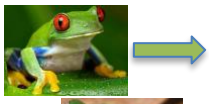
AIMSSEC

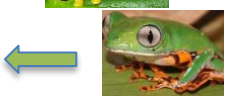
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

PATTERNS is the theme for this INCLUSION AND HOME LEARNING GUIDE
This Guide suggests related learning activities for all ages from 4 to 17+
Choose whatever seems suitable for your group of learners

FROGS PUZZLE


Five red-eye frogs and five orange frogs line up in a row with a space between them.
They must change places.
They can hop one frog over another frog or slide to an empty place next to them.

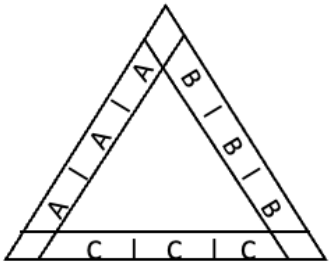
Red-eye frogs can only move to the right 

Orange frogs can only move to the left 

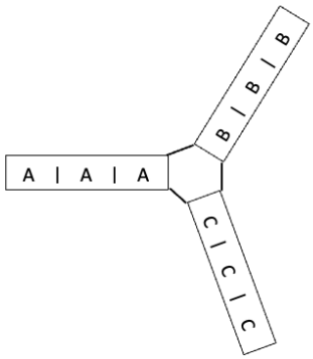
How can they change places following these rules?
Try the puzzle with 2 frogs on each side first, then with 3 frogs on each side …
How many moves will it take?

HELP Think about the order in which the frogs should move to avoid blocking in any of the frogs so that they can't move.

NEXT TRIANGLE AND TRIBAR FROGS PUZZLES



In these puzzles, the frogs slide and hop as in the standard Frogs puzzle. For the Triangle Puzzle there are 12 cells, 3 on each edge, and one at each vertex that is empty at the start. For the Tribar puzzle there are 10 cells. For example, the A and B frogs can change places as follows:



1. Frog A slides into the empty cell leaving an empty space.
2. Frog B hops over Frog A and lands in the empty cell on the other side.

Show that there are 2 different layouts with 3 frogs on each edge, discounting the different positions arising from rotating the triangle or tribar with the frogs 'riding' in their cells. Work out how to start from the layouts shown and move to the other layout, and how it can be done with the smallest number of moves.

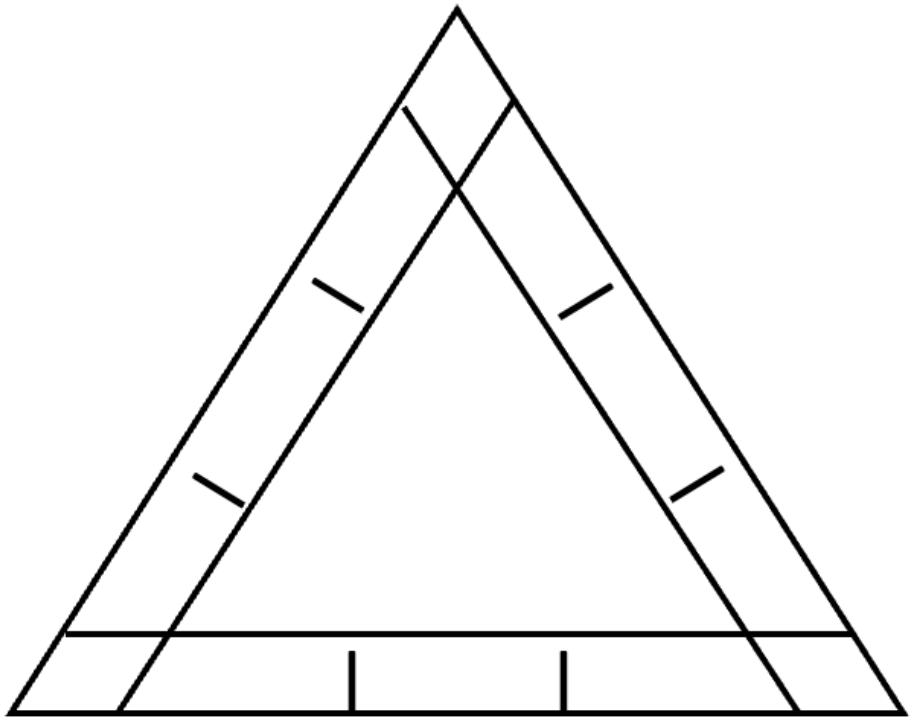
Resources: Counters or cut-out paper frogs different colours.

