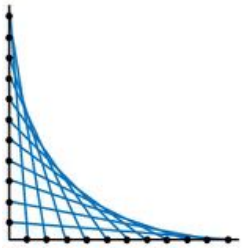
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

JUST TOUCHING is the theme
for this INCLUSION AND HOME LEARNING GUIDE
This Guide suggests related learning activities for all ages from 4 to 17+
Just choose whatever seems suitable for your group of learners
The CONSTRUCT WITH LINES activity was designed for Lower Secondary

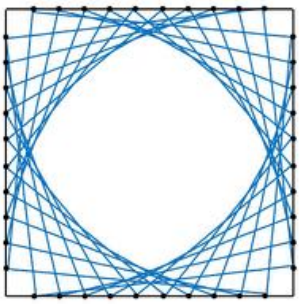
CONSTRUCT WITH LINES



ENVELOPE PATTERNS WITH STRAIGHT LINES

Draw two base lines and mark the same number of points at equal distances along each line. Join the outermost point on one base line to the innermost point on the other base line. Work your way inwards on one base line and outwards on the other base line joining corresponding points.

The curve that you see is called the **ENVELOPE** to the family of straight lines you have drawn. The lines are tangents to the envelope curve.



Experiment with different angles between the base lines, also using several pairs of base lines in the same diagram and using different colours.

HELP

The patterns could be drawn on squared paper to make it easier.

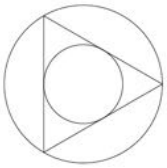
NEXT

Experiment with your own designs using different base lines and different angles.

You could do this as craft work. You'll need some scrap card, some wool and a darning needle, as well as a ruler.

Draw the lines and mark equally spaced points along the line. Make holes at the points. Then use wool to make the lines instead of drawing the lines. Thread the darning needles and sew through the holes one by one pulling the wool gently taut to form the required lines. Experiment with different colours.

TANGENT LINES WORKSHEET Start at any point on the outer circle. Draw a line that **just touches** the inner circle and then goes on to another point on the outer circle.



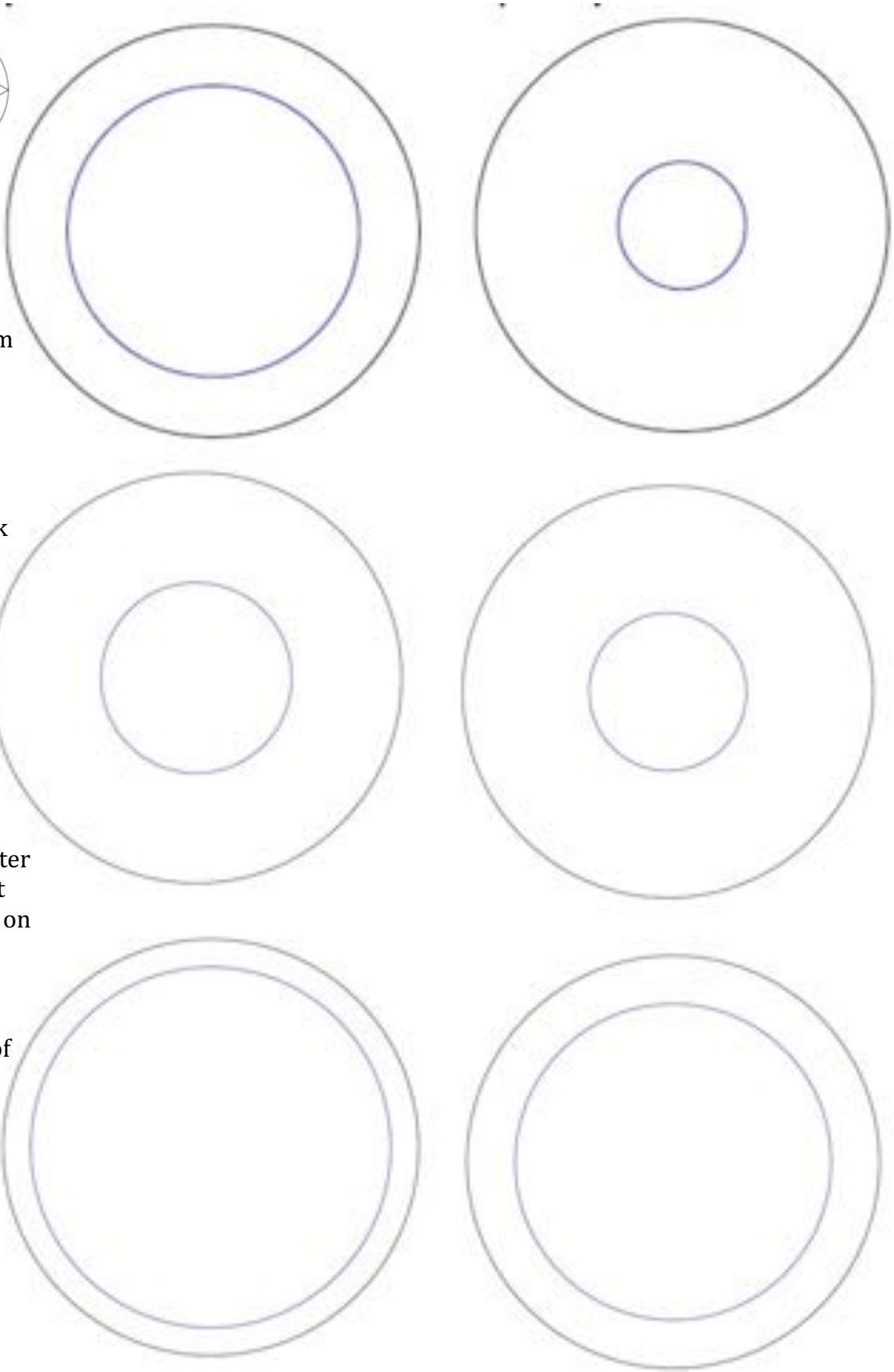
Repeat this a few more times.

