AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

NUMBER PATTERNS is the theme

for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 4 to 17+

Just choose whatever seems suitable for your group of learners

The TIMES NINE activity was designed for Upper Primary and Secondary

TIMES NINE

Complete these calculations.

What do you notice?

To explain how the patterns of numbers arise:

Either change from multiplying by 9 to multiplying by (10 – 1)

or use the table below first completing the calculations on each line.

The picture shows the pyramids in the background because this number pattern, and others like, it were known to the ancient Egyptians.

100 000 000 × 9 =	
20 000 000 × 9 =	
3 000 000 × 9 =	
400 000 × 9 =	
50 000 × 9 =	
6 000 × 9 =	
700 × 9 =	
80 × 9 =	
9 × 9 =	
123 456 789 × 9 =	

HELP

The Game is to help you to understand how and why the pattern works – there is no competition here. Play with a friend if you can.

PLAY THE GAME: Cut out the strips on page 3.

Write down the multiples of 9 from 9 to 99 before you start.

Mix up the strips.

Fill in the answer in the last box on the right hand end of each strip.

Arrange the strips in order.

Talk about how the patterns come about and why they occur.

NEXT

What do you notice about this pattern of numbers?

Can you explain why the pattern occurs?

Work out the numbers to replace the ? marks.

? × 9 + 7 = 88

? × 9 + 6 = 888

? × 9 + 5 = 8888

? × 9 + 4 = 88888

? × 9 + 3 = 888888

? × 9 + 2 = 8888888

? × 9 + 1 = 88888888

? × 9 + 0 = 888888888

COPY AND CUT THESE CARDS INTO 10 STRIPS FOR THE GAME

0 × 9 + ? =	1	? =
1 × 9 + ? =	11	? =
12 × 9 + ? =	111	? =
123 × 9 + ? =	1 111	? =
123 4 × 9 + ? =	11 111	? =
123 45 × 9 + ? =	111 111	? =
123 456 × 9 + ? =	1 111 111	? =
1 234 567 × 9 + ? =	11 111 111	? =
12 345 678 × 9 + ? =	111 111 111	? =
123 456 789 × 9 + ? =	1 111 111 111	? =

INCLUSION AND HOME LEARNING GUIDE

THEME: NUMBER PATTERNS

Early Years

Count your fingers (including thumbs) and toes.

8 fingers + 2 thumb = 10

10 fingers + 10 toes = 20.

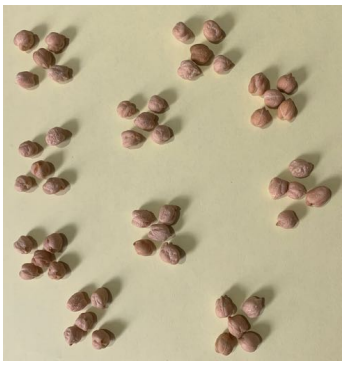
Make some handprints on a large sheet of paper.

Count the number of hands.

Count the fingers in fives.

Count out some stones or dried peas into sets of 5.

Count 5, 10, 15, 20, … 50.



Fingers and Toes
16th Century Nursery Rhyme

Every Lady in this Land

Hath 20 nails on *each* Hand;

Five & twenty on Hands and *Feet*;

And this is true, without deceit.

Different punctuation changes the meaning:

Every lady in this land hath 20 nails.
On each hand five; and twenty on hands and feet.

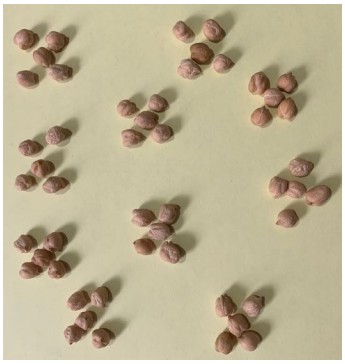
Lower Primary

Fingers and Toes
16th Century Nursery Rhyme

Every Lady in this Land
Hath 20 nails on *each* Hand;
Five & twenty on Hands and *Feet*;
And this is true, without deceit.

Different punctuation changes the meaning:
Every Lady in this Land hath 20 nails.
On each hand five; and twenty on hands and feet.

Count your fingers (including thumbs) and toes.
8 fingers + 2 thumb = 10
10 fingers + 10 toes = 20.
Count the number of hands in this picture?
Count the fingers in fives.
Count out some stones or dried peas into sets of 5.
Count 5, 10, 15, 20, ... 50.
(There are 14 handprints or parts of handprints, 14 × 5 = 70)



Count out some stones or dried peas and arrange them in sets of 9.

Count the stones in 9s

9
18
27
36
45
54
63
72
81

What do you notice about the number pattern on the right? Make a list.

Use stones and counting to check that the calculations in the patterns are correct.

