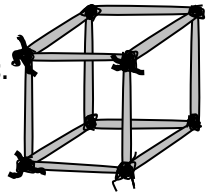
AIMSSEC

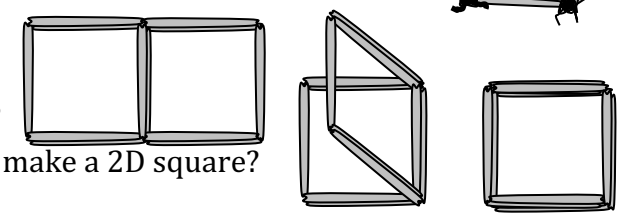
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

COLLAPSIBLE CUBE



1. Make a 3D cube out of rolled paper sticks, tied together at the ends.
   Push your cube gently down and sideways.
   It will collapse into a 2D shape!

   Can you collapse your cube into
   a 2D rectangle made of 2 squares?
   Can you fold the rectangle over to make a 2D square?



2. Find another 2D shape that you can collapse your 3D cube into.
   Sketch your new 2D shape.
   Does it have a mathematical name? Or can you describe it?

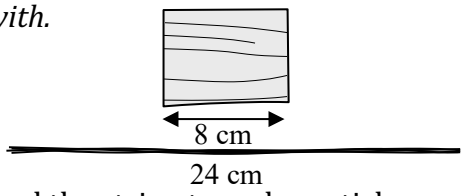
3. What other 2D shapes that can you make with your collapsible 3D cube?
   Sketch each one you find, and name it or describe it carefully.

HELP

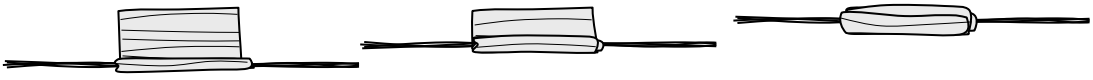
Follow the instructions below, or from this video: https://youtu.be/iaJ6EitIGKU **on how to make rolled paper sticks.**

You need: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

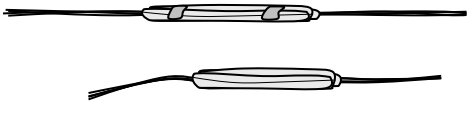
1. Cut a rectangle of paper,
   8 cm long and about 6 cm wide.
   Cut a length of string about 24 cm long.



2. Carefully roll the rectangle of paper around the string to make a stick.
   Roll it as tightly as you can.



3. About 8 cm of string will hang out from each end of the stick.
   Fasten the paper with sticky tape.
4. You have made your first paper stick!
   Make more paper sticks.
   How many will you need to make a cube?
   (Each edge of the cube will need one paper stick.)



NEXT

Think of a different 3D shape that you can make with paper sticks, all the same length.

Will it collapse into a 2D shape? Or will it be rigid? Or will it do something different? *Before* you make your new 3D shape, try to visualise it and describe how it will behave.

Then check your predictions. Record what you find with sketches and words.

Resources: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

NOTES FOR TEACHERS

SOLUTION

The paper stick cube can be collapsed into a number of 2D shapes, including a triangle, rectangle, regular hexagon, irregular hexagon, square, rhombus, and trapezium.

Other 3D shapes made with paper sticks behave in different ways. Deltahedra (shapes with triangle faces) are rigid and will not change their shapes.

Some 3D shapes with differently shaped faces, such as a square-based pyramid or a triangle prism, can make a 2D shape, but some others, such as a pentagonal pyramid, can be folded into a tetrahedron but not into a 2D shape.

There are many shapes to explore, and a great deal to discover.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the questions one at a time on the board. Say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C or 4 fingers for D".

Tandi made a cube out of paper sticks tied together at the corners.