The diagram shows one possible result.

Do you get always back to your starting point?

Does it matter which point you choose on the outer circle?

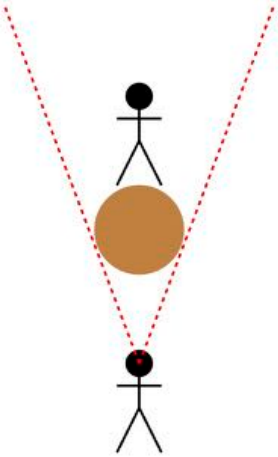
What sort of shapes do you draw?



INCLUSION AND HOME LEARNING GUIDE

THEME: JUST TOUCHING

Early Years and Lower Primary



Is anyone hiding behind this tree?

Talk about hide-and seek.

We cannot tell if anyone is hiding behind the tree. Why?

Talk about the reasons why.

The red dotted lines show our lines of vision. We can see past the tree on each side, but we cannot see around the tree and we cannot see through the tree.

There is a lot of room to hide

Get a ruler and a can of beans or some other cylinder.

Put the can on a table. Ask the children to place the ruler so it just touches the can, as in the picture. Ask them if they can put the ruler beside the can so that it touches somewhere else. Can they do this again? And again? How many times?

Draw the circle around the base of the can. Then move it away. Ask the children to place the ruler so it just touches the circle, then make it touch at other points.

Talk about what is happening. Some children will say that there are lots of points around the edge of the can, on the circle, where the ruler can just touch the circle. Some older children may realise that there are infinitely many ways to do this, there are infinitely many tangents to a circle.

TANGENT LINES

Start at any point on the outer circle. Draw a line that **just touches** the inner circle and then goes on to another point on the outer circle.

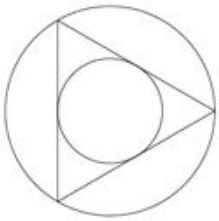
Repeat this a few more times. The diagram shows one possible result.

Do you get back to your starting point?