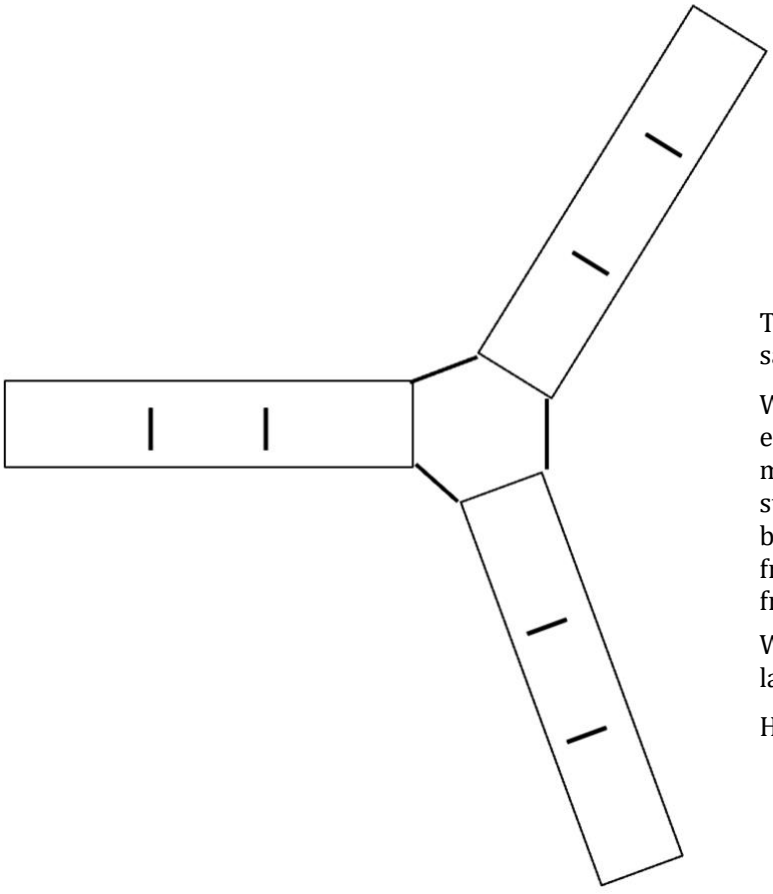
TRIANGLE FROGS PUZZLE

The frogs move according to the same rules as in the other frog puzzles. One frog can hop over another frog into an empty cell.

With differently coloured frogs on each edge, and empty cells at the vertices, there are 2 different possible starting layouts with 3 frogs on each edge, discounting other layouts arising from rotating the triangle with the frogs 'riding' in their cells.

Work out how to start from one layout and move to the other layout. How many moves would it take?





