AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

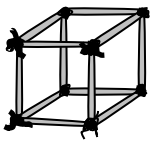
VISUALISATION is the theme for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 4 to 17+

Just choose whatever seems suitable for your group of learners

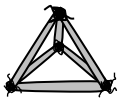
The FLEXIBLE SHAPES activity was designed for Upper Primary and Secondary

FLEXIBLE SHAPES



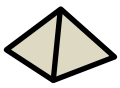
If you have not already done so, try the
Collapsible Cube activity and watch the video.

1. Make a tetrahedron out of rolled paper sticks. Press it gently. See that it does not change its shape – it is rigid.



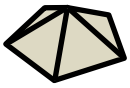
2. Make the five 3D shapes below with paper sticks, all the same length.
   Before you make each 3D shape, imagine it in your mind. Try to visualise how it will behave. Do you think that it will be:
   a) Rigid, like the tetrahedron, so it will not change its shape at all?
   b) Collapsible, like the cube, so it will collapse into one or more flat shapes?
   c) Not rigid, but will not collapse into a flat shape?

3. Write down your predictions.

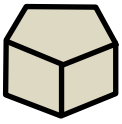


Square pyramid

Triangular prism



Pentagon pyramid



Pentagonal prism

Boat octahedron

4. Sketch, and name or describe, the shapes that each of the 3D shapes can make.

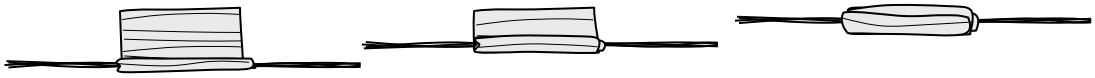
HELP

Follow the instructions below or [watch this video] to learn how to make rolled paper sticks.

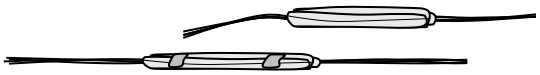
You need: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

1. Cut a rectangle of paper,
   8 cm long and about 6 cm wide.
   Cut a length of string about 24 cm long.

2. Carefully roll the rectangle of paper around the string to make a stick.
   Roll it as tightly as you can.



3. About 8 cm of string will hang out from each end of the stick.
   Fasten the paper with sticky tape.
   You have made your first paper stick!



Make more paper sticks. How many will you need to make your 3D shape?
(Each edge of your shape will need one paper stick.)

NEXT

When you have made several 3D shapes, begin to compile your results into a table. This could show: the mathematical name of the 3D shape (if it has one); the shape(s) of its faces; the number of its edges, faces and vertices; whether it is rigid or can change its shape; what shapes it can change into. Remember to include the cube and the tetrahedron on your table.

Then think of another 3D shape you can try.

Try to predict, *before* you make each 3D shape, whether it will be rigid or whether you will be able to change its shape.
Try to visualise and describe how your 3D shape will behave.
Record what you predict, and what you find out.

If you are working with a group of learners then you can share your results. Build up a complete record of everything you have discovered about the shapes you have made.

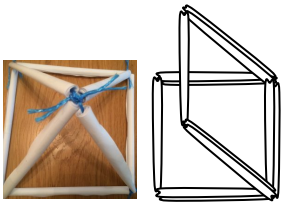
Resources: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

INCLUSION AND HOME LEARNING GUIDE

THEME: VISUALISATION

Before starting Flexible Shapes the learners should have worked on the Collapsible Cube activity which precedes this one. You might also watch the Flexible Shapes video.

Early Years and under 7's



Make a paper stick cube and a square based pyramid and give them to the child to play with. Encourage them to squash these 3D shapes and 'play' with them to see what flat 2D shapes they can make.

Talk about these 3D shapes and that they are like skeletons. What shapes are their faces? How many edges do they have? How many vertices? How many faces? Have they seen shapes like this anywhere else before? Where?

They could cut out paper shapes and stick them on to cover the 3D shapes like skin. They could paint the shapes to make them pretty and hang them up. If they hang the cube up from one vertex and then with 4 strings from 4 vertices, what happens? Why? If they hang the pyramid up from different vertices does the same happen? Why or why not?

There is a lot to talk about. In so doing you are helping children to develop visualisation and communication skills and a mathematical vocabulary.

Primary

Make sure that all the paper sticks used to make the 3D shapes are the same length. If the learners have done the Collapsible Cube activity mentioned above then they will have already made and explored a flexible cube, and they may have found and made some more 3D shapes of their own.

If they are ready to move on then they can make some of the 3D shapes shown on the Flexible Shapes instructions on page 1, and explore these in the same way. Encourage the learners to record all their findings, from the cube and the tetrahedron as well as from the other 3D shapes they make.

Lower Secondary

Make sure that all the paper sticks used to make the 3D shapes are the same length. If learners are working in a small group then they can share the task of making the paper sticks, but it is important that every learner should have the opportunity to handle all the 3D shapes that they work on.

Learners should try to imagine each 3D shape in their minds **before** they make it, and describe what they see. Encourage them to discuss how they think it will behave, and whether it will be rigid or will be able to change its shape. The important point here is not for learners to get the 'right' answer, but rather to visualise what the shapes may look like and to predict what may happen.

When learners explore shapes that they have made they should share anything they discover with others in the group. For example, if some learners have found a 3D shape that is not rigid but does not collapse into a flat shape, they could show this to the others.

When learners explore the shapes they have made you could invite them to demonstrate what they discover to the rest of the class. For example, if some learners have found a 3D shape that is not rigid but does not collapse into a flat shape, you could ask them to show this to the other learners.

