AIMSSEC

**AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES**
**SCHOOLS ENRICHMENT CENTRE (AIMSSEC)**

**AIMING HIGH**

**EVIDENCE FOR DECISIONS is the theme**

**for this INCLUSION AND HOME LEARNING GUIDE**

**This Guide suggests related learning activities for all ages from 4 to 17+**

**Just choose whatever seems suitable for your group of learners**

The EPIDEMIC activity was designed for Upper Secondary

## EPIDEMIC

Twenty per cent of the inhabitants of a city have been inoculated against a certain disease. A city is hit by an epidemic. The chance of infection amongst those inoculated is 10% but amongst the rest it is 75%.

Copy and fill in the contingency table and the Venn diagram below and use them to answer the questions.

1. a. In the contingency table, split the 20% who are inoculated into the percentages who get infected and the percentages who are not affected (remain immune).

b. Split the 80% who are not inoculated into three quarters who get infected and one quarter who are not infected (who remain immune).

c. Fill in the totals in the right hand column.

2. What proportion are infected?

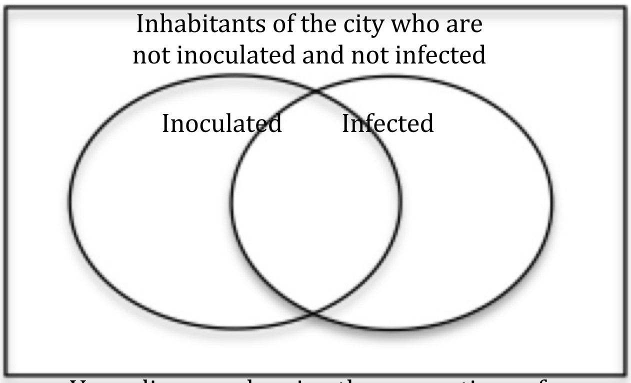
3. If a man is chosen at random and found to be infected, what is the chance of his having been inoculated?

| | Inoculated | Not inoculated | Totals |
|---|---|---|---|
| Infected | | | |
| Not infected | | | |
| Totals | 20% | 80% | 100% |

*Note: fractions, decimals and percentages are 3 alternative ways of writing the same number.*

*This example refers to proportions of the population not to actual numbers.*

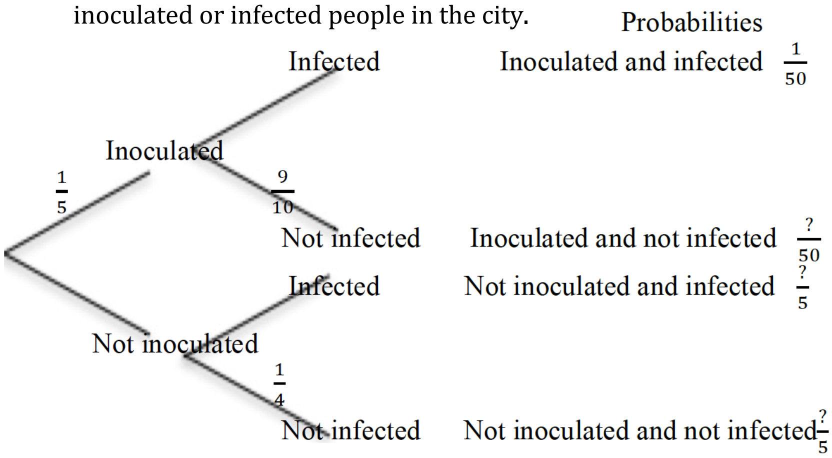
*Sometimes fractions are convenient, but we often use decimals for calculations.*




Venn diagram showing the proportions of inoculated or infected people in the city.

4. Complete this Venn diagram, and also complete the tree diagram below.

The contingency table, Venn diagram and tree diagram give the same information in different ways.




Fill in the given percentages as fractions, and to show the probabilities of an individual chosen at random from the population being inoculated or not inoculated, and being infected or not infected.

5. The chance of an infected person having been inoculated is $\frac{1}{50}/\frac{31}{50} = \frac{1}{31}$.

What is the chance of a person not infected having been inoculated?

## HELP

If possible work in a group where different people bring different experience, skills and knowledge to contribute to the group discussion. Talk to other people about this problem but aim for everyone to help everyone else.

There is much talk in the media about the corona virus.  Some of what you hear is inaccurate and misleading. For example, when people speak of ‘safe’ they should often say ‘lower risk’ because going into places where there are other people is not perfectly safe.

When people speak of medical test results you may believe the results are accurate. Test are not always accurate. They can show a person does not have the virus when they do have it or that a person is free of the virus when they are not (called false-positives and false-negatives). Nothing is certain except that what happens in the future will depend on probability.

## NEXT

Thinking of your own gender and height which of the 4 boxes in the contingency table are you in?