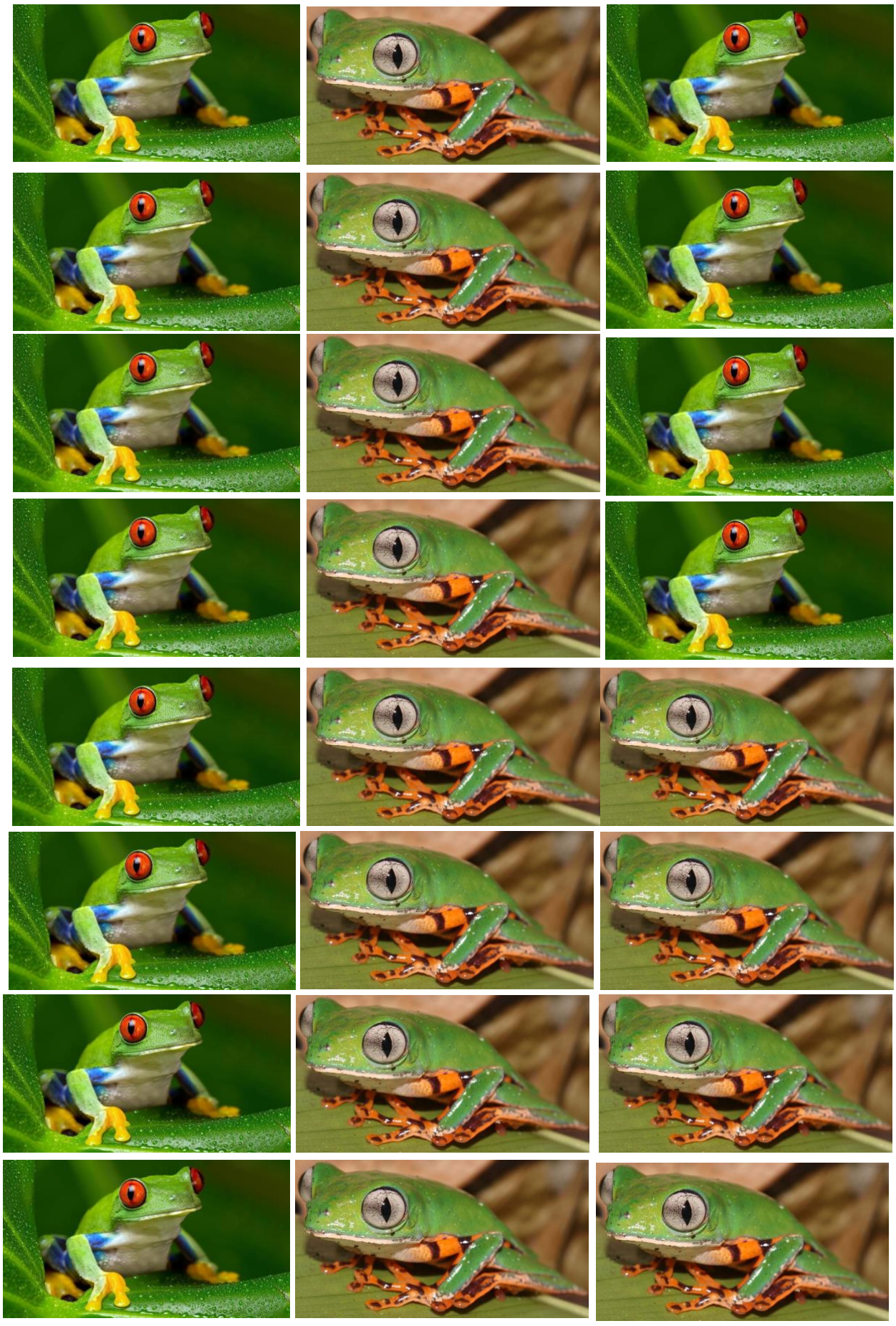
TRIBAR FROGS PUZZLE

The frogs move according to the same rules as in the other puzzles.

With differently coloured frogs on each bar, and an empty cell in the middle, there are 2 different possible starting layouts with 3 frogs on each bar, discounting other layouts arising from rotating the tribar with the frogs 'riding' in their cells.

Work out how to start from one layout and move to the other layout.

How many moves would it take?



INCLUSION AND HOME LEARNING GUIDE
THEME: PATTERNS

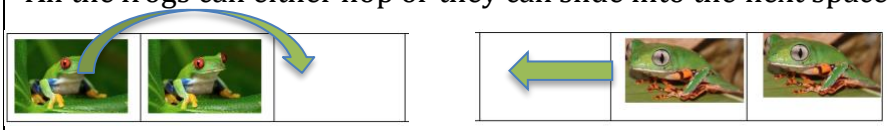
Early Years and Lower Primary

Try to solve the FROGS puzzle by pretending to be frogs. Start by choosing 2 people to be red-eye frogs and 2 people to be orange frogs. You need 5 chairs. The 2 red-eye frogs sit at one end and the 2 orange frogs at the other end with an empty chair in the middle.

