AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

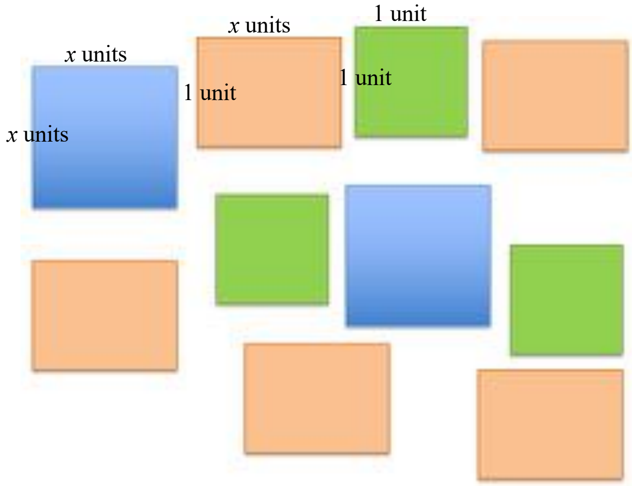
This INCLUSION SCHOOL AND HOME LEARNING GUIDE suggests related learning activities for all ages from 4 to 18 on the theme of THE CONNECTION BETWEEN MULTIPLICATION AND AREA

Just choose whatever seems suitable for your group of learners

The ALGEBRAREA activity was designed for Years 7 to 10.

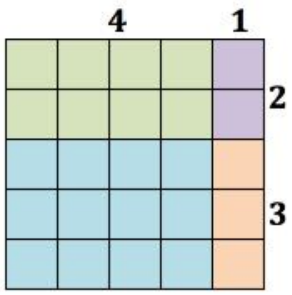
ALGEBRAREA Product of two brackets and area

1. Describe the three different types of pieces shown in the diagram.
2. Cut out the pieces on page 3. Using **all ten pieces**, make one BIG shape, draw it in your notebook and find its area in square units in terms of x.
3. Find as many different BIG shapes as you can, including a rectangle, that are made with the 10 pieces joined edge to edge. Draw them and find their areas in square units in terms of x.



4. What do you notice about the areas of the BIG composite shapes?
5. What mathematical relationships can you find from the area of the rectangle made with the 10 pieces?

HELP



Using this diagram, and counting squares, explain how you would work out (4 + 1) multiplied by (2 + 3) in different ways.

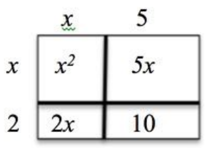
Apply your newly developed method to find different ways to write down the area of the big rectangle made from the 10 pieces shown above.

If you are having difficulties then think about the importance of subdividing the geometrical shapes into simpler units and then finding their areas.

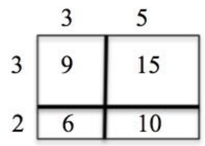
See https://aiminghigh.aimssec.ac.za/grades-7-to-9-partitioning/

NEXT

Compare the algebraic expression $(x + 2)(x + 5)$ to $(3 + 2)(3 + 5)$.



$(x + 2)(x + 5)$
$= x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$



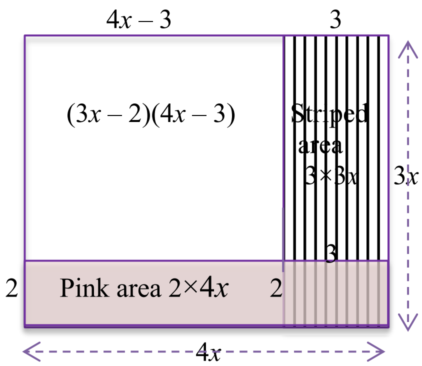
$(3 + 2)(3 + 5)$
$= 9 + 15 + 6 + 10$
$= 40$

Explain how these diagrams illustrate the products of the binomial expressions.

We talk about polynomials that have many terms, but when there are 2 terms we call them *binomial* and when there are 3 terms we call them *trinomial.*

Make up some examples for yourself and work out more problems involving expansion of brackets where all the terms are positive, such as $(3x + 2)(2x + 3)$, using a diagram if you find it helpful.

When you are confident about the method without needing to draw a diagram, then multiply binomials with negative terms such as $(2x - 3)(2x + 4)$. The illustration representing $(3x - 2)(4x - 3)$ is given as an example. Remember the rules that multiplying two positive numbers or two negative numbers gives a positive number and multiplying a positive and a negative gives a negative number.