Can you explain why you are in that box and not in any of the other three boxes?

| | Under 175 cm in height | 175 cm in height or taller | Totals |
|---|---|---|---|
| **Male** | Number of males who are under 175 cm in height | Number of males who are 175 cm in height or taller | |
| **Female** | Number of females who are under 175 cm in height | Number of females who are 175 cm in height or taller | |
| **Totals** | | | |

Thinking about complementary sets, would you say that the sets of males and females are complementary sets? Why? Would you say that the sets of people under 175 cm in height and of people 175 cm in height or taller are complementary sets? Why?

Either using real data from a class in your school or some made up data, make up a contingency table using these attributes. When you have found the totals of males and females what relative frequencies can you calculate and what are they?

When you have found the total number of people under 175 cm in height and the total number of people 175 cm or taller what relative frequencies can you calculate and what are they?

Suppose you recorded this data from a Year 12 class, would these relative frequencies be the same as if you took a random sample of 100 learners from your **whole school**? Explain your answer.

**Draw a Venn diagram to represent the data in your contingency table.**

## INCLUSION AND HOME LEARNING GUIDE

## THEME: EVIDENCE FOR DECISIONS

### Early Years Lower Primary

**Would it be a good idea to plan a picnic for tomorrow**? What would help us to decide? We might ask 'Is it going to rain?' How do we find out? We should look at a weather forecast because it is going to rain then we will not plan a picnic. .

Talk about making decisions and evidence for decisions that has nothing to do with what we want or how we feel. Often, like the weather and a picnic, the evidence for a decision depends on probability.

**Talk about how the COVID19** pandemic has affected the lives of the children. Some of the following will apply.

- Why can't we play with our friends?
- Why do we have to stay at home?
- Why was the school closed?
- Why are shops and cafes closed?
- Why can't we go to visit … my grandparents or …?
- Why are the grown-ups working at home instead of going to work?

**The evidence for the decisions that led to these restrictions is the number of people getting seriously ill from the virus and the risk of spreading the disease to other people.**

**DIAGNOSTIC ASSESSMENT FOR SECONDARY** Use this before the lesson.
Show this question and say:
**"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".**

A box contains 20 nails.
The table shows information about the length of each nail.

| Length of nail (mm) | 25 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|
| Number of nails | 1 | 8 | 4 | 5 | 2 |

Jamila puts all 20 nails into a bag.
She takes at random one of the nails and records its length.
She replaces the nail in the bag.
She then takes at random a second nail from the bag and records its length.

Calculate the probability that the two nails she takes

each have a length of 60 mm

A: 4/400

B: 4/20

C: 2/20

D: 2/400

1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who who gave right and wrong answers.
5. Again ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
6. The concept is needed for the lesson to follow so explain the right answer.

The correct answer is: **A** The probability is 2/20x2/20
Possible misconceptions
B. It could be that the learner has added the probabilities rather than multiplying.
C. This is the probability of picking a 60mm nail each time.
https://diagnosticquestions.com

## Why do this activity?

Everyone has been affected by the Covid19 pandemic and even **very young children** are aware that it has affected their lives. Examples like this help learners to have a better appreciation of how expert knowledge and understanding of probability provide evidence that leads to the major decisions made by politicians that change the lives of the whole population during a pandemic.

**For Primary and Lower Secondary learners** this Epidemic learning activity gives learners an experience of using and applying school mathematics in real world situations. This may help them to recognise some of the misinterpretation of data that are often met in the media and advertising.

This example helps learners to understand a little about the ways in which immunisation can be used to protect people from viruses.

The activity helps **Upper Secondary learners** to review their knowledge and understanding of the concepts and language of probability by connecting contingency tables, Venn diagrams and tree diagrams as representations of a set of data. Working on this example should help learners to understand more about the way that probability and statistics are used in decision making about immunisation and public health.

## Upper Primary and Lower Secondary

**Talk about immunisation and why it's so important.** Ask questions so that the learners say what they think, and give them a chance to ask questions so that you can inform them or you can find the answers together.

Talk about why scientists are searching for a vaccine to protect people against the corona virus. Talk about how, not long ago, most children got Measles, Mumps and Rubella, which were very common and unpleasant childhood illnesses, how sometimes adults were affected even more seriously than children, and there were many deaths. But now these diseases are rare because babies are given the 'Measles, Mumps and Rubella' (MMR) vaccine. Talk about how other vaccines against smallpox, polio and influenza have kept these diseases under control and even eliminated them altogether in some places.

*This exercise is all about reading and understanding what information you are given and then deciding how to use that information.*

**UNDERSTANDING THE INFORMATION AND USING A CONTINGENCY TABLE**

**Getting started**: Make sure the question on page 1 is written on a board that all can see or give everyone a copy of the question. If possible ask the learners to work in pairs, to read the question and to decide, if they can, how to fill in the figures (in percentages) in the contingency table corresponding to the information (given in maroon). Then they should fill in the totals in the right hand column.

When they have had time to try this for themselves, lead a discussion about putting the information into the table. Ask the learners to suggest how the contingency table could be filled in. Then check this together until everyone agrees on the figures and understands them.