Red-eye frogs move this way → and orange frogs this way ←

The frogs have to change places

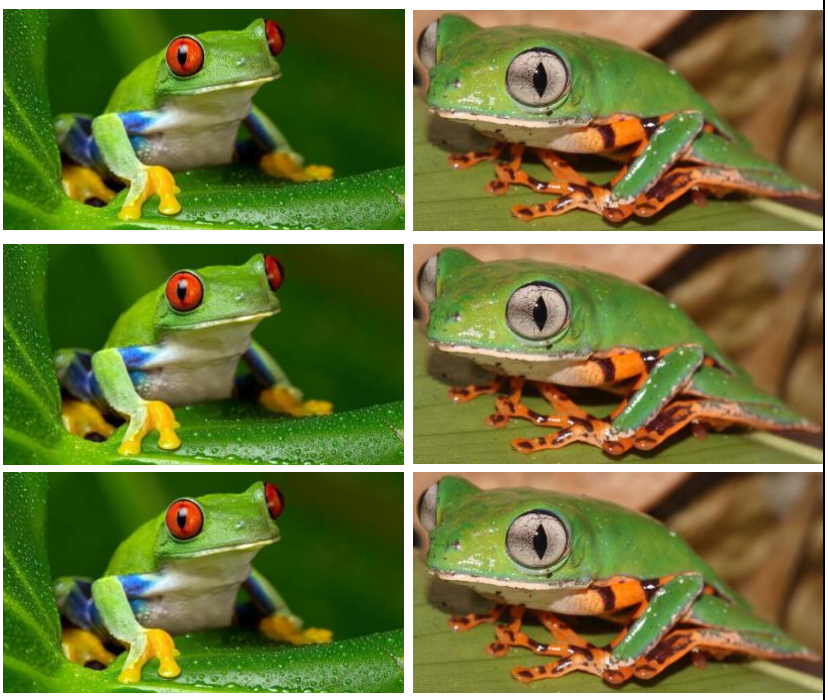
All the frogs can either hop or they can slide into the next space if it is empty.



Can you and your friends solve this puzzle? Can you find out how to follow the rules and make the right moves so that you change places?

Here are 6 frogs that you can cut out. Can you solve the puzzle with the paper frogs in the same way as when you acted as frogs in the game?

Can you solve the puzzle for 3 frogs on each side?



Upper Primary

Start by trying to solve the puzzle and to find the right moves for 5 frogs on one side and 5 on the other as on page 1. When this proves difficult talk about problem solving methods. In particular work on the simplest version of the problem with 2 frogs on each side.

This is not a script for a home learning session, rather it would be best to adapt this plan according to responses from the participants.

- If you have enough people, act out the problem with 3 girl frogs and 3 boy frogs and 7 chairs, or fewer if you have fewer people in your group.
- If the learners find it difficult, try 2 boy frogs and 2 girl frogs on the chairs and make sure everyone understands the rules.
- Perhaps try 2 boy frogs and 3 girl frogs next.
- This is collaborative, not competitive – everyone should try to help the players to make the right move so as to solve the puzzle.
- When they know what they have to do learners can try this in pairs using cut out paper frogs or counters.

When learners have worked out the sequence of moves so that they can do the change-overs without getting stuck, suggest that they record their results in a table as below.

Think about simple cases: e.g. 2 boy and 2 girl frogs **Fill in a table of results.** **How many moves?** **How many hops?** **How many slides?** **Is there a pattern?** **Can you explain the pattern?**	Number of boy frogs	Number of girl frogs	Number of hops	Number of slides	Number of moves
	1	2			
	2	2			
	2	3			
	3	3			
	3	4			
	4	4			
	4	5			
	5	5			

Before the end of the session have a whole group discussion about the strategies for solving the puzzle and the number patterns that the learners have found.

Only go as far as the learners are comfortable with. Do not tell them the rule or formula. You can suggest that they might like to play the game again just for fun and try to find more patterns and answers, and maybe a rule. The 'rule' can be given in words, it does not have to be in a formula.

Lower and Upper Secondary

For secondary learners this exercise leads towards algebra. A good answer from the learners would be a formula in words like:

"'you multiply the number of frogs of each type to get the number of hops and you add the number of frogs of each type to get the number of slides".

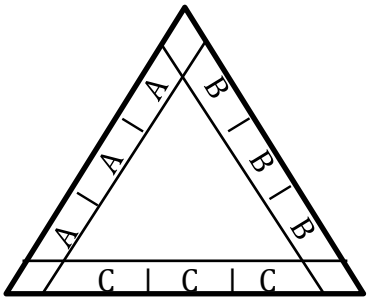
Some younger 'high flyers', and the year 9 learners, may be able to express the formula algebraically and even give a proof.

- With older players introduce the activity by 5 boys and 5 girls acting it out, with help from the class.
- When they find it too difficult ask for suggestions for working on the problem.
- If the players have experience of problem solving they should suggest **trying simple cases**, if not the teacher will need to suggest it (and point out that this is a **general strategy** for problem solving).

Key questions

- Can you find a formula (rule) in words for the number of hops?
- Can you find a formula (rule) in words for the number of places moved?
- Can you find a formula (rule) in words for the number of slides?
- Can you find a formula (rule) for the number of moves?
- Can you prove the formula always works?
- Can you find a formula in symbols?