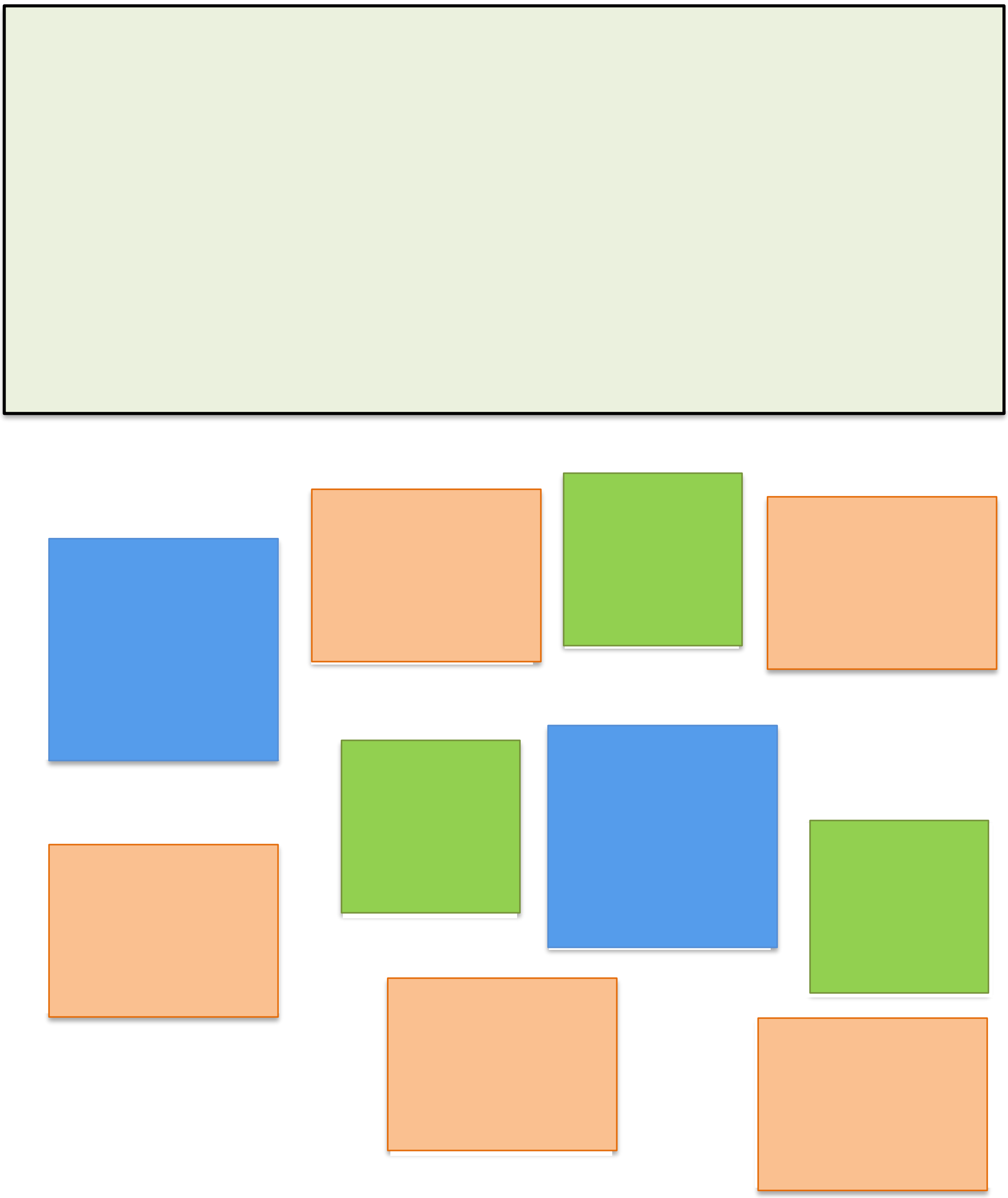


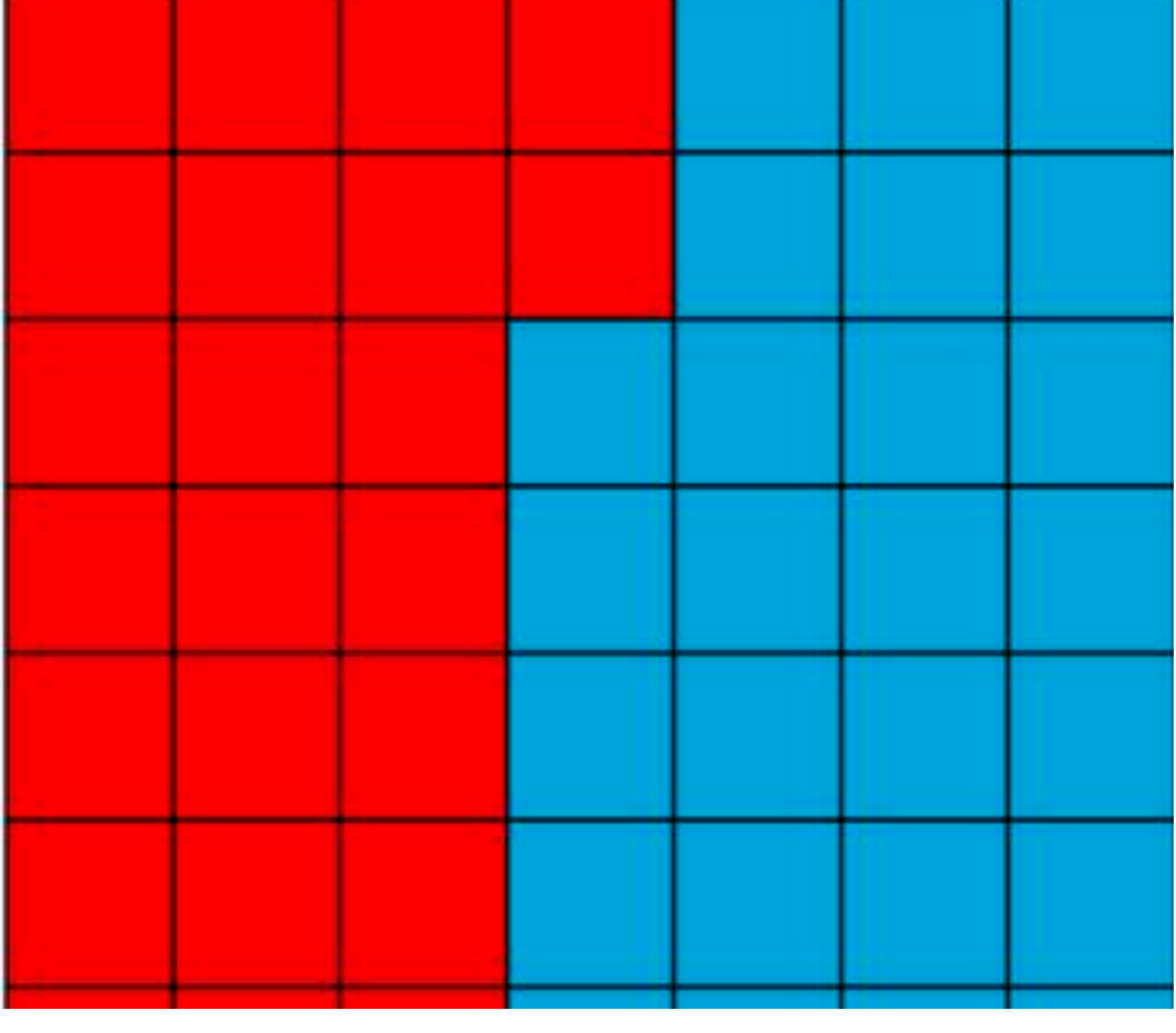
The unshaded area is $(3x - 2)(4x - 3)$, given by the area of the large rectangle minus the pink area, minus the striped area plus the overlapping area that has been deducted twice.

