AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

DIMENSION is the theme for this INCLUSION AND HOME LEARNING GUIDE

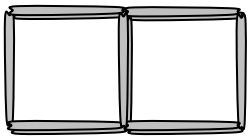
This Guide suggests related learning activities for all ages from 4 to 17+

Just choose whatever seems suitable for your group of learners

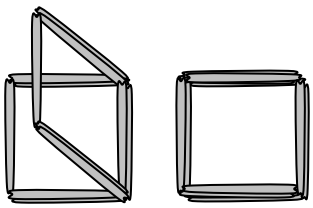
The COLLAPSIBLE CUBE activity was designed for Primary and Lower Secondary

COLLAPSIBLE CUBE

1. Make a 3D cube out of rolled paper sticks, tied together at the ends.
   Push your cube gently down and sideways. It will collapse into a 2D shape!
   Can you collapse your cube into a 2D rectangle made of 2 squares?

   Can you fold the rectangle over to make a 2D square?

2. Can you collapse your 3D cube into other 2D shapes?
   Sketch your new 2D shape.
   Does it have a mathematical name? Or can you describe it?

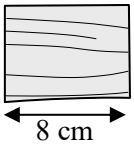
3. What other 2D shapes that can you make with your collapsible 3D cube?
   Sketch each one you find, and name it or describe it carefully.

HELP

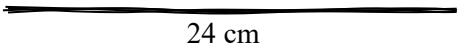
Follow the instructions below, or from this video: https://youtu.be/iaJ6EitIGKU **on how to make rolled paper sticks.**

You need: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

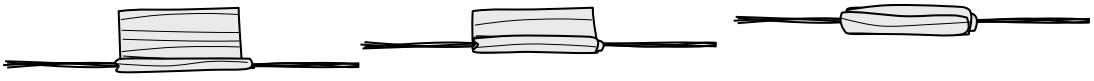
Cut a rectangle of paper,
8 cm long and about 6 cm wide.



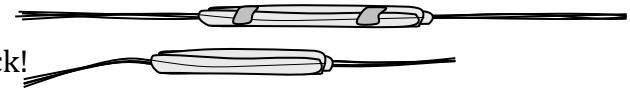
Cut a length of string about 24 cm long.



Carefully roll the rectangle of paper around the string to make a stick. Roll it as tightly as you can.



About 8 cm of string will hang out from each end of the stick.
Fasten the paper with sticky tape.



You have made your first paper stick!
Make more paper sticks. How many will you need to make a cube?
(Each edge of the cube will need one paper stick.)

NEXT

Think of a different 3D shape that you can make with paper sticks of equal length.

Will it collapse into a 2D shape?

Or will it be rigid?

Or will it do something different?

Before you make your new 3D shape, try to visualise it and describe how it will behave.

Then check your predictions.

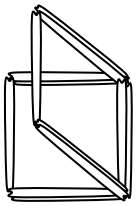
Record what you find with sketches and words.

Resources: An old magazine or scrap paper to cut up (or you can use dried banana fibre); string; sticky tape; scissors or a blade to cut with.

INCLUSION AND HOME LEARNING GUIDE

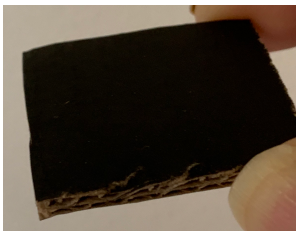
THEME: DIMENSION

Young children 5 and under.



Make a paper stick collapsible cube and give it to the child to play with. Encourage them to squash it and 'play' with it to see what flat 2D shapes that they can make. They could paint the cube to make it pretty. Hang the cube up from one vertex and then from 4 vertices. What happens? Why?

Talk about flat shapes in the room, like a sheet of paper. Talk about flat things and 3 dimensional things. Introduce the language of 2D and 3D. What is the difference?



Take a piece of thick card, say a rectangle of card, no this is not piece of chocolate, it's a cardboard cuboid! Is it 2-dimensional or 3-dimensional?

There is a lot to talk about. In so doing you are helping children to develop language skills and a mathematical vocabulary. You are also sowing the seeds of understanding of deep mathematical concepts.

Lower Primary

Young children may need help in constructing the collapsible cube. They should be encouraged to squash it and 'play' with it to see what flat 2D shapes that they can make.

Talk about flat shapes in the room, like a sheet of paper. Talk about flat things and 3 dimensional things. Introduce the language of 2D and 3D. What is the difference? Talk about flat surfaces in the room. Talk about a 3D shape and its faces.

Take a piece of thick card, say a rectangle of card, no this is not piece of chocolate, it's a cardboard cuboid! Is it 2-dimensional or 3-dimensional? What about a pamphlet of 3 or 4 pages thick? There is a lot to talk about. In so doing you are helping children to develop language skills and a mathematical vocabulary. You are also sowing the seeds of understanding of deep mathematical concepts.

Ask the learners to sketch some 2D shapes made by the collapsible cube, and label them: 'square' or 'rectangle' or 'triangle' or …

They could be asked question 1 from the **Diagnostic Assessment** below.

Upper Primary

Upper Primary learners may be able to roll the sticks and make a collapsible cube for themselves. They could search for the 2D shapes that can be made, sharing their discoveries.

Whatever the size of the group, or the ages of the learners, everyone should be encouraged to share their discoveries and tell each other what they have found out. For example, if a learner makes a square with their collapsible cube and then 'squeezes' it into a rhombus, they could be encouraged to show their new shape to others in the group. If another learner can then give the correct mathematical name to the shape they should be encouraged to do so. Let older learners share their knowledge with younger members of the group.

Introduce the idea of dimension as for younger learners (see above). Your aim should be to introduce the concept of dimension and some idea of 1D, 2D and 3D and the differences between them.