1) How many corners (vertices) does the cube have?
   A. 4 B. 6 C. 8 D. 10

2) Tandi wants to make some 2D shapes with her cube.
   Which ONE of these 2D shapes CAN'T she make?
   A. Square B. Rectangle C. Pentagon D. Hexagon

3) Tandi makes a regular hexagon with her cube.
   Then she folds the regular hexagon over along one diagonal to make another 2D shape.

a. How many sides does her new 2D shape have?
   A. 3 B. 4 C. 5 D. 6

b. What is the mathematical name of the new 2D shape?
   A. Square B. Rectangle C. Trapezium D. Heptagon

2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. If a learner has changed from a wrong to a right answer, you could ask them to explain what made them change their mind. This will help them to understand the reasoning for themselves.
5. For each question, you could demonstrate the correct answer with a paper stick cube before going on to the next.

The correct answers are:
q1) C. *8* q2) C. *Pentagon*; q3a) B. *4*; q3b) C. *Trapezium*

Why do this activity?

Learners will explore the properties of a cube and will find a range of 2D shapes.
They may be able to name some of these, but they will need to sketch and describe less familiar ones such as the irregular hexagon.
When learners go on to explore other 3D shapes they will visualise how these may change into different 3D or 2D shapes. They can discover for themselves the effect of triangles on rigidity.

Learning objectives

Explore the properties of a cube. Carefully observe, identify and record a range of 2D shapes. Understand how one 2D shape can transform into another.
Visualise and create different 3D shapes. Explore their properties. Predict their transformations, and then test the predictions.

Generic competences

In doing this activity students will have an opportunity to:

- **search systematically** and identify new examples;
- **visualise** and develop the skill of interpreting and creating visual images;
- **predict** transformations and test their predictions;
- **record** their own discoveries.

Suggestions for teaching

Emphasise that all the paper sticks should be the same length. If learners work in small groups then they can share the task of making the paper sticks, but it is important that every learner should have the opportunity to handle the cube, and any other 3D shapes that they work on.

When learners go on to explore other 3D shapes you could invite them to demonstrate what they discover to the rest of the class. For example, if some learners have found out that a tetrahedron is rigid and will not change its shape, you could ask them to show this to the other learners.

When the learners have collected and recorded a number of examples between them, you could begin to compile the results with a table on the board. This could show: the mathematical name of the 3D shape (if it has one); the number and shape(s) of its faces; whether it is rigid or can change its shape; what shapes it can change into.

Key questions

For the cube – Key Questions to develop understanding
What do you notice about your cube?
How can you describe the 2D shapes you are making with your cube?
What is the same about the shapes? What is different?
How can you record what you have found out?

For the cube – Key Questions to check knowledge and understanding
How many edges/vertices does the cube have? How many faces does it have? What shape are the faces?

Do you know a mathematical name for the 2D shape you have made with your cube?
Can you sketch your 2D shape? What mathematical words could you use to describe it?
Can you squeeze or fold it into different 2D shape?

For new 3D shapes – Key Questions to develop understanding

Can you think another example of a 3D shape? What is the same? What is different?
What is the same about the 2D shapes you are making with your cube?
What is the same about the shapes? What is different?
How can you record what you have found out?

For new 3D shapes – Key Questions to check knowledge and understanding

What shape were the faces of the cube? What other shaped face could you try? Could you make a shape with faces of two different shapes?
What are the properties of your new 3D shape? How many edges/vertices/faces does it have? What shape(s) are the faces?
Can you change your new 3D shape? What can you make with it?
Can you use the same shaped faces to make another different 3D shape? Does that behave in the same way as your first one?

Follow up

Shadows Activity https://aiminghigh.aimssec.ac.za/years-4-8-shadows-activity/
Cube Nets https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
How do you see it? https://aiminghigh.aimssec.ac.za/years-4-to-9-how-do-you-see-it/
Three Views https://aiminghigh.aimssec.ac.za/years-4-8-three-views/

MATHS TOYS

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the MATHS TOYS YouTube Channel
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13