So $(3x - 2)(4x - 3) = 3x \times 4x$
$-2 \times 4x$
$-3 \times 3x$
$+2 \times 3$

COMMON TASK FOR A MIXED-AGE GROUP

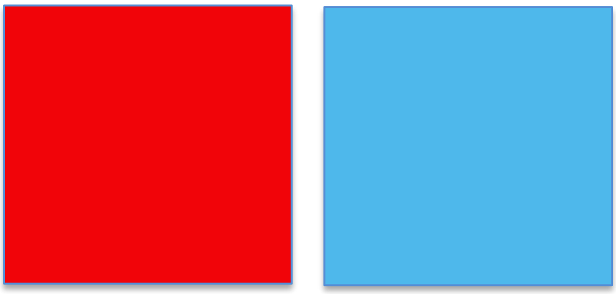
Cut out the 10 shapes below.
Solve the puzzle by fitting the smaller pieces into the big rectangle?
Older learners could help younger learners to solve the puzzle.





Early Years

Print and cut out the squares on page 4 or cut out 54 squares measuring 3 cm by 3 cm from scrap card. Let the children play with the squares. After some free play suggest that they might use them like tiles and make some patterns.



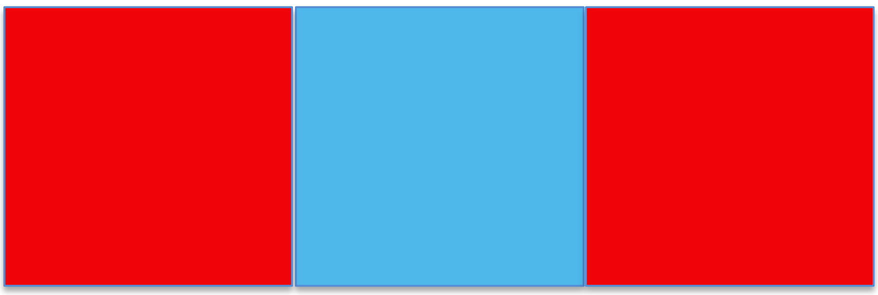
Look together at the patterns below.

Ask "What is the same and what is different about the patterns?"

Ask the children to arrange their tiles into similar patterns.

Look out for tiling patterns wherever you go, for example in a shopping mall. Are the tiles square or, if not, what shape are they?

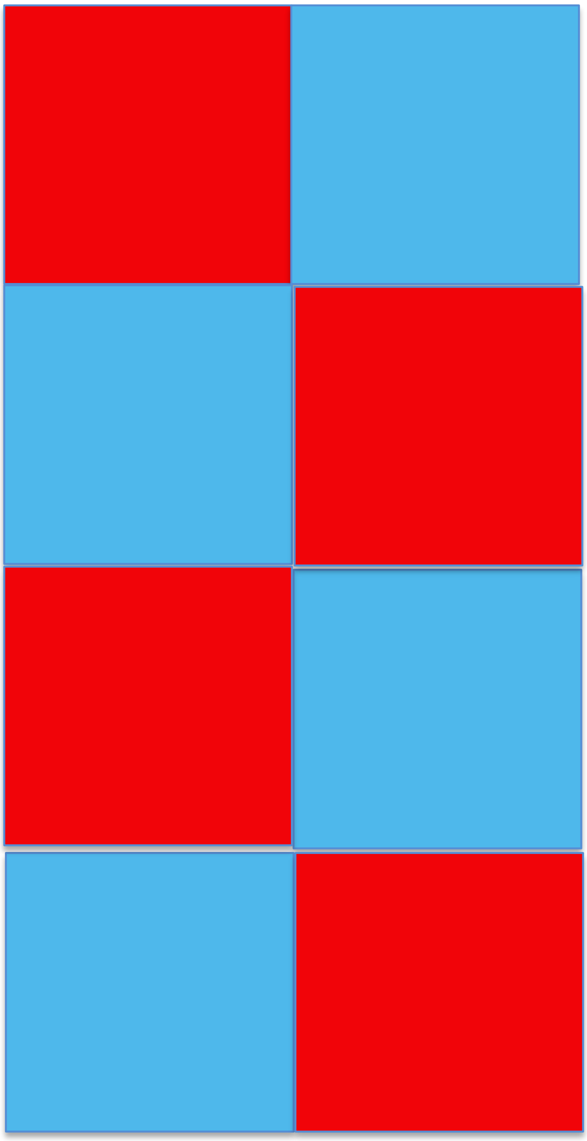
Ask the children to make some rectangles with their tiles and to count the squares in their rectangles.



3 tiles.

1 row of 3.

2 red tiles, 1 blue tile



8 tiles.

4 rows of 2.