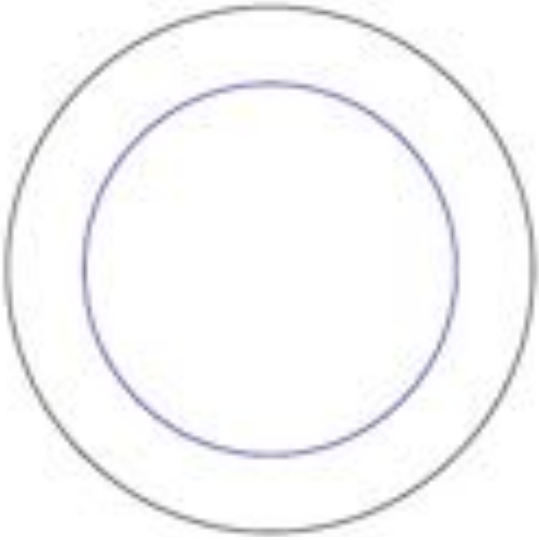
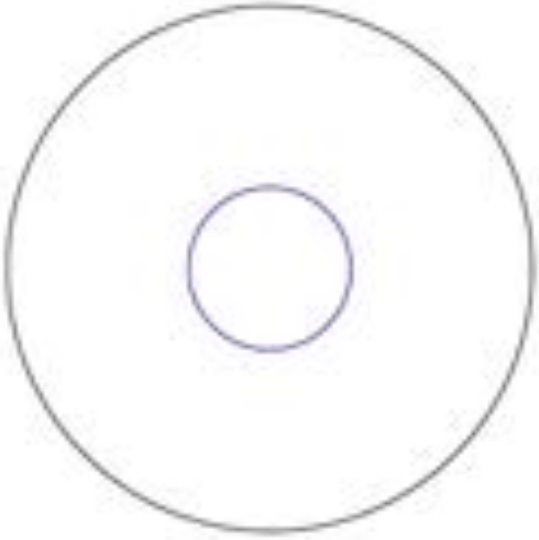
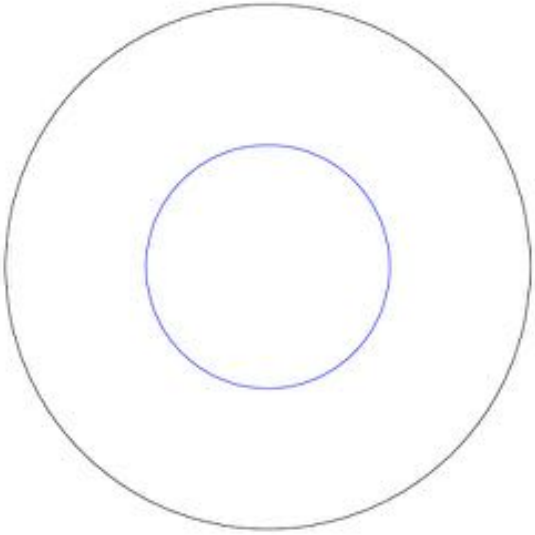
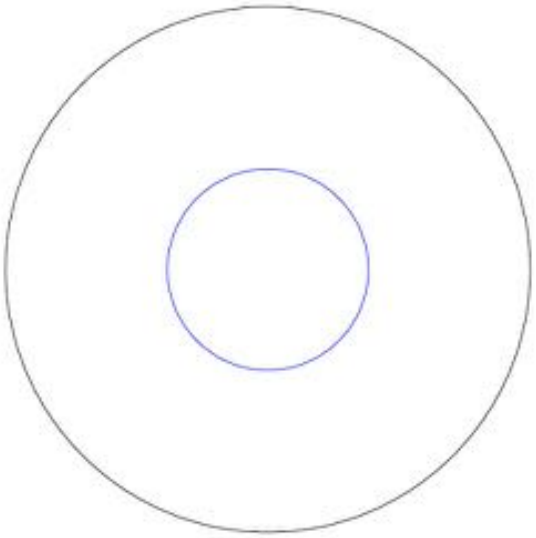
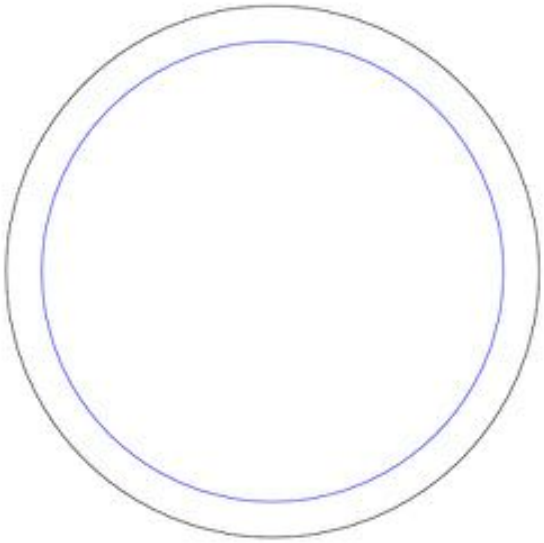
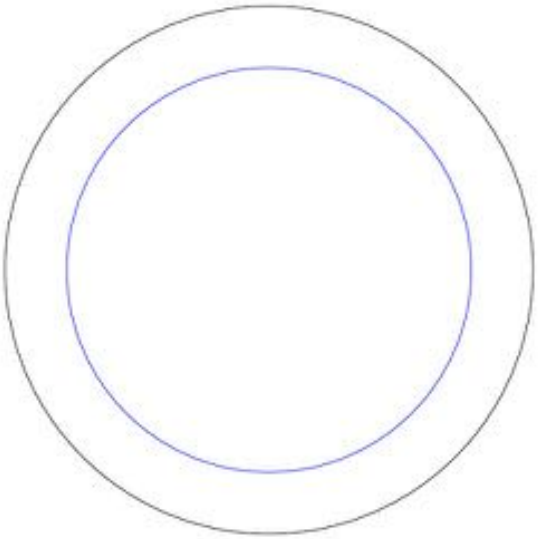
Does it matter which point you choose on the outer circle?

