

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

COUNTING OUTCOMES OF EVENTS is the theme for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 4 to 17+

Just choose whatever seems suitable for your group of learners

The TWO ACES activity was designed for Upper Secondary

TWO ACES

For this activity you need six cards, four Aces and the Jack of hearts and Queen of spades. If playing cards are not available, then make some coloured cards instead. (See the Two Red Cards version below)



You pick 2 cards at random and tell me "I have an Ace". What is the probability that you have two aces?

Next suppose I pick 2 of the 6 cards at random and tell you "I have the Ace of spades". What is the probability that I have two Aces?

Are these two probabilities of two Aces the same? Explain your answer.

TWO RED CARDS

We have six cards, four red cards and one yellow card and one green card.

You pick 2 cards at random and tell me "I have a red card". What is the probability that you have two red cards?

Next suppose I pick 2 of the 6 cards at random and tell you "I have the A1 card". What is the probability that I have two red cards?

Are these two probabilities of two red cards the same? Explain your answer.

HELP

If possible, discuss the problem with someone else and help each other.

Try to answer questions (1) and (2). Only study the answer below if you can't do it yourself. When you understand this family example then answer questions (3) to (9) below to solve the Two Aces problem.

- (1) You meet a mother who tells you "I have 2 children" and you know one of them is a boy. What is the probability that her other child is a boy?
- (2) Now suppose she tells you "I have 2 children and the oldest is a boy," What is the probability that the other one is a boy?

Case (1) The possible families in order of age are BB, BG, GB so the probability that the other child is a boy is $\frac{1}{2}$.

Case (2) The possible families are BB, BG so the probability that the other child is a boy is $\frac{1}{2}$.

- (3) If you choose two cards from the six cards, how many different pairs of cards are there? Make a list of all these possible outcomes.
- (4) How many of the pairs contain one or more Aces (red cards)?
- (5) How many of the pairs have 2 Aces (2 red cards)?
- (6) If you have already picked 2 cards and one is an Ace what is the probability that both are Aces?
- (7) How many pairs contain the Ace of spades (A1 card)?
- (8) If you have already picked 2 cards and one is the Ace of spades (A1) what is the probability that both are Aces?
- (9) Are the answers to questions 2 and 6 the same? If not, why not?

NEXT

Seeing the example below, make up a similar problem to this for a family of 3 children.

- (1) You meet a mother who tells you "I have 2 children" and you know one of them is a boy. What is the probability that her other child is a boy?
- (2) Now suppose she tells you "I have 2 children and the oldest is a boy," What is the probability that the other one is a boy?

Cut out these cards or make your own set.



INCLUSION AND HOME LEARNING GUIDE

THEME: COUNTING OUTCOMES OF EVENTS

In the TWO ACES example the event is choosing two cards and there are 15 possible outcomes. You could choose both cards at the same time by putting the 6 cards face down on a table, mixing them up, and picking one up with each hand. It does not matter which card is in which hand or which card is chosen first.



Like many mathematical activities for young children this one can be made to seem like a puzzle or game. Playing is one of the best ways to learn.

Find 5 toys or any set of 5 different objects.

Start with just 3 toys. Tell the child to pick just two toys from the three.

Ask "how many ways can you pick two toys?"

Then bring in toy number 4. Ask "how many ways can you pick two toys now?"

For many children that will be sufficient for one session.

Later bring in toy number 5 and challenge the children to find how many ways they can choose two toys from 5.

The children might record their answers by drawing or writing the letters D for dog, E for elephant, M for mouse, C for cat and H or horse. So the ten pairs are:

50 L	neter	i pan	s are:	This list gives the 10 possible pairs.	
DE	DM	DC	DH 10 is EH The f	10 is a triangle number.	
	EM	EC MC		The first few triangle numbers are 1, 3, 6, 10 What next?	
			MH		
			СН		

Lower Primary

Do the same activity as described for Early Years with 3 cards, then 4, then 5 cards, then 6 cards.

Record all possible pairs each time. You could use coloured pens or write R for red, O for orange, Y for yellow, G for green, B for blue and V for violet.



Upper Primary

Start with just 3 cards. Tell the child to pick just 2 cards from 3.