Encourage the learners to enter their findings on a table, and to share their results in order to build up as complete a record as possible. Point out that the table can never be finalised – there will always be other 3D shapes that could be added. Ask, 'What else might we do, to add to our results?' Explain that this is how mathematics progresses, as mathematicians share and build on one another's discoveries.

Key

Key Questions to develop understanding

- Look at the pictures of the five 3D shapes given. Which ones do you think will be rigid?
- Which ones do you think will change their shapes?
- Imagine and describe what you think will happen. Then make the shapes and see.
- What could you change in order to make a different 3D shape?
- Can you imagine a different 3D shape that you think will be rigid?
- Can you imagine one that you think will change its shape?
  Sketch your new shapes, then make them and find out what happens.
- How can you record what you have found out?

Key Questions to check knowledge and understanding

- Look at the cube and the tetrahedron made with paper sticks.
  Which one is rigid? What makes it rigid? Why is the other one not rigid?
- What are the properties of the 3D shapes you have made? How many edges/vertices/faces do they have? What shape(s) are the faces?
- Can you show everything you have found so far on a table?
- Have you included all the 3D shapes you have made on your table?
- Compare two 3D shapes that you have made.
  What is the same about them? What is different?
- Which of the 3D shapes you have made can be changed into flat shapes? Which cannot?
- Can two different 3D shapes both make the same flat shape?
  Find some examples.
- What is the same about the flat shapes that one of your 3D shapes will make? What is different?
- Can you use the same shaped faces to make two different 3D shapes? Do they both behave in the same way?

Upper Secondary

Do the activity on page 1. If possible, work in a small group to share the task of making the paper sticks and the 3D shapes. You may be leading a group with younger learners, in which case you will make some discoveries for yourself and where possible share them with everyone else. One always learns from explaining things to other people so everyone benefits.

Try to imagine each 3D shape in your minds **before** you make it and then describe what you see. Talk about how you think it will behave, and whether it will be rigid or able to change its shape. The important point here is not to get the 'right' answer, but rather to visualise what the shapes may look like and to predict what may happen.

When you investigate the shapes that you have made you should talk to others in your group and everyone should share anything they discover. For example, if someone finds a 3D shape that is not rigid but does not collapse into a flat shape, they could show this to the others.

Make a record of your findings in a table. Share your results in order to build up as complete a record as possible. The table can never be finalised – there will always be other 3D shapes that could be added. Ask, 'What else might we do, to add to our results?' This is how mathematics progresses, as mathematicians share and build on one each other's discoveries.

SOME SOLUTIONS

3D shapes made with paper sticks change in different ways. These include (but are probably not limited to) those shown in the table below.

3D Shape	Behaviour	Changes into
Cube	Collapsible	equilateral triangle; square; rectangle; regular hexagon; irregular hexagon; rhombus; trapezium
Tetrahedron	Rigid	
Square pyramid	Collapsible	rhombus; equilateral triangle
Triangular prism	Collapsible	trapezium; irregular pentagon; equilateral triangle
Pentagon pyramid	Neither rigid nor fully collapsible	tetrahedron
Pentagonal prism	Collapsible	trapezium; equilateral triangle; irregular pentagon; irregular hexagon; irregular heptagon
Boat octahedron	Rigid	

Why do this activity?

As learners make and explore the 3D shapes they will discover how some of these can be collapsed into flat shapes, while others cannot. They will use their experience of manipulating the shapes to begin to visualise and predict their behaviour. They can discover for themselves the effect of triangles on rigidity.

Learning objectives

In doing this activity students will have an opportunity to:

- visualise and create a range of 3D shapes;
- explore and record their properties;
- visualise and predict how the shapes may (or may not) change;
- test their predictions;
- record what they find out;
- analyse their results, and make further predictions.

Generic competences

In doing this activity students will have an opportunity to:

- **search systematically** for new examples;
- **visualise** 3D shapes and predict how they may change;
- **test** their own predictions;
- **share, organise and classify** their findings;
- **record** their own discoveries;
- **think creatively** to find new examples of 3D shapes to try.

DIAGNOSTIC ASSESSMENT This can be used before or after the lesson.

Show each question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1) Tandi makes a collapsible cube out of paper sticks.
She collapses her cube into a flat regular hexagon.
She folds her regular hexagon in half along one diagonal.
Then she folds the resulting shape into three.
What flat shape has she made now?
A. Triangle B. Tetrahedron
C. Trapezium D. Square

2) Eddy made these four shapes out of paper sticks, all the same length, tied together at the corners.

A. Square pyramid B. Triangular prism C. Pentagon pyramid D. Tetrahedron

Which of the shapes can he collapse into a flat trapezium?
A. Square pyramid B. Triangular prism
C. Pentagon pyramid D. Tetrahedron

1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
2. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
3. Again, ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. If a learner has changed from a wrong to a right answer, you could ask them to explain what made them change their mind. This will help to embed their reasoning so they can recall it later.
4. For each question, you could demonstrate the correct answer with the relevant shapes made out of paper sticks.

The correct answers are: q1) A. *Triangle*; q2) B. *Triangular prism*

Follow up

Shadows Activity https://aiminghigh.aimssec.ac.za/years-4-8-shadows-activity/
How do you see it? https://aiminghigh.aimssec.ac.za/years-4-to-9-how-do-you-see-it/
Three Views https://aiminghigh.aimssec.ac.za/years-4-8-three-views/