Talk about 1-dimensional lines, 2-dimensional flat areas (or surfaces) and 3-dimensional objects. When they draw lines, are they drawing 1-dimensional or 2-dimensional things? Does the thickness of the pencil change what they draw? Yes! Does a very hard pencil draw a 1-dimensional line? No!

Here you are sowing the seeds of understanding of mathematical abstraction in geometry. You don't have to explain this, it's not like teaching something you want children to remember for a test. Just ask questions and engage the children in talking about the ideas.

They might sketch the shapes they find, giving their mathematical names if they can, or describing them in simple language.

They could try to find as many different 2D shapes as possible and talk about what is the same and what is different between pairs of shapes.

Key Questions

For the cube - Hold your cube up straight.

- How many edges does it have?
- How many corners/vertices does it have?
- How many faces does it have?
- What shape are the faces?
- Can you collapse your cube to make a 2D shape?
- Can you tell me the mathematical name of the 2D shape you made with your cube?
- Can you sketch your 2D shape? Can you describe it?
- Can you squeeze or fold or pull it into different 2D shape?

Learners could be asked questions 1 and 2 from the **Diagnostic Assessment** below.

Secondary

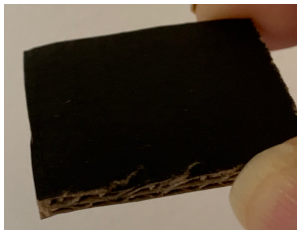
Secondary learners could find and record most of the shapes that can be made from a collapsible cube. Then they might go on to make one or more different 3D shapes with paper sticks and explore and record their properties.

Key Questions about the new shapes

- What other 3D shape could you try?
- What shape were the faces of the cube? What other shaped face could you try?
- Could you make a shape with faces of two different shapes?
- What are the properties of your new 3D shape? How many edges/vertices/faces does it have? What shape(s) are the faces?
- Can you change your new 3D shape? What can you make with it? How can you record what you have found out?
- Can you use the same shaped faces to make another different 3D shape? Does that behave in the same way as your first one?

The learners could be asked all the questions from the **Diagnostic Assessment.**

Collapsible shapes are an excellent means of motivating the **theme of dimension** which is fundamentally important in mathematics and physics. Your aim should be to introduce the concept of dimensions in space and you could refer to the number line and coordinates in 2 and 3 dimensions.



Introduce the language of dimensions by asking the learners to tell you what dimension is. Have they heard the word before? Can they explain what it means? What is the difference between 1-dimension, 2-dimensions and 3-dimensions? Talk about 1-dimensional lines, 2-dimensional flat areas (or surfaces) and 3-dimensional objects.

Ask: "When you draw lines are you drawing one or two dimensional things?"
" Does the thickness of the pencil change what you draw?" Yes!
"Does a very hard pencil draw a one-dimensional line?" No!

Here you are sowing the seeds of understanding of mathematical abstraction in geometry. You don't have to explain this, it's not like teaching something you want the learners to remember for a test. Just ask questions and encourage the learners to talk about the ideas.

Ask the learners if they can think of different meanings of the word 'dimension'. It has three **different meanings**. It can mean 'measurement' such as 'length'. It can mean 'units of measurement' such as metres. It can refer to **the focus of this theme, the number of directions in space.**

Ask: In how many directions can you go along a line? On a line you can reach any point by going in one direction or the other (actually in one direction or the negative of that direction). So mathematically speaking a line is one-dimensional.

Ask: In how many directions can you go on a flat surface (a plane)? On a plane you can go anywhere by going the right distances in two directions: North and East. Mathematically, West is the negative direction to East and South is the negative direction to North. So mathematically speaking a plane is two-dimensional. It is important to appreciate that you can get to any point on an infinite plane by going the right distances in 2 directions.

Ask: In how many directions can you go in space? If you can go in three directions North, East and Up in space, you can go anywhere like a bird. Mathematically, West is the negative direction to East, South is the negative direction to North and Down is the negative direction to Up. Mathematically speaking space is three-dimensional. It is important to appreciate that you can get to any point in space by going distances in 3 directions.

Years 12 and 13

Think about polyhedra made from paper sticks tied together at the vertices. Which polyhedra can be flattened, theoretically, into 2D? What different shapes do they make in 2D? If you have time, make some and try it out. You can yourself learn a lot by doing this with younger brothers and sisters or other children.

Vectors may or may not be on the syllabus for your school leaving exams in mathematics or physics. Whatever the case you can easily appreciate that the concept of dimension is important, and that getting from one point to another involves direction as well as distance. You might like to read some of the above written for teachers of younger learners.

What do you know about space-time? You might like to investigate this on the internet. Unlike the other dimensions, apart from science fiction, we can only travel in one dimension in time. Space-time is a special case in 4 dimensions.

Whatever mathematicians prove in 3 dimensions they often ask “is the corresponding theorem true in general in 4 dimensions and in higher dimensions?”.

Similar questions about dimension and the research that such questions lead to, apply to algebra as well as geometry. For example, if mathematicians prove something about quadratic and cubic equations they will ask “is this true for all polynomial equations?” If mathematicians prove something about polynomial functions (or certain types of series), they will ask “is this true in general about all functions (or all types of series)?” In many scientific fields the term ‘degrees of freedom’ is used and this is formalized in mathematics as ‘dimension’.

Diagnostic Assessment You could ask these questions after the lesson.

For which diagnostic questions might suit your learners, see suggestions for home learning above in the sections for different age groups.

Show the questions one at a time. For each question in turn, say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Tandi made a cube out of paper sticks tied together at the corners.

1) How many corners (vertices) does the cube have?
A. 4 B. 6 C. 8 D. 10

2) Tandi wants to make some 2D shapes with her cube.
Which ONE of these 2D shapes CