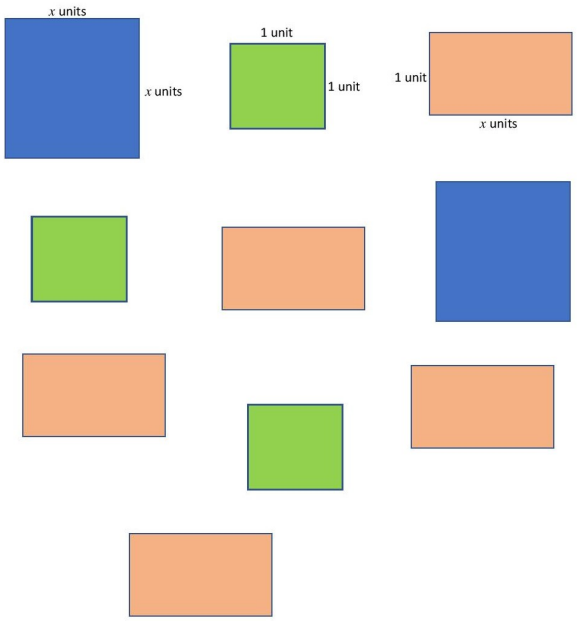
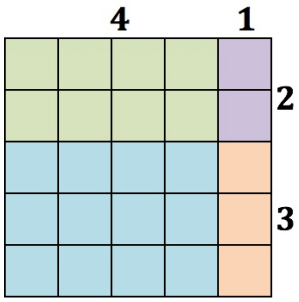
ALGEBRAREA Product of two brackets and area

1. Describe the three different types of pieces shown in the diagram and find their areas in square units.
2. Using **all ten pieces**, make one BIG shape, draw it in your notebook and find its area.
3. Find as many different BIG shapes as you can made with the 10 pieces joined edge to edge, including a rectangle. Draw them and find their areas.
4. What do you notice about the areas of the BIG shapes?
5. What mathematical relationships can you find from the area of the rectangle made with the 10 pieces?



HELP



Using this diagram, and counting squares, explain how you would work out (4 + 1) multiplied by (2 + 3) in different ways.

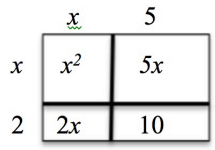
Apply your newly developed method to find different ways to write down the area of the big rectangle made from the 10 pieces shown above.

If you are having difficulties then think about the importance of subdividing the geometrical shapes into simpler units and then finding their areas.

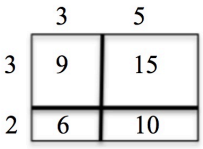
See https://aiminghigh.aimssec.ac.za/grades-7-to-9-partitioning/

NEXT

Compare the algebraic expression $(x + 2)(x + 5)$ to $(3 + 2)(3 + 5)$.



$(x + 2)(x + 5)$
$= x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$



$(3 + 2)(3 + 5)$
$= 9 + 15 + 6 + 10$
$= 40$

Explain how these diagrams illustrate the products of the binomial expressions.

Make up some examples for yourself and work out more problems involving expansion of brackets where all the terms are positive, such as $(3x + 2)(2x + 3)$, using a diagram if you find it helpful.

When you are confident about the method without needing to draw a diagram, then multiply binomials with negative terms such as $(2x - 3)(2x + 4)$ and $(3x - 2)(4x - 3)$. Remember the rules that multiplying two positive numbers or two negative numbers gives a positive number and multiplying a positive and a negative gives a negative number.

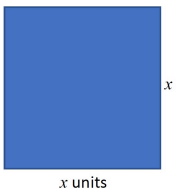
NOTES FOR TEACHERS

SOLUTION

1. The geometrical shapes that can be identified are squares and rectangles (2D shapes):

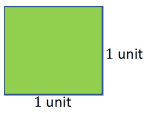
3 identical small squares (green)
2 identical bigger squares (blue)
5 identical rectangles (brown)

Identical shapes are said to be **congruent.** Teachers can discover whether the learners are familiar with the terminology.