4 red tiles, 4 blue tiles

Make a poster to show rectangles and the number of squares in each rectangle.

Talk about each small square having an area of 1 unit. What is the total area of each rectangle?

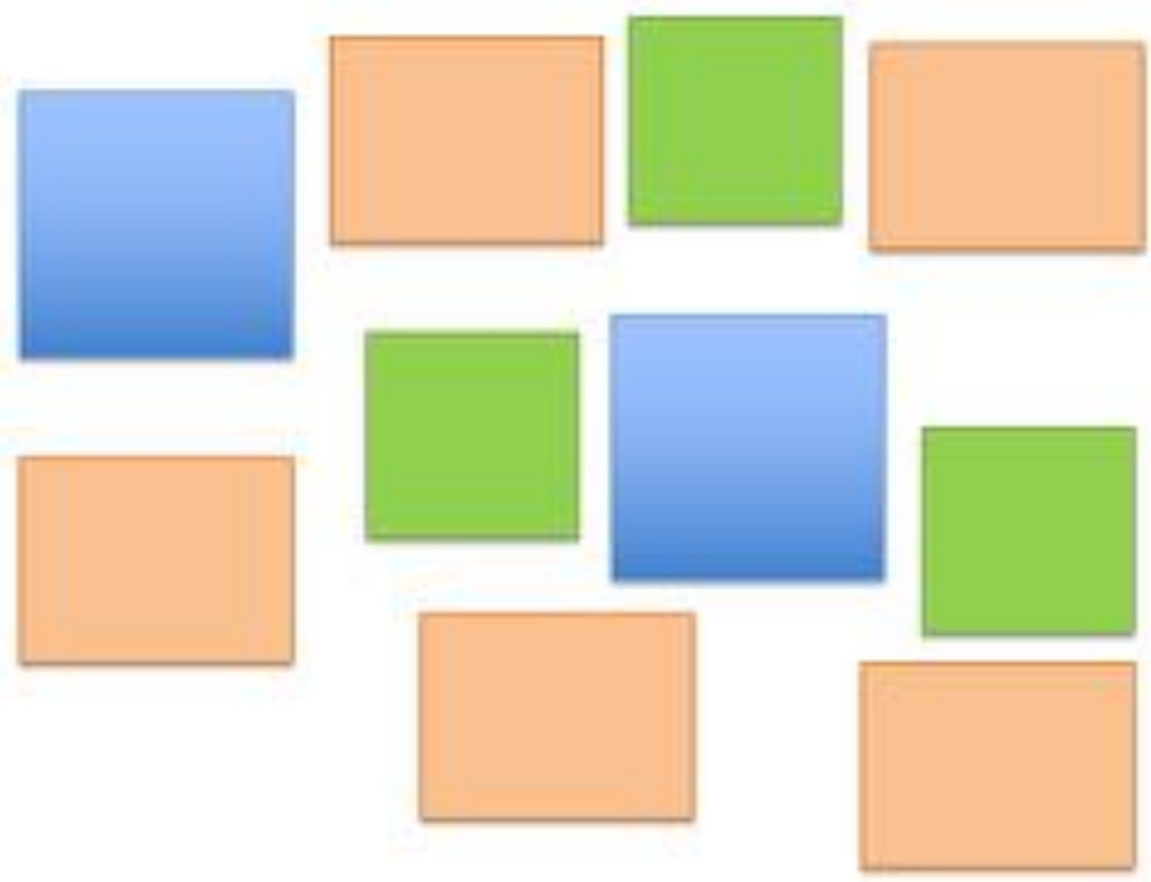
Lower Primary

The learners should do the activities described for the Early Years if they have not already done them.

Cut out these pieces. Ask the children to describe what they see and tell you about the different colours and shapes.

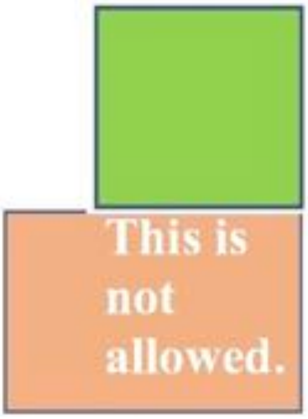
Ask “what is the same and what is different about the blue shape and the green shape?”

Maybe the learners could draw 10 small pictures in the shapes, then make a pattern with the shapes, and stick them in a notebook or on a backing sheet.



Cut out the shapes on page 3. Ask the learners to solve the puzzle by fitting the 10 smaller pieces into the large pale green rectangle. You might fit the pieces into the pale green rectangle to show the children how it is done, then take them out and ask the children to put the pieces back.

Talk about the shapes. Ask the learners: “How many other shapes can you make with these 10 pieces matching them edge to edge?”

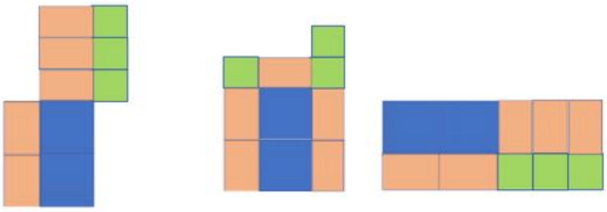


Talk about the **10 shapes together making the same area** whatever way you arrange them.

Tell the learners to draw the shapes that they make.

For example they might make shapes like this:

The last shape here is the solution to the puzzle.



