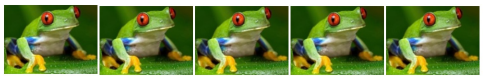
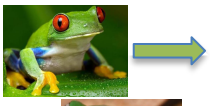
FROGS PUZZLE

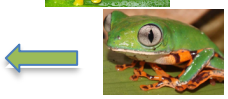



Five red-eye frogs and five orange frogs line up in a row with a space between them.

They must change places.

They can hop one frog over another frog or slide to an empty place next to them.


Red-eye frogs can only move to the right 

Orange frogs can only move to the left 

How can they change places following these rules?

Try the puzzle with 2 frogs on each side first, then with 3 frogs on each side …

How many moves will it take?

HELP

Think about the order in which the frogs should move to avoid blocking in any of the frogs so that they can't move.

NEXT FROGS AND TOADS

This is a competitive game for 2 players with almost the same rules as Frogs. Place some frogs at one end and some toads at the other end of a strip of cells with just one empty space between them. The rules for moving are the same as for frogs. The players take it in turn to move any one of the pieces. A player loses if he cannot move any of the pieces when it is his turn.

When you have tried the puzzle with an equal number of frogs on each side, try it a number on one side different from the other side.

Resources: Cut out paper frogs 2 colours.

NOTES FOR TEACHERS

SOLUTION

The formula for the number of moves for x frogs on one side and y on the other is

$x + y + xy$.

This is $(x + y)$ slides and xy hops.

Proof

Each frog must hop exactly once over all the frogs of the other type (or be hopped over) so there are xy hops.

The x frogs on one side must move $(y+1)$ places in one direction and y frogs on the other side must move $(x+1)$ places in the other direction. So the total number of places moved is $x(y+1) + y(x+1) = x + y + 2xy$.

With each hop a frog moves 2 places so with xy hops the frogs move a total of $2xy$ places. The number of places moved is $x + y + 2xy$ so there are $(x + y)$ slides.

Why do this activity?

Patterns are at the heart of mathematics and, in this activity, learners develop their problem solving skills. The problems to solve are first to find a pattern of moves that will succeed in the 2 sets of frogs swapping places, then secondly to find number patterns in the numbers of moves and thirdly to find a formula (or rule) that gives the number of moves as a function of the numbers of each type of frog. The physical engagement (people maths activity) promotes kinaesthetic learning. If learners work in pairs and small groups they develop their communication and reasoning skills through discussion. Using a table to record results helps learners to work systematically.

Learning objectives

In doing this activity students will have an opportunity to:

- develop logical reasoning and problem solving skills;
- develop communication skills through talk in groups and as a class;
- (for older learners) practise describing a number pattern in an algebraic formula.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically**, reason logically and give explanations**;
- **think flexibly** and apply knowledge and skills;
- interpret and **solve problems**;
- **collaborate and** work with a partner or group.

Suggestions for teaching

This is not a script for a lesson. Adapt the plan according to learners' responses.

- Start as a whole class activity to act out the problem - 3 girl frogs and 3 boy frogs and 7 chairs.

- If they find it difficult try 2 boy frogs and 2 girl frogs on the chairs and make sure everyone understands the rules.
- Perhaps try 2 boy frogs and 3 girl frogs next.
- This is collaborative, not competitive – everyone should try to help the players to make the right move so as to solve the puzzle.
- When they know what they have to do learners can try this in pairs using counters as the frogs.

When learners have worked out the sequence of moves so that they can do the change-overs without getting stuck, suggest that they record their results in a table as below.

Think about simple cases: e.g. 2 boy and 2 girl frogs Fill in a table of results. How many moves? How many hops? How many slides? Is there a pattern? Can you explain the pattern?	Number of boy frogs	Number of girl frogs	Number of hops	Number of slides	Number of moves
	1	2			
	2	2			
	2	3			
	3	3			
	3	4			
	4	4			
	4	5			
	5	5			

Before the end of the session have a whole class discussion about the strategies for solving the puzzle and the number patterns that the learners have found.

Only go as far as the learners are comfortable with. Do not tell them the rule or formula. You can suggest that they might like to play the game again before the next lesson and try to find more patterns and answers, and maybe a rule. The 'rule' can be given in words, it does not have to be in a formula.

For secondary learners this exercise leads towards algebra. A good answer from the learners would be a formula in words like:

"'you multiply the number of frogs of each type to get the number of hops
and you add the number of frogs of each type to get the number of slides".

Some younger 'high flyers', and the year 9 learners, may be able to express the formula algebraically and even give a proof.

- With older players introduce the activity by 5 boys and 5 girls acting it out, with help from the class.
- When they find it too difficult ask for suggestions for working on the problem.
- If the players have experience of problem solving they should suggest **trying simple cases**, if not the teacher will need to suggest it (and point out that this is a **general strategy** for problem solving).

Key questions

- Can you find a formula (rule) in words for the number of hops?
- Can you find a formula (rule) in words for the number of places moved?
- Can you find a formula (rule) in words for the number of slides?
- Can you find a formula (rule) for the number of moves?
- Can you prove the formula always works?
- Can you find a formula in symbols?

Follow up

FROGS AND TOADS

This is a competitive game for 2 players with almost the same rules as Frogs. Place some frogs at one end and some toads at the other end a strip of cells with just one empty space between them. The rules for moving are the same as for frogs. The players take it in turn to move any one of the pieces. A player loses if he cannot move any of the pieces when it is his turn.

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13