Blue square:

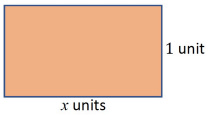
Area = x units × x units
= x^2 square units



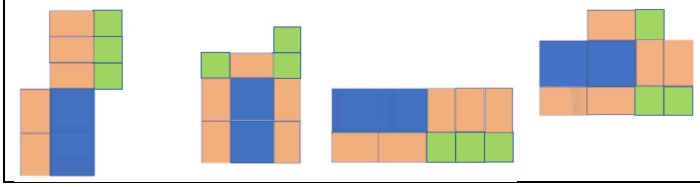
Green square:

Area = 1 unit × 1 unit
= 1 square unit

Brown rectangle:



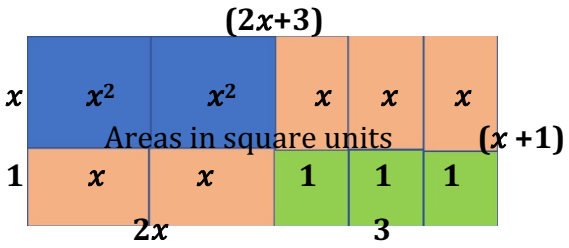
Area = x units × 1 unit
= x square units



2 and 3. Some images of possible combinations.
All have area $2x^2 + 5x + 3$

4. There is **conservation** of area in this case; logically, we are using the same pieces whose area remains the same.
Each shape should give us the following results:
Area = $x^2 + x^2 + x + x + x + x + x + 1 + 1 + 1$ units^2
= $2x^2 + 5x + 3$ units^2

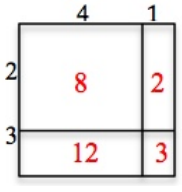
5.
Area of rectangle = Length × Width
= $(2x + 3)(x + 1)$
square units.



From 4. the area was found to be $2x^2 + 5x + 3$
From 5. the area is $(2x + 3)(x + 1)$.
This gives the mathematical relationship $(2x + 3)(x + 1) = 2x^2 + 5x + 3$

HELP SECTION

$(2 + 3)(4 + 1) = (5)(5) = 25$
$(2 + 3)(4 + 1) = 2(4 + 1) + 3(4 + 1) = 2(5) + 3(5) = 10 + 15 = 25$
or $(2 + 3)(4 + 1) = 8 + 2 + 12 + 3 = 25$ as shown by the diagram.

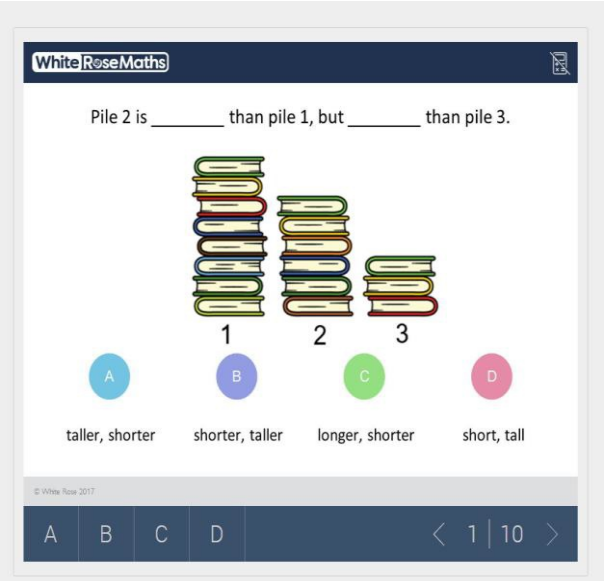


$(2x + 3)(x + 1) = 2x(x + 1) + 3(x + 1)$
$= 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer again by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is **B.**

Learners choosing other answers do not understand the language used. **https://diagnosticquestions.com**

Why do this activity?

In order to develop conceptual understanding of the product of two binomials, this activity links algebra with area problems in some common 2D shapes like squares and rectangles. Learners develop skills of manipulating a product of two binomials to obtain a quadratic expression; work which prepares them for solution of quadratic equations that follow in subsequent lesson activities.

The activity consolidates learner's knowledge and understanding of conservation, and of tessellation, in geometry, especially when they are playing with the pieces to obtain different 2D shapes. For those who will go into the construction industry or do their own home improvement, for example, the work will resonate with problems of tiling floors and laying patio tiles.

The activity is a preparation for work that involves the application of quadratic equations in solving area problems in 2D shapes; we focus on the connections between areas of simple geometrical shapes like squares and rectangles and the product of two binomials.

