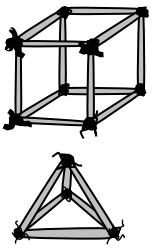
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

FLEXIBLE SHAPES

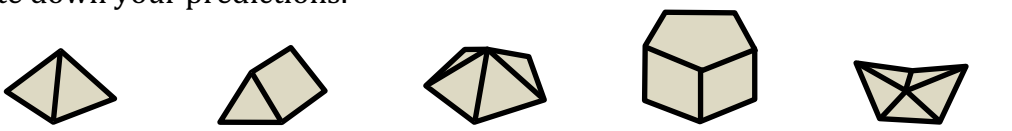


If you have not already done so, try the
Collapsible Cube activity and watch the video.

1. Make a tetrahedron out of rolled paper sticks. Press it gently. See that it does not change its shape – it is rigid.

2. Make the five 3D shapes below with paper sticks, all the same length.
   Before you make each 3D shape, imagine it in your mind. Try to visualise how it will behave. Do you think that it will be:
   a) Rigid, like the tetrahedron, so it will not change its shape at all?
   b) Collapsible, like the cube, so it will collapse into one or more flat shapes?
   c) Not rigid, but will not collapse into a flat shape?

3. Write down your predictions.



Square pyramid	Triangular prism	Pentagon pyramid	Pentagonal prism	Boat octahedron

4. Sketch, and name or describe, the shapes that each of the 3D shapes can make.

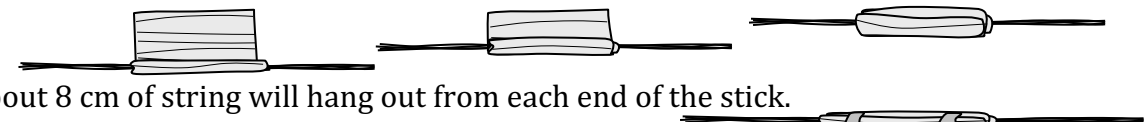
HELP

Follow the instructions below, or watch this video: https://youtu.be/iaJ6EitIGKU, to learn how to make rolled paper sticks.

You need: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

Cut a rectangle of paper,
8 cm long and about 6 cm wide.
Cut a length of string about 24 cm long.

Carefully roll the rectangle of paper around the string to make a stick.
Roll it as tightly as you can.



About 8 cm of string will hang out from each end of the stick.
Fasten the paper with sticky tape. You have made your first paper stick!

Make more paper sticks. How many will you need to make your 3D shape?
(Each edge of your shape will need one paper stick.)

NEXT

When you have made several 3D shapes, begin to compile your results into a table. This could show: the mathematical name of the 3D shape (if it has one); the shape(s) of its faces; the number of its edges, faces and vertices; whether it is rigid or can change its shape; what shapes it can change into. Remember to include the cube and the tetrahedron on your table.

Then think of another 3D shape that you can try.

Try to predict, *before* you make each 3D shape, whether it will be rigid or whether you will be able to change its shape.
Try to visualise and describe how your 3D shape will behave.
Record what you predict, and what you find out.

If you are working with a group of learners then you can share your results. Build up a complete record of everything you have discovered about the shapes you have made.

Resources: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

NOTES FOR TEACHERS

SOME SOLUTIONS

3D shapes made with paper sticks change in different ways. These include (but are probably not limited to) those shown in the table below.

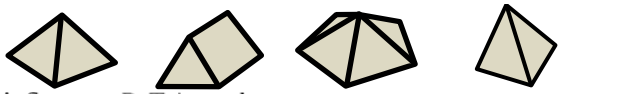
3D Shape	Behaviour	Changes into
Cube	Collapsible	equilateral triangle; square; rectangle; regular hexagon; irregular hexagon; rhombus; trapezium
Tetrahedron	Rigid	
Square pyramid	Collapsible	rhombus; equilateral triangle
Triangular prism	Collapsible	trapezium; irregular pentagon; equilateral triangle
Pentagon pyramid	Neither rigid nor fully collapsible	tetrahedron
Pentagonal prism	Collapsible	trapezium; equilateral triangle; irregular pentagon; irregular hexagon; irregular heptagon
Boat octahedron	Rigid	

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write each question on the board. Say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1) Tandi makes a collapsible cube out of paper sticks.
She collapses her cube into a flat regular hexagon.
She folds her regular hexagon in half along one diagonal.
Then she folds the resulting shape into three.
What flat shape has she made now?
A. Triangle B. Tetrahedron
C. Trapezium D. Square

2) Eddy makes these four shapes out of paper sticks, all the same length, tied together at the corners.