Upper Primary

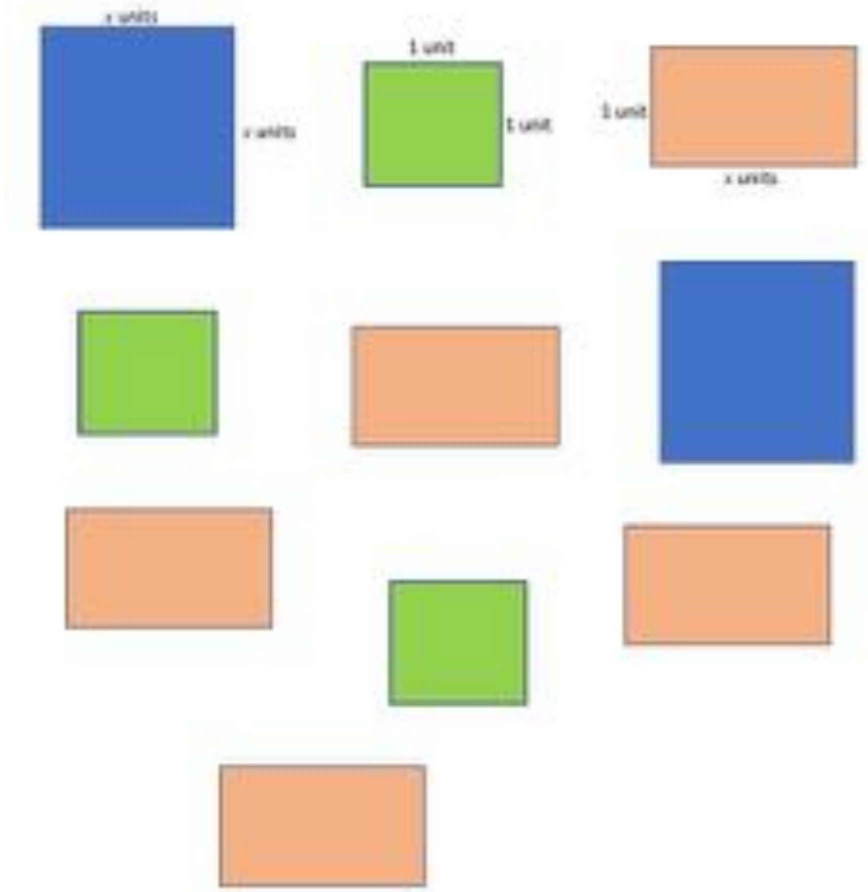
Cut out the shapes on page 3 or make your own shapes. Chose a unit of length. For example, a convenient unit is 3 cm. Cut out 2 squares measuring x cm by x cm for any value of x, 3 squares measuring 1 unit by 1 unit for your chosen unit of length, and 5 rectangles measuring x units by 1 unit.

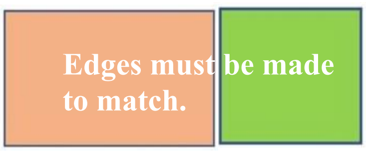
Talk about x being just a name that could be any number. Talk about the different shapes.

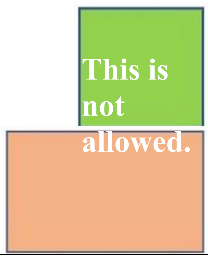
Ask the learners to solve the puzzle by making a rectangle with the 10 pieces.

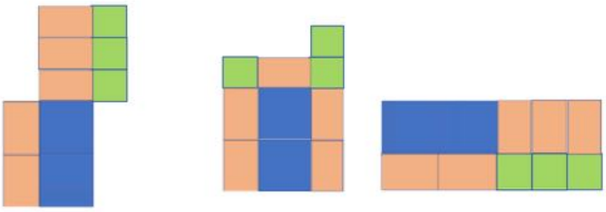
How many other shapes can your learners make with these 10 pieces matching the pieces edge to edge?

Learners should draw the shapes that they make and write down their areas in square units in terms of x.









These are some of the arrangements that the learners might make. The last one is the solution to the puzzle.

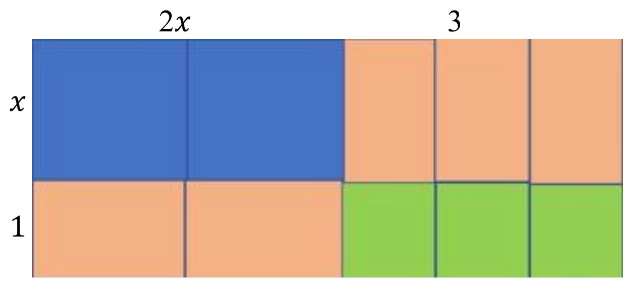
Lower Secondary – Years 7 and 8

To prepare for this work, your learners could do, the Partitioning learning activity https://aiminghigh.aimssec.ac.za/years-7-9-partitioning/. It is a good starter task.

Try to make the lesson as practical and engaging as possible. Learners can either work individually in their exercise books or work in small groups.

To enable all learners to tackle this question and experience success (planning for differentiation) you can give the HELP section to those learners who struggle to get started and give the NEXT section to learners who finish ahead of the rest of the class.

Let learners explore all the options; there are still more combinations they can come up with other than those shown in the solutions section. When most of the group have found several combinations, and worked out the areas, the teacher can conduct a discussion. Learners could be asked to draw on the board the different combinations that they have found so that they share their ideas. Everyone can copy the shapes that they had not found previously into their exercise books.



Adding the areas of the individual pieces is a common strategy for finding the total area. Ideally, most learners should notice that ALL the composite shapes have the same area. Most learners will notice that the **rectangle** can be used to calculate the area using the length and width of the shape.