9	= 1 × 9	= 10 - 1
18	= 2 × 9	= 20 - 2
27	= 3 × 9	= 30 - 3
36	= 4 × 9	= 40 - 4
45	= 5 × 9	= 50 - 5
54	= 6 × 9	= 60 - 6
63	=7 × 9	= 70 -7
72	= 8 × 9	= 80 -8
81	= 9 × 9	= 90 - 9

Upper Primary, Lower and Upper Secondary

Work as a group or in pairs.

Ask: 'What do you notice about the calculations in the picture?'

1. Complete the calculations in the picture
2. Complete the calculations on each line in the table below.

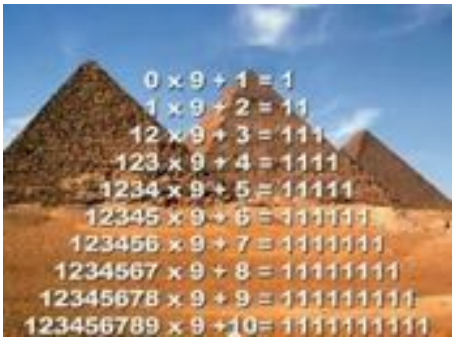
100 000 000 × 9 =	
20 000 000 × 9 =	
3 000 000 × 9 =	
400 000 × 9 =	
50 000 × 9 =	
6 000 × 9 =	
700 × 9 =	
80 × 9 =	
9 × 9 =	
123 456 789 × 9 =	

PLAY THE GAME: Cut out the strips on page 3.
Mix up the strips.
Fill in the answer in the last box on the right hand end of each strip.
Arrange the strips in order.
Talk about how the patterns come about and why they occur.

Key questions

- How do the multiples of 9 appear in the calculations?
- How do the multiples of 9 appear in the pattern?
- What do you notice?
- Can you explain why the pattern comes out like that?

SOLUTION



You could explain why this pattern appears in different ways.

If we think of multiplying by 9 as multiplying by 10-1 then, for example

12 345 678 × (10 – 1) + 9
= 123 456 789 – 12 345 678
= 111 111 111

Looking at another calculation we see the pattern of 1's appearing again:

12 345 × (10 – 1) + 6

= 123 456 – 12 345

= 111 111

The table below shows part of the last calculation

100 000 000 × 9 =	900 000 000
20 000 000 × 9 =	180 000 000
3 000 000 × 9 =	27 000 000
400 000 × 9 =	3 600 000
50 000 × 9 =	450 000
6 000 × 9 =	54 000
700 × 9 =	6 300
80 × 9 =	720
9 × 9 =	81
123 456 789 × 9 =	**1 111 111 101**

Another way to explain the patterns uses the fact that the patterns come from the 9 times table, that is multiples of 9:

9, 18, 27, 36, 45, 54, 63, 72, 81

We see the bracketed pairs of numbers in the same list all add up to 10:

(9, 1)(8, 2)(7, 3)(6, 4)(5, 5)(4, 6)(3, 7)(2, 8)1

or, re-writing these pairs:

9
18
27
36
45
54
63
72
81

So, as in the table, we have these pairs of numbers occurring in the units, tens, hundreds, thousands, ten thousands … places where each column adds up to 10 with 1 carried over from the column on the right of it.

Why do this activity?

Learners get practice in doing calculations and the activity re-enforces their understanding of the concept of place value. The pattern is pleasing. Explaining why the pattern occurs develops learners' logical reasoning and number sense. The calculations can be done by multiplying by 9 for each line first, then doing the additions, or doing the whole calculation line by line.

This can be game for a whole class as described below. Or all the calculations can be done as a class effort where a learner gives an answer and then names the learner to give the next answer.

The activity can be structured to cater for everyone in a class; simpler calculations and more challenging tasks like giving explanations and doing longer calculations can be distributed by the teacher according to the abilities of the learners to cater for all attainment levels.

Learning objectives

In doing this activity students will have an opportunity to:

- practise multiplication, calculation and solving equations;
- develop visualisation, logical reasoning and number sense;
- re-enforce understanding of the concept of place value.

Generic competences

In doing this activity students will have an opportunity to practise pattern spotting and explaining why the patterns occur, which has applications to solving problems in a range of situations.

DIAGNOSTIC ASSESSMENT This can be done as a group as described below or the question can be answered individually.
Show this question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Which of the follow is equivalent to $201 \times 9 - 9 \times 102$?

A. 99×9
B. 101×9
C. 0
D. 9

1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: A "I chose A because I subtracted 201 and 102 and got 99"
Possible misconceptions – all quotes are reasons for these answers given by learners.
B. "its almost halving each other"
C. "you take away what you have calculated therefore it equally 0"
D. "201 times 9 is 1809 -1818 is 9 https://diagnosticquestions.com

Follow up

Two by Two Puzzle https://aiminghigh.aimssec.ac.za/years-4-7-two-by-two-puzzle/
Multiplication Squares
https://aiminghigh.aimssec.ac.za/years-4-7-multiplication-squares/
Magic 1387 https://aiminghigh.aimssec.ac.za/years-6-10-magic-13837/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13