Ask "how many ways can you pick 2 cards?"

Then include a 4th card. Ask "how many ways can you pick 2 cards now?"



Later include a 5th card and challenge the children to find how many ways they can choose 2 cards from 5. Then include a 6th card.

Record all possible pairs each time. You could use coloured pens or write R for red, O for orange, Y for yellow, G for green, B for blue and V for violet.

RO	RY	RG	RB	RV						
	OY OG YG	0.0	0 P	OV	This list gives the 15 possible pairs					
		OG	OB		Can the children explain the system used to record all the pairs?					
		YG YB	YB	YV	can the enhanced explain the system used to record an the pairs.					
					10 is a triangle number .					
			GB	GV	The first few triangle numbers are 1, 3, 6, 10, 15					
					The first few thangle humbers are 1, 5, 6, 16, 15					
		Include a seventh colour Indigo so that you have all the colours of the								
		rainbow. Record all possible pairs of 2 colours chosen from 7.								

Lower Secondary

A1 A2 A3 A4 J Q

Your group could make their own sets of cards and work in pairs if you have more than one learner at home. In order that learners get **practice in reading questions**, and thinking for themselves how to tackle problems, simply give the learners the questions in the box headed 'TWO RED CARDS' on page 1 and ask the learners to find the answers, or write the question out for them to read.

If you have four or more learners together, the learners can start by working in pairs or groups of three. Observe what the learners are doing and if they are not making progress ask one of the key questions. It is important for learners to understand that you will not tell them how to solve problems, but you are asking questions to help and guide them to the answers.

After some of the learners have found answers, you could ask the learners to work with another group and to compare answers, and to explain their reasons for giving those answers. If they have not found an answer, or have found different answers, then they should try to explain what they have done, and why, and help each other to agree on an answer.

When most of the learners have found answers ask for a vote as to who thinks the two probabilities are different and who thinks they are the same. Record the number of votes cast each way.

Then ask learners to come to explain how they found their answers. Ask different learners to give explanations. Draw the 15 possibilities on a board, go over the answers and summarize what has been learned.

Key questions

- (1) If you choose two cards from the six cards, how many different pairs of cards are there? Make a list of all these possible outcomes.
- (2) How many of the pairs contain one or more Aces (red cards)?
- (3) How many of the pairs have 2 Aces (2 red cards)?
- (4) If you have already picked 2 cards and one is an Ace (red card) what is the probability that both are Aces?
- (5) How many pairs contain the Ace of spades (A1 card)?
- (6) If you have already picked 2 cards and one is the Ace of spades (A1) what is the probability that both are Aces?
- (7) Are the answers to questions 2 and 6 the same? If not, why not?

Upper Secondary

Try the question yourself as given on page 1. Record your answers in a triangular pattern as described in the box on page 3 for Young Children.

In probability we refer to events. We need to count the total number of possible outcomes of the event and the number of favourable outcomes.

Probability is defined as <u>Number of favourable outcomes</u> Total number of possible outcomes.

In this TWO ACES (or TWO RED CARDS) example the event is choosing two cards and there are 15 possible outcomes.

The 'favourable event' in question is two aces (or equivalently 2 red cards).

You could choose both cards at the same time by putting the 6 cards face down on a table, mixing them up, and picking one up with each hand. It does not matter which card is in which hand or which card is chosen first.

Everything so far has been done by younger learners.

THE RACE TO WIN A GOLD OR SILVER MEDAL

The extra challenge for you is to compare this to an example where there is a race for 6 runners, with a gold medal for the winner and silver medal for the runner up in second place. What is the difference to the TWO RED CARDS problem?

Suppose you and your sister run in the race with 4 other people and your parents hear that you got a medal. What is the probability that both you and your sister got a medal?

Suppose you and your sister run in the race with 4 other people and your parents hear that you got the silver medal. What is the probability that both you and your sister got a medal?

The theme for this sequence of activities is COUNTING OUTCOMES OF EVENTS. There are 2 types situations where you might have to solve probability problems like this:

- (1) Counting outcomes where the order in which the selected set is chosen does not matter. **In the Bag** is an example of this type. <u>https://aiminghigh.aimssec.ac.za/years-9-12-in-the-bag/</u>
- (2) Counting outcomes where the order in which the selected set is important and makes a difference. **Another example of this type is**: How many 2-digit pin codes can be made from the 9 digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if you are not allowed to repeat a digit?