A. Square pyramid B. Triangular prism C. Pentagon pyramid D. Tetrahedron

Which of the shapes can he collapse into a flat trapezium?
A. Square pyramid B. Triangular prism
C. Pentagon pyramid D. Tetrahedron

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
2. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. Again ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. If a learner has changed from a wrong to a right answer, you could ask them to explain what made them change their mind. This will help to embed their reasoning so they can recall it later.

4. For each question, you could demonstrate the correct answer with the relevant shapes made out of paper sticks.

The correct answers are: q1) A. *Triangle*; q2) B. *Triangular prism*

Why do this activity?

As learners make and explore the 3D shapes they will discover how some of these can be collapsed into flat shapes, while others cannot. They will use their experience of manipulating the shapes to begin to visualise and predict their behaviour. They can discover for themselves the effect of triangles on rigidity.

Learning objectives

In doing this activity students will have an opportunity to:

- visualise and create a range of 3D shapes;
- explore and record their properties;
- visualise and predict how the shapes may (or may not) change;
- test their predictions;
- record what they find out;
- analyse their results and make further predictions.

Generic competences

In doing this activity students will have an opportunity to:

- **search systematically** for new examples;
- **visualise** 3D shapes and predict how they may change;
- **test** their own predictions;
- **share, organise and classify** their findings;
- **record** their own discoveries;
- **think creatively** to find new examples of 3D shapes to try.

Suggestions for teaching

Before starting Flexible Shapes the learners should have worked on the Collapsible Cube activity which precedes this one and watch the Flexible Shapes video.

Emphasise that all the paper sticks should be the same length. If learners work in small groups then they can share the task of making the paper sticks, but it is important that every learner should have the opportunity to handle all the 3D shapes that they work on.

Ask learners to imagine each 3D shape in their minds **before** they make it, and to describe what they see. Encourage them to discuss how they think it will behave, and whether it will be rigid or will be able to change its shape. The important point here is

not to get the 'right' answer, but to begin to visualise what the shapes may look like and to predict what may happen.

When learners explore shapes that they have made you could invite them to demonstrate what they discover to the rest of the class. For example, if some learners have found a 3D shape that is not rigid but does not collapse into a flat shape, you could ask them to show this to the other learners.

When learners begin to enter their findings on a table you could encourage them to share their results in order to build up as complete a record as possible. Point out that the table can never be finalised – there will always be other 3D shapes that could be added. Ask, 'What else might we do, to add to our results?' Explain that this is how mathematics progresses, as mathematicians share and build on one another's discoveries.

Key questions

Key Questions to develop understanding

- Look at the pictures of the five 3D shapes given on the worksheet.
  Which ones do you think will be rigid? Which ones do you think will change their shapes?
- Imagine and describe what you think will happen. Then make the shapes and see.
- What could you change in order to make a different 3D shape?
- Can you imagine a different 3D shape that you think will be rigid?
- Can you imagine one that you think will change its shape?
- Sketch your new shapes, then make them and see.
- How can you record what you have found out?

Key Questions to check knowledge and understanding

- Look at the cube and the tetrahedron made with paper sticks.
  Which one is rigid? What makes it rigid? Why is the other one not rigid?
- What are the properties of the 3D shapes you have made? How many edges/vertices/faces do they have? What shape(s) are the faces?
- Can you show everything you have found so far on a table?
  Have you included all the 3D shapes you have made on your table?
- Compare two 3D shapes that you have made.
  What is the same about them? What is different?
- Which of the 3D shapes you have made can be changed into flat shapes? Which cannot?
- Can two different 3D shapes both make the same flat shape?
  Find some examples.
- What is the same about the flat shapes that one of your 3D shapes will make? What is different?
- Can you use the same shaped faces to make two different 3D shapes? Do they both behave in the same way?

Follow up

Shadows Activity https://aiminghigh.aimssec.ac.za/years-4-8-shadows-activity/
How do you see it? https://aiminghigh.aimssec.ac.za/years-4-to-9-how-do-you-see-it/
Three Views https://aiminghigh.aimssec.ac.za/years-4-8-three-views/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13