TRIANGLE FROGS PUZZLE



In this puzzle, the frogs slide and hop as in the standard Frogs puzzle and there are 12 cells, 3 on each edge and one at each vertex. The frogs can occupy the rhombus shaped spaces at the vertices of this triangular track which are empty at the start.

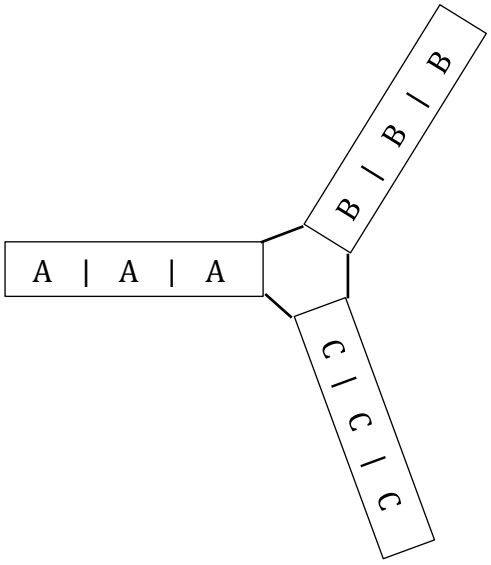
For example, the A and B frogs at the top can change places as follows:

1. Frog A slides into the empty rhombus shaped cell leaving an empty space.
2. Frog B hops over Frog A in the corner cell and lands in the empty cell.

At the start there are 3 frogs of each type on the edges of triangle with no spaces between them except at the vertices, and 2 different possible starting layouts, discounting the layouts arising from rotating the triangle with the frogs staying in their cells and riding on the rotating triangle.

Work out how to move from one layout to the other and the number of moves taken.

TRIBAR FROGS PUZZLE



In this puzzle there is only one empty cell in the hexagon at the centre.

The frogs move according to the same rules as in the other puzzles.

Again, there are 2 different possible starting layouts.

Work out how to start from the layout shown and move to the other layout. How many moves would it take?

Edouard Lucas (1842-1891) invented this puzzle which is also known as the Lucas puzzle. He was a French mathematician, well known for his inventions of mathematical puzzles and recreations, and he also invented the Tower of Hanoi puzzle. https://mathshistory.st-andrews.ac.uk/Biographies/Lucas/

RESULTS FOR FROGS GAME

Think about simple cases: e.g. 2 boy and 2 girl frogs

Fill in a table of results.

How many moves?

How many hops?

How many slides?

Is there a pattern?

Can you explain the pattern?

Number of boy frogs	Number of girl frogs	Number of hops	Number of slides	Number of moves
1	2	2	3	5
2	2	4	4	8
2	3	6	5	11
3	3	9	6	15
3	4	12	7	19
4	4	16	8	24
4	5	20	9	19
5	5	25	10	35

SOLUTION

The formula for the number of moves for R red-eye frogs on one side and G orange frogs on the other is $R + G + RG$.

This is $R + G$ slides and RG hops.

Proof

Each frog must hop exactly once over all the frogs of the other type (or be hopped over) so there are RG hops.

The R frogs on one side must move $(G+1)$ places in one direction and G frogs on the other side must move $(R+1)$ places in the other direction. So the total number of places moved is $R(G + 1) + G(R + 1) = R + G + 2RG$.

With each hop a frog moves 2 places so with RG hops the frogs move a total of $2RG$ places. The number of places moved is $R + G + 2RG$ so there are $(R + G)$ slides, RG hops and $R + G + RG$ moves.

Why do this activity?

Patterns are at the heart of mathematics and, in this activity, learners develop their problem solving skills. The problems to solve are first to find a pattern of moves that will succeed in the 2 sets of frogs swapping places, then secondly to find number patterns in the numbers of moves and thirdly to find a formula (or rule) that gives the number of moves as a function of the numbers of each type of frog. The physical engagement (people maths activity) promotes kinaesthetic learning. If learners work in pairs and small groups they develop their communication and reasoning skills through discussion. Using a table to record results helps learners to work systematically.

Learning objectives

In doing this activity students will have an opportunity to:

- develop logical reasoning and problem solving skills;
- develop communication skills through talk in groups and as a class;
- (for older learners) practise describing a number pattern in an algebraic formula.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically**, reason logically and give explanations**;
- **think flexibly** and apply knowledge and skills;
- interpret and **solve problems**;
- **collaborate and** work with a partner or group.

Follow up

Try a similar puzzle with 4 teams of frogs on the edges of a square.

Invent other Frogs puzzles.

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