Ask the class why all the shapes they have made with the 10 pieces have the same area. Let the learners explain their reasons. It may be obvious to some learners that; logically, we are using the same pieces whose area remains the same. This is called **conservation of area.**

Key questions

- Can you make a rectangle with the 10 pieces?
- Can you subdivide the rectangle into 4 simpler subunits and find the 4 areas?
- You have made some different shapes using the 10 pieces. Do you notice anything about the areas?
- Could you draw a rectangle to multiply 28 by 34 and multiply the tens separately from the units?
- Suppose you need to multiply $(p + q)$ by $(r + s)$, can you draw a rectangle with edges $(p + q)$ and $(r + s)$ and find 4 areas: and in this way find the product of the two expressions?
- Suppose you need to multiply $(3x + 2)$ by $(4x + 5)$, can you draw a rectangle with edges $(3x + 2)$ and $(4x + 5)$ and find 4 areas and in this way find the product of the two expressions?
- Can you apply the distributive law to multiply a binomial by another binomial?

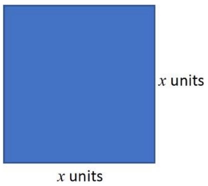
The diagram on the previous page and the solutions to parts 4 and 5 and the HELP section make a good summary of lesson.

SOLUTION for Years 7 and 8

1. The geometrical shapes that can be identified are squares and rectangles (2D shapes):

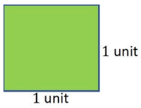
3 identical small squares (green)
2 identical bigger squares (blue)
5 identical rectangles (brown)

Identical shapes are said to be **congruent.** Teachers can discover whether the learners are familiar with the terminology.



Blue square:

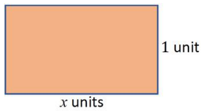
Area = x units × x units
= x^2 square units



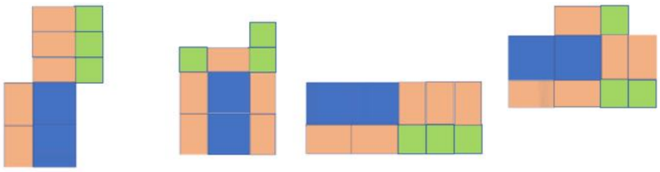
Green square:

Area = 1 unit × 1 unit
= 1 square unit

Brown rectangle:



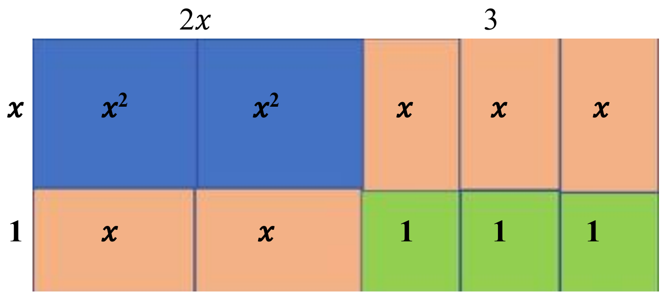
Area = x units × 1 unit
= x square units



2 and 3. Some images of possible combinations.
All have area $2x^2 + 5x + 3$

4. There is **conservation** of area in this case; logically, we are using the same pieces whose area remains the same.
Each shape should give us the following results:
Area = $x^2 + x^2 + x + x + x + x + x + 1 + 1 + 1$ units^2
= $2x^2 + 5x + 3$ units^2

5.
Area of rectangle = Length × Width
= $(2x + 3)(x + 1)$
square units.



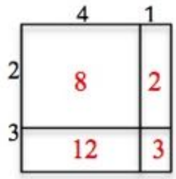
From 4. the area was found to be $2x^2 + 5x + 3$
From 5. the area is $(2x + 3)(x + 1)$.
This gives the mathematical relationship $(2x + 3)(x + 1) = 2x^2 + 5x + 3$

HELP SECTION

$(2 + 3)(4 + 1) = (5)(5) = 25$

$(2 + 3)(4 + 1) = 2(4 + 1) + 3(4 + 1) = 2(5) + 3(5) = 10 + 15 = 25$

or $(2 + 3)(4 + 1) = 8 + 2 + 12 + 3 = 25$ as shown by the diagram.



$(2x + 3)(x + 1) = 2x(x + 1) + 3(x + 1)$

$= 2x^2 + 2x + 3x + 3$

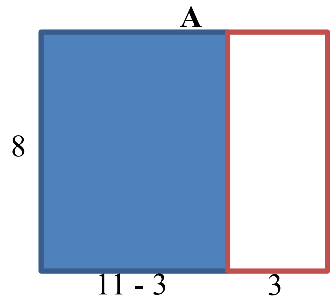
$= 2x^2 + 5x + 3$

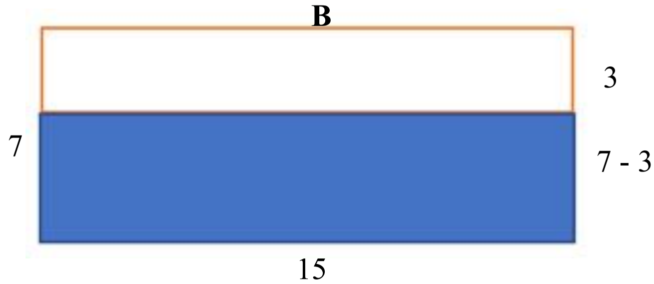
Years 9 and 10

1. Match each expression in the first column with its value in the second column.

Expression	Value
$(7 + 2)\,(5 + 3)$	20
$(7 + 2)\,5 + 3$	23
$7 + 2 \times 5 + 3$	72
$7 + 2 \times (5 + 3)$	48

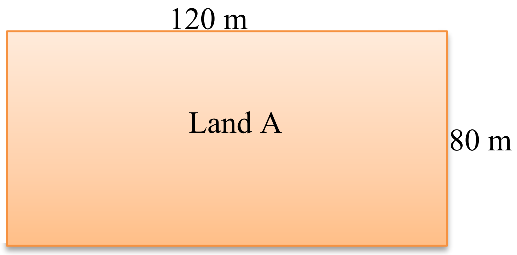
2.