What sort of shapes do you draw?

Talk about the lines you have drawn that just touch the inner circle. We say that these lines are **tangents** to the inner circle.

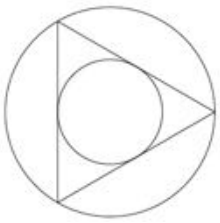


Try this with other circles. You can colour in the patterns you make.

Upper Primary

1. **TANGENT LINES**



Use the worksheet on page 2. Start at any point on the outer circle. Draw a line that **just touches** the inner circle and then goes on to another point on the outer circle.

Repeat this a few more times.

The diagram shows one possible result.

Do you get back to your starting point?

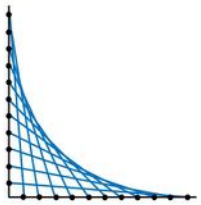
Does it matter which point you choose on the outer circle?

What sort of shapes do you draw?

Talk about the lines you have drawn that just touch the inner circle. We say that these lines are **tangents** to the inner circle.

Try this with other circles. You can colour in the patterns you make.

2. ENVELOPE PATTERNS WITH STRAIGHT LINES



Draw two base lines and mark the same number of points at equal distances along each line. Join the outermost point on one base line to the innermost point on the other base line. Work your way inwards on one base line and outwards on the other base line joining corresponding points.

Lower Secondary

1. TANGENT LINES

Use the worksheet on page 2. Start at any point on the outer circle. Draw a line that **just touches** the inner circle and then goes on to another point on the outer circle.

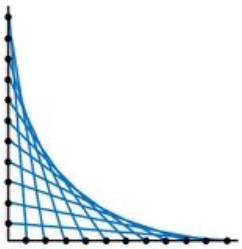
Repeat this a few more times. Do you get back to your starting point?

Does it matter which point you choose on the outer circle?

What sort of shapes do you draw?

Talk about the lines you have drawn that just touch the inner circle. We say that these lines are **tangents** to the inner circle.

2. ENVELOPE PATTERNS WITH STRAIGHT LINES

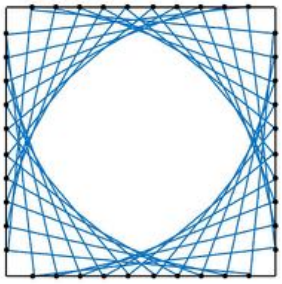


Show your learners some pictures. Tell them that they are going to copy some of the designs and then to create some of their own.

Ask the learners to draw two lines perpendicular to each other and to mark 10 points equally spaced along each line.

Then they must join points 1 to 10, 2 to 9, 3 to 8, 4 to 7, 5 to 6, 6 to 5, 7 to 4, 8 to 3, 9 to 2 and 10 to 1.

Once they have achieved this pattern, to engage the learners in accurate measurement of angles, you can ask them to draw lines at 60^{o} and repeat the pattern.



Experiment with different angles between the base lines, also using several pairs of base lines in the same diagram and using different colours.
Create your own patterns.

Key question

What other patterns could you make in a similar way?

Upper Secondary

Do the activity as described on page 1 and also the **TANGENT LINES** worksheet on page 2.

Answer these questions:

1.