This activity becomes useful as prior knowledge for learners when they get into topics like 'nets' of 3D shapes. The connection between geometry and the algebraic terrain cannot be over emphasized.

Learning objectives

In doing this activity students will have an opportunity to:

- construct quadratic expressions through area problems in 2D shapes;
- calculate areas of squares and rectangles using dimensions given in algebraic form;
- establish a mathematical relationship between area (algebraic expression) and product of two binomials;
- find the product of two binomials by expanding the brackets;
- develop deep conceptual understanding of the construction of quadratic expressions by the product of two binomials.

Generic competences

In doing this activity students will have an opportunity to:

- develop algebraic manipulative skills and to recognize the equivalence between different representations of the same relationship;
- co-operate, collaborate and work in a team;
- have empathy with others, listen to different points of view.

Suggestions for teaching

As preparation for this work you might like the class to do the Partitioning learning activity https://aiminghigh.aimssec.ac.za/partitioning/ first.

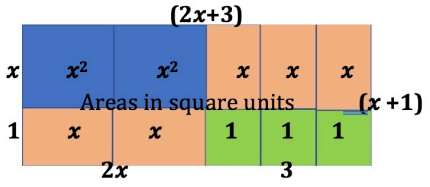
Try to make the lesson as practical and engaging as possible. Learners can either work individually in their exercise books or work in groups of two to four students per group.

To enable all learners to tackle this question and experience succes (planning for differentiation) you can give the HELP section to those learners who struggle to get started and give the NEXT section to learners who finish ahead of the rest of the class.

Let learners explore all the options; there are still more combinations they can come up with other than those shown in the solutions section. When most of the class have found several combinations and worked out the areas the teacher can conduct a class discussion. Learners could be asked to draw on the board the different combinations that they have found.

Adding the areas of the individual pieces is a common strategy for finding the total area. Ideally, most learners should notice that ALL the shapes have the same area. Most learners should easily notice that the **rectangle** can be used to calculate the area using the length and width of the shape.

Ask the class why the shapes all have the same area. It may be obvious to some learners that logically, we are using the same pieces whose area remains the same. Let the learners explain their reasons. We call this conservation of area.



This diagram and the solutions to parts 4 and 5 and the HELP section make a good summary of lesson.

Key questions

- Can you make a rectangle with the 10 pieces?
- Can you subdivide the rectangle into 4 simpler subunits and find the 4 areas?
- You have made some different shapes using the 10 pieces. Do you notice anything about the areas?
- Could you draw a rectangle to multiply 28 by 34 and multiply the tens separately from the units?
- Suppose you need to multiply $(p + q)$ by $(r + s)$, can you draw a rectangle with edges $(p + q)$ and $(r + s)$ and find 4 areas: and in this way find the product of the two expressions?
- Suppose you need to multiply $(3x + 2)$ by $(4x + 5)$, can you draw a rectangle with edges $(3x + 2)$ and $(4x + 5)$ and find 4 areas and in this way find the product of the two expressions?
- Can you apply the distributive law to multiply a binomial by another binomial?

Follow up

PRIMARY

MD https://aiminghigh.aimssec.ac.za/md/

Multiplication Squares
https://aiminghigh.aimssec.ac.za/multiplication-squares/

Two by Two Puzzle https://aiminghigh.aimssec.ac.za/two-by-two-puzzle/

Can you help these farmers?
https://aiminghigh.aimssec.ac.za/can-you-help-these-farmers/

LOWER SECONDARY

Partitioning https://aiminghigh.aimssec.ac.za/partitioning/

Pair Products https://aiminghigh.aimssec.ac.za/pair-products/

Use Area to find x https://aiminghigh.aimssec.ac.za/use-area-to-find-x/

Muggles Magic https://aiminghigh.aimssec.ac.za/muggles-magic/

UPPER SECONDARY

Quadratic Matching 1 and Quadratic Matching 2
https://aiminghigh.aimssec.ac.za/quadratic-matching-1/
https://aiminghigh.aimssec.ac.za/quadratic-matching-2/

Quadratic Equations
https://aiminghigh.aimssec.ac.za/quadratic-equations/

Quadratic Functions
https://aiminghigh.aimssec.ac.za/quadratic-functions/

Graphing Quadratic Equations
https://aiminghigh.aimssec.ac.za/graphing-quadratic-equations/