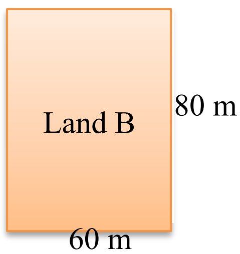




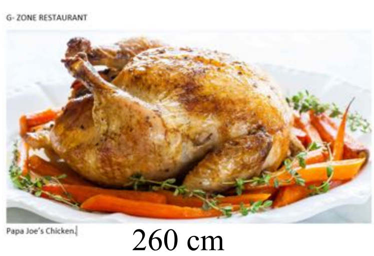
Find the area of the shaded rectangle in **two ways** in each case.

3.





The two rectangles represent pieces of land, Land A and Land B, which is up for sale. The assessed **values** of the property are **proportional** to their **areas.** The value of L and B is R95 000. What is the value of Land A?



4. The length of a **billboard** is 260 cm; its height if 50 cm less than the length.

(a) Write an expression for the area of the billboard.

(b) Use the distributive law to rewrite the expression.

(c) Find the area of the billboard in square centimetres.

SOLUTION for Years 9 and 10

1. Area of shaded part $= 8 \times 11 - 8 \times 3$
$= 88 - 24$
$= 64$

or Area of shaded part $= 8\,(11 - 3)$
$= 8 \times 8$
$= 64$

B: Area of shaded part $= 7 \times 15 - 3 \times 15$
$= 105 - 45$
$= 60$

or Area of shaded part $= 15 \times (7 - 3)$
$= 15 \times 4$
$= 60$

2.

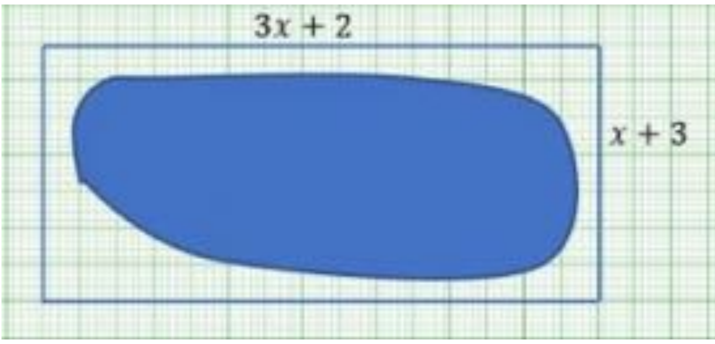
Expression	**Value**
(7 + 2) (5 + 3) →	72
(7 + 2) 5 + 3 →	48
7 + 2 × 5 + 3 →	20
7 + 2 × (5 + 3) →	23

BIDMAS may help you to remember the order of operations:
Brackets/**I**ndices/**D**ivision/**M**ultiplication/**A**ddition/**S**ubtraction
Some people remember this as **BODMAS**
where the **O** stands for **O**rder.

3. Land A and Land B have equal widths, and B is twice as long as A, so the areas are in the ratio 2 : 1
Value of Land A : value of Land B = 2 : 1
Value of Land B = R95 000 so value of Land A is R190 000

4. (a) Area of billboard $= 260 \times (260 - 50)$ cm^2
(b) $260 \times (260 - 50) = 260\,(260) - 260\,(50)$
(c) $= 67\,600 - 13\,000$
$= 54\,600$ cm^2

Years 11 and 12



For security John plans buy the minimum amount of Diamond Razor Mesh fencing material to fence around his dam and discourage entry.

He sent his 15 year old son Mzu to measure the dam.

Mzu decided to test his Dad to find out if he can solve quadratic equations. He tells his Dad that, to help him, he has drawn this rectangular shape of the dam and the rectangular fence.

Mzu has given the dimensions in algebraic form, in metres.

Find an expression for the area of this rectangle.

If the area of the rectangle is 66 m^2 can you write down and solve a quadratic equation to find *x*?

The solution to this Quadratic Equations problem, and more learning activities, can be found by clicking the link below.

https://aiminghigh.aimssec.ac.za/quadratic-equations/

Key questions

- What do you understand by the expression 'binomial factors'?
- What is the expansion of the two brackets $(\boldsymbol{x}+\mathbf{2})(\boldsymbol{x}+\mathbf{3})$? Expand and simplify the algebraic expression.
- Can you solve the quadratic equation $\boldsymbol{x}^{\mathbf{2}}+\mathbf{5}\boldsymbol{x}+\mathbf{6}=\mathbf{0}$ through factorisation?
- What is the leading coefficient in the equation $\mathbf{3}\boldsymbol{x}^{\mathbf{2}}+\mathbf{11}\boldsymbol{x}-\mathbf{60}=\mathbf{0}$?
- If you factorize $\mathbf{3}\boldsymbol{x}^{\mathbf{2}}+\mathbf{11}\boldsymbol{x}-\mathbf{60}=\mathbf{0}$ what first terms would you put in the brackets?
- If you factorize $\mathbf{3}\boldsymbol{x}^{\mathbf{2}}+\mathbf{11}\boldsymbol{x}-\mathbf{60}=\mathbf{0}$ what can you say about the signs in the brackets?
- What pairs of numbers when one is subtracted from the other, make 11? Can you make 11 in different ways.

Why do this activity?

In order to develop conceptual understanding of the product of two binomials, this activity links algebra with area problems in some common 2D shapes like squares and rectangles. Learners develop skills of manipulating a product of two binomials to obtain a quadratic expression; work which prepares them for solving quadratic equations that follow in subsequent lesson activities.