You have a heavy long narrow bookcase and you need to carry it along a corridor that is exactly 1 metre wide, and around a right-angled corner. The bookcase is so tall that it almost touches the ceiling and it is 220 millimetres deep from front to back. What is the longest bookcase that you can get around the corner in the corridor?

2.

If you drew the tangents accurately in the **TANGENT LINES** activity on page 2 your tangents will have made a square, a 5-pointed star, an 8-pointed star, a hexagon and a 9-pointed star and one of them will have gone on and on and never gone back to the starting point.

What happens depends on the ratio of the radii of the two circles. Draw circles such that in one case you draw an equilateral triangle, and in another a 8-pointed star.

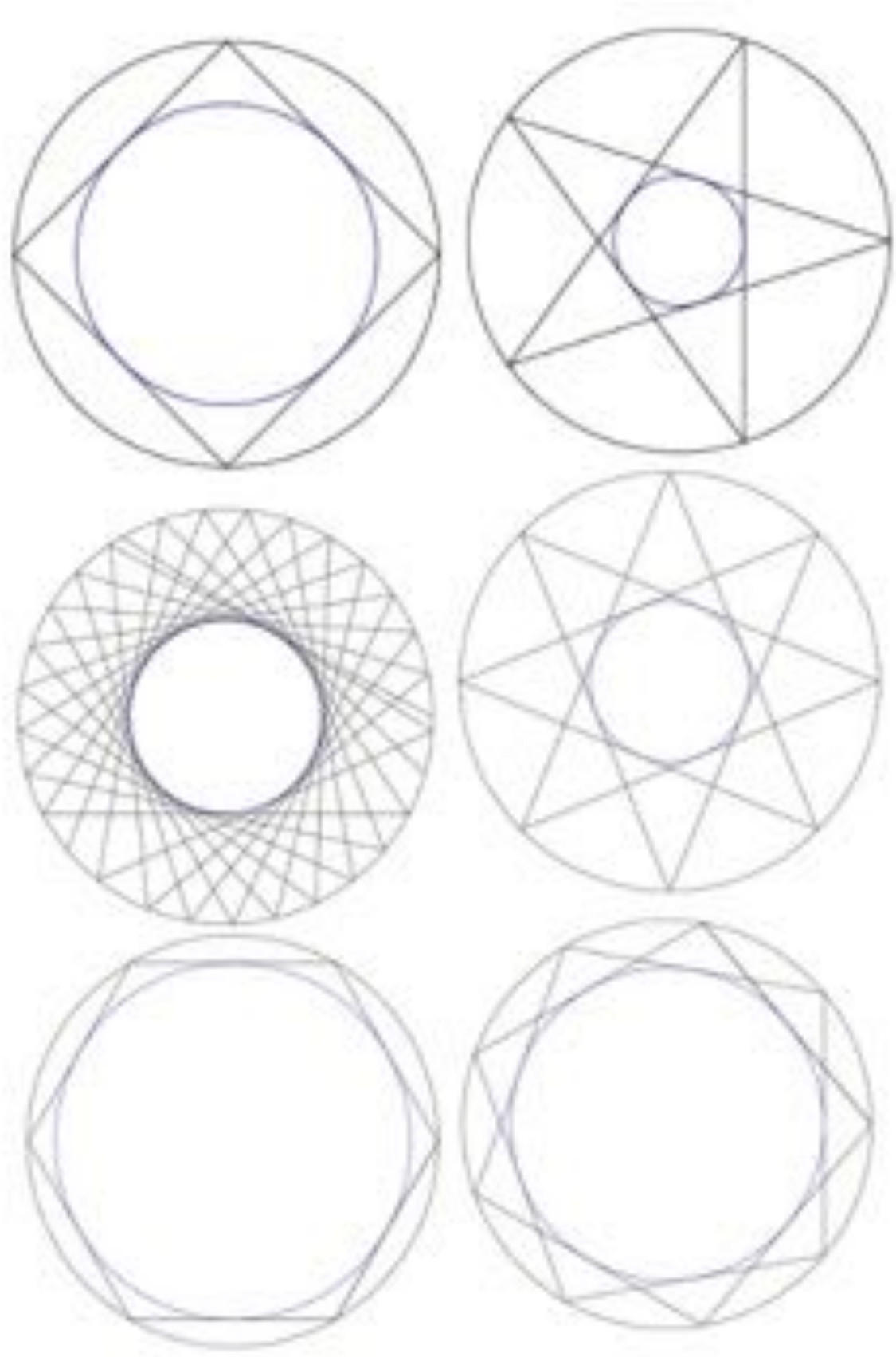
Why do this activity?