Twenty per cent of the inhabitants of a city have been inoculated against a certain disease. This information is in the table on the bottom row.

| | Inoculated | Not inoculated | Totals |
|---|---|---|---|
| Infected | | | |
| Not infected | | | |
| Totals | 20% | 80% | 100% |

The city is hit by an epidemic.

The chance of infection amongst those inoculated is 10%

1a. In the table, fill in for those who have been inoculated the percentages who get infected and the percentages who are not affected (remain immune).

Amongst those not inoculated the chance of infection is 75%.

1b. Split the 80% who are not inoculated into three quarters who get infected and one quarter who are not infected (who remain immune).

1c. Fill in the totals in the right hand column.

2. What proportion are infected?

3. If a man is chosen at random and found to be infected, what is the chance of his having been inoculated?

Discuss this as a group and encourage the learners to read the questions and to talk about how to answer them.
Ask who can find an answer? Ask them to explain how they worked it out.
Fill in the boxes one by one and ask the group:
'Who would like to explain this working and the answers in the cells of the table.

*Note: This example refers to proportions of the population not actual numbers. Fractions, decimals and percentages are 3 alternative ways of writing the same number. Sometimes fractions are convenient, but we often use decimals for calculations*

**SOLUTIONS**

| | Inoculated | Not inoculated | Totals |
|---|---|---|---|
| Infected | 10% of 20% = $\frac{1}{10} \times \frac{1}{5} = \frac{1}{50}$ =2% | 75% of 80% = $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$ =60% | 2% + 60% = 62% |
| Not infected | 90% of 20% = $\frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$ =18% | 25% of 20% = $\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$ = 20% | 18% + 20% =38% |
| Totals | 20% | 80% | 100% |

2.

62% are infected. This can also be given as 0.62 or $\frac{31}{50}$.

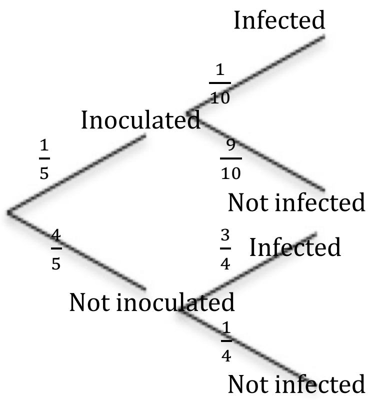
3.

If a man is chosen at random and found to be infected, the chance of his having been inoculated is $\frac{2}{62} = \frac{1}{31}$ = 0.03 to 2 decimal places.
It can be written 3% or $\frac{3}{100}$

**Stop at this point for younger learners. For Year 9, discuss the tree diagram below to see how it can be used to give the same information**

**For Year 9**

**4.**




**Probabilities**

Inoculated and infected $\frac{1}{50}$

Inoculated and not infected $\frac{9}{50}$

Not inoculated and infected $\frac{3}{5}$

Not inoculated and not infected $\frac{1}{5}$

5. The chance of a person not infected having been inoculated is $\frac{9}{50}/\frac{19}{50} = \frac{9}{19} = 0.47$ to 2 decimal places.

## Upper Secondary

Read the information on pages 1 and 2 and answer the questions.

If you do not at first understand the Venn diagram, be warned that a very common source of confusion and misunderstanding is not to appreciate that the region within the rectangular frame but outside the circles in the Venn diagram, represents the complement of the sets shown inside the circles.

Answering the following questions may help you to see what to do:

- If 20% of the population were inoculated, what percentage were not inoculated?

There are 4 sets to consider:

1. What percentage of the whole population is inoculated and infected?
2. What percentage is inoculated and not infected?
3. What percentage of the whole population is not inoculated and infected?
4. What percentage is not inoculated and not infected?

- Can you give all the percentages as fractions?

The solutions are on page 6.

## Learning objectives

In doing this activity older students will have an opportunity to review and consolidate knowledge and understanding of the use of contingency tables, Venn diagrams, and tree diagrams as aids to solving probability problems (where events are not necessarily independent).

## Generic competences

In doing this activity students will have an opportunity to develop their understanding of probability and their problem solving skills.

## Follow up

The table gives the percentages of right and left handed males and females a random sample of people from a large population.

| Sex \ Handed-ness | Right handed | Left handed | Total |
|---|---|---|---|
| Male | 43 | 9 | 52 |
| Female | 44 | 4 | 48 |
| Total | 87 | 13 | 100 |

What is the probability that a female chosen at random from this population is left handed?

What is the probability that a male chosen randomly from the population is left handed?

Carry out a survey of 100 people, either in your school or in your home area and record your data in a similar way. You might do this as a group project so that each person in your group just asks a few people and takes care to avoid the same person being asked more than once. Compare your data with the data given here.

Two Aces https://aiminghigh.aimssec.ac.za/years-9-12-two-aces/
If this then that https://aiminghigh.aimssec.ac.za/years-10-12-if-this-then-that/
Lucky Numbers https://aiminghigh.aimssec.ac.za/grades-7-to-12-lucky-numbers/
Mathsland Lottery
https://aiminghigh.aimssec.ac.za/grades-9-to-12-mathsland-lottery/.