The activity consolidates learner's knowledge and understanding of conservation or area, and of tessellation, in geometry, especially when they are playing with the pieces to obtain different 2D shapes. For those who will go into the construction industry or do their own home improvement, for example, the work will resonate with problems of tiling floors and laying patio tiles.

The activity is a preparation for work that involves the application of quadratic equations in solving area problems in 2D shapes; we focus on the connections between areas of simple geometrical shapes like squares and rectangles and the product of two binomials.

This activity becomes useful as prior knowledge for learners when they get into topics like 'nets' of 3D shapes. The connection between geometry and the algebraic terrain cannot be over emphasized.

Learning objectives

In doing this activity students will have an opportunity to:

- construct quadratic expressions through area problems in 2D shapes;
- calculate areas of squares and rectangles using dimensions given in algebraic form;
- establish a mathematical relationship between area (algebraic expression) and product of two binomials;
- find the product of two binomials by expanding the brackets;
- develop deep conceptual understanding of the construction of quadratic expressions by the product of two binomials.

Generic competences

In doing this activity students will have an opportunity to:

- develop algebraic manipulative skills and to recognize the equivalence between different representations of the same relationship;
- co-operate, collaborate and work in a team;
- have empathy with others, listen to different points of view.

DIAGNOSTIC ASSESSMENT This should be done before the lesson in the group or class as described below, or the question can be answered individually.
Show this quiz question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

1.Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2.It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3.With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

QUIZ 1
The correct answer is: C
If the learners don't recognise this then time spent discussing how the diagram provides a method of long multiplication will be a valuable learning experience for those involved.

QUIZ 2
The correct answer is: D
The statements given in the other choices A. B. & C. are true but the correction is not given
https://diagnosticquestions.com

QUIZ 1

×	20	9
50	1000	450
1	20	9

The table shows:

A. 52 × 19 = 988
B. 25 × 19 = 475
C. 51 × 29 = 1479
D. 51 × 19 = 969

QUIZ 2

SPOT THE MISTAKE
How do these methods work?
Find the mistake and put it right.

$$\begin{array}{r} 29 \\ \times 51 \\ \hline 145 \\ 290 \\ \hline 435 \end{array}$$

×	20	9
50	1000	450
1	20	9

29 × 51
= 1000 + 450 + 20 + 9
= 1479

A. 435 is the wrong answer
B. Units digit in the multiple of 50 should be 0
C. 1 × 29 should be 29 not 290
D. The multiplication algorithm is wrong – it should produce 1450 + 29 = 1479

Follow up

PRIMARY

MD https://aiminghigh.aimssec.ac.za/md/

Multiplication Squares
https://aiminghigh.aimssec.ac.za/multiplication-squares/

Two by Two Puzzle https://aiminghigh.aimssec.ac.za/two-by-two-puzzle/

Can you help these farmers?
https://aiminghigh.aimssec.ac.za/can-you-help-these-farmers/

LOWER SECONDARY

Partitioning https://aiminghigh.aimssec.ac.za/partitioning/

Pair Products https://aiminghigh.aimssec.ac.za/pair-products/

Use Area to find x https://aiminghigh.aimssec.ac.za/use-area-to-find-x/

Muggles Magic https://aiminghigh.aimssec.ac.za/muggles-magic/

UPPER SECONDARY

Quadratic Matching 1 and Quadratic Matching 2
https://aiminghigh.aimssec.ac.za/quadratic-matching-1/
https://aiminghigh.aimssec.ac.za/quadratic-matching-2/

Quadratic Equations
https://aiminghigh.aimssec.ac.za/quadratic-equations/

Quadratic Functions
https://aiminghigh.aimssec.ac.za/quadratic-functions/

Graphing Quadratic Equations
https://aiminghigh.aimssec.ac.za/graphing-quadratic-equations/

MATHS TOYS

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

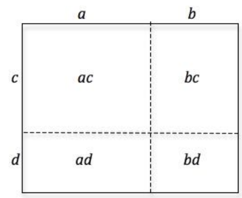
Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13

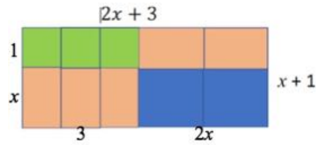
Appendix

List of learning tasks in this Guide:

1. **Puzzle Activity for All:** Fit the 10 pieces into the rectangular frame (page 3)
2. **Early Years:** Make tiling patterns and rectangles with square tiles.
3. **Lower Primary:** Do the puzzle and compare it with other patterns made from the 10 pieces by matching edges. Notice that the area stays the same when the pieces move to make different shapes.
4. **Years 7 and 8.** Finding areas of rectangles and composite shapes made from rectangles, some with lengths given in numbers other in terms of x. For example rectangles split into 4 smaller rectangles to show (4 + 1)(2 + 3) and (x + 2)(x + 5)



5. Multiplication of 2 digit numbers using area
   $23 \times 45 = (20\times40) + (20\times5) + (3\times40) + (3\times5) = 800 + 100 + 120 + 15 = 1035$
6. Matching expressions with their values e.g. $(7 + 2)(5 + 3) = 72$
7. Putting the Algebrarea jigsaw pieces together to make a rectangle.
8. Area of the rectangle produced from the Algebrarea jigsaw
   $(2x+3)(x+1) = 2x^2 + 5x + 3$



9. **Years 9 and 10:** Multiplication of binomial and trinomial algebraic expressions using area diagrams.
10. Applying the area method for multiplication with negative terms.
11. Application to areas of land in square metres.
12. Application to area of a billboard in square centimetres.
13. **Years 11 and 12:** Reading information, interpreting information in mathematical terms, forming a quadratic equation, solving the equation, and interpreting and applying it to solve the original problem. For example, from information given form and solve the equation: $(3x + 2)(x + 3) = 66$ (see page 15).