This activity gives learners practice in using a ruler and protractor to draw accurate geometrical constructions. To make the patterns learners will need to measure lengths and angles, to follow instructions and to draw accurately. The activity offers opportunities for talking about the geometrical properties of shapes.

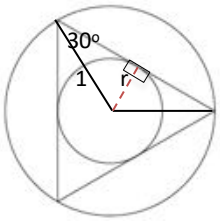
Teachers can plan for learners of different abilities by giving learners different patterns to draw. The activity may improve learners' attitude to mathematics by appealing to some learners who do not like mathematics and to others who find it difficult. The activity encourages creativity and learners will enjoy experimenting with different ways to adapt the designs to make their own patterns.

For older learners there are challenging applications to drawing tangents to circles and to moving furniture in restricted spaces.

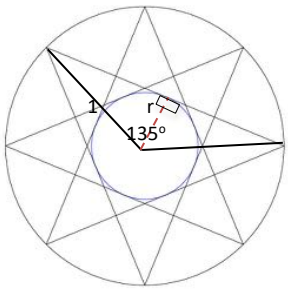
SOLUTION



What happens depends on the ratio of the radii of the two circles.



To draw circles with an equilateral triangle you need each tangent to subtend an angle of 120° at the centre. If the outer circle has radius 1 unit then the inner circle has radius r = 0.5



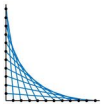
To draw circles with an 8-pointed star (8 tangents), where the path goes round the circle 3 times, you will have drawn 8 arcs on the circle, each subtending an angle $(3 \times 360^o)/8 = 135^o$.

The radius of the inner circle $r = \cos\frac{1}{2} \times 135^o = 0.383$ (to 3 dp)

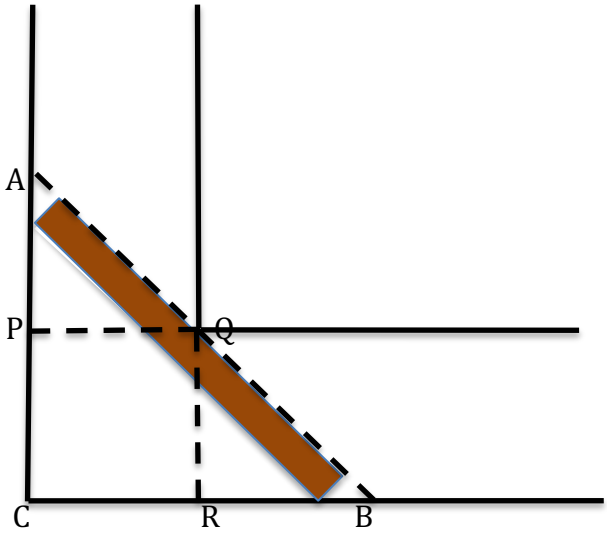
If the tangents make a shape with N edges in m revolutions then:

- N and m must be coprime (else the tangent lines are repeated);
- N edges and m revolutions give the same result as N edges and N – m revolutions, with the difference that the path is traced in the reverse direction;
- the path will return to the original starting point on the circumference of the outer circle if and only if the angle subtended by each tangent at the centre is a rational multiple of 360°.

BOOKCASE PROBLEM



As the bookcase goes around the corner the position where there is least space is when it is at 45° to the walls as in the diagram.



PQRC is a square with edge length 1 metre because the corridor is 1 metre wide. So AC= CB = 2 metres and $AB = 2\sqrt{2} \times 1000 = 2828$ mm (to nearest mm).

Notice the two small 45° isosceles right angled triangles where the bookcase touches the walls at A and B. The depth of the bookcase is 220 mm metres from front to back so you have to allow for an extra 220 mm at each end to get around the corner.

The bookcase has to be less than (2828 – 440) mm ≈ 2.388 metres long to go around the corner.

To avoid scratching the wall it should be less than 2 or 2.1 metres.