SOLUTION

The probability that you have two Aces given the information that one of your two cards is and Ace is different from the probability that you have two Aces if the information is known that you have one particular Ace, say the Ace of Spades.

There are 15 possible pairs of cards as shown below. If you say you have an Ace then you have one of 14 pairs of cards, and 6 pairs contain 2 Aces, so the probability that you have 2 Aces is 6/14 = 3/7 = 0.429 (to 3 decimal places).

If you say that you have the Ace of spades then you have one of 5 possible pairs of cards, and 3 pairs contain 2 Aces, so the probability that you have 2 Aces is 3/5 = 0.6



Notice that the number of pairs is the TRIANGLE NUMBER

 $1 + 2 + 3 + 4 + 5 = \frac{1}{2}(6 \times 5) = 15$

Each of the 6 cards can be paired with 5 other cards making $6 \times 5 = 30$ pairings but this counts each pair twice so the number of pairs is 15.

Why do this activity?

This activity appears to be straightforward. It requires listing and counting the number of different pairs (combinations) of two objects chosen from six distinct objects, and then working out two probabilities.

However, the answer may be a little surprising as it might be expected that the probability of two Aces is the same in each case. But this is not so, and the activity shows learners that they need to think clearly about the information given and not jump to conclusions without justifying the answer that they give.

This is an example of conditional probability that can be found simply by counting cases and it does not require the use of a conditional probability formula.

Learning objectives

In doing this activity students will have an opportunity to:

- practice reading comprehension, that is interpreting information given
- count the number of possibilities, and to use this information to work out probabilities.
- develop problem solving skills.

Generic competences

In doing this activity students will have an opportunity to interpret information, apply mathematical reasoning and **solve problems**.

DIAGNOSTIC ASSESSMENT This can be used after the lesson for Upper Secondary. Show this question and say: "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D". 1.Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer. 2.It is important for learners to explain the reason for their You spin the above fair spinners twice answer so that, by putting their thinking into words, they develop What is the probability of getting a total of 2 when you communication skills and get a add the scores together? better understanding. 3.If you have a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong. 4.Again ask the learners to vote 1 2 2 9 for the right answer by putting up 20 9 20 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers. **A.** is the correct answer. The probability of a 1 with the 4-sector spinner is $\frac{1}{4}$. In those cases only $\frac{1}{5^{th}}$ of the spins with the 5-sector spinner will be 1s. So the probability of two 1s is $1/4 \times 1/5 = 1/20$. Another way to understand this is that there are 20 possible outcomes and only one of them is a double 1 so the probability is 1/20. **Common Misconceptions** B. Learners may be using this incorrect reasoning "There are 9 chances of landing a number. There are

two 2's. The probability is 2/9"

C. Learners may be using this incorrect reasoning "1/4 + 1/5 = 0.45 = 4/9"

D. Learners may reason correctly that there are 20 possible outcomes but use 2 as the number of ways of getting a total of 2 whereas there is only one possible way to get 2, which is the total when you add the two spinners together which is 1 + 1."

https://diagnosticquestions.com

Follow up

In the Bag <u>https://aiminghigh.aimssec.ac.za/years-9-12-in-the-bag/</u> If this then that <u>https://aiminghigh.aimssec.ac.za/years-10-12-if-this-then-that/</u>



Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: <u>http://aiminghigh.aimssec.ac.za</u> Subscribe to the **MATHS TOYS YouTube Channel** <u>https://www.youtube.com/c/mathstoys</u> Download the whole AIMSSEC collection of resources to use offline with

the AIMSSEC App see <u>https://aimssec.app</u> or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. New material will be added for Secondary 6.

For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339

Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12									
	Lower Primary	Upper Primary	Lower Secondary	Upper Secondary					
	Approx. Age 5 to 8	Age 8 to 11	Age 11 to 15	Age 15+					
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12					
East Africa	Nursery and Primary 1 to	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6					
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12					
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13					