrahedra for older learners:
https://aiminghigh.aimssec.ac.za/grade-9-and-10-tet-trouble/
https://aiminghigh.aimssec.ac.za/years-9-11-triangles-to-tetrahedra/

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the MATHS TOYS YouTube Channel https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13