Learning objectives

In doing this activity students will have an opportunity to:

MEASURING ANGLES

- use a protractor to measure angles accurately;
- classify angles: angles < 90 ^{o} (acute angles); right-angles; angles > 90^{o} (obtuse angles); straight angles; > 180^{o} (reflex angles)

CONSTRUCTIONS

- use a compass, ruler and protractor appropriately to construct geometric figures accurately, including: angles, to one degree of accuracy.

Generic competences

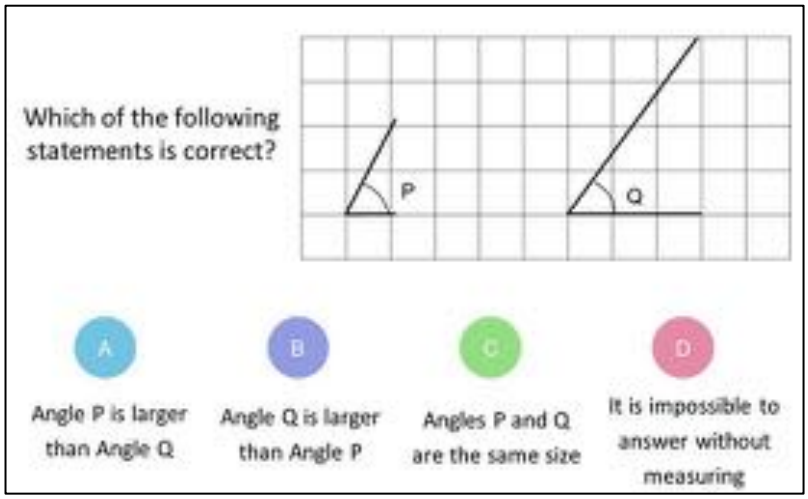
We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- be creative and innovative - to apply knowledge and skills;
- develop the skill of interpreting and creating visual images;
- engage in independent learning to develop the manual dexterity and the skill of using measuring and drawing instruments.

DIAGNOSTIC ASSESSMENT This can be done as a group as described below or the question can be answered individually. Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is A because the gradient of the sloping arm is 2 and the gradient of the sloping arm of angle Q is 1.5

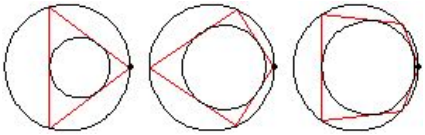
Possible misconceptions:

B. Students might think that, because the arms of angle Q are longer, it is the larger angle.

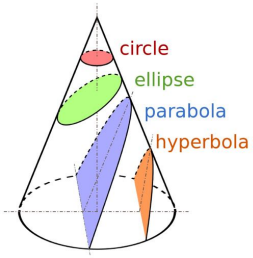
C. Students might not have looked closely at the angles.

D. Students giving this answer do not have a good understanding about measuring angles. It is not necessary to know about gradient, but students usually know about the steepness of hills so they should know that, with one arm horizontal, as the angle increases, the slope of the other arm gets steeper. https://diagnosticquestions.com

Follow up



Read about Jean Victor Poncelet and Poncelet's Porism.
https://mathworld.wolfram.com/PonceletsPorism.html
He was an officer in Napoleon's army when they invaded Russia in 1812. He did a lot of creative mathematics while he was in prison as a prisoner of war. He was forced to march for nearly five months across frozen plains to Saratov Prison on the banks of the Volga. When recovered he resolved to use his time in prison to recall all he could of his mathematical education.



He was interested in shadows cast on his prison wall. He observed that the shadow of an ellipse can look exactly like a circle when projected at an angle and then he proved properties of the ellipse by first proving the corresponding properties for a circle.

Poncelet showed that a conic section (conic) is a projective figure and he solved difficult problems in conics by projecting the conic, solving the problem for the circle then doing an inverse projection.
Find out more about Jean Victor Poncelet and Poncelet's Porism which is the basis for the Tangent Lines activities in this Guide.
See https://mathshistory.st-andrews.ac.uk/Biographies/Poncelet/
and https://www.encyclopedia.com/science/encyclopedias-almanacs-transcripts-and-maps/jean-victor-poncelet

Try these activities from the AIMING HIGH website.
Construct with circles
https://aiminghigh.aimssec.ac.za/years-7-9-constructions-with-circles/
Construct Circle and Line Patterns
https://aiminghigh.aimssec.ac.za/years-7-9-construct-circle-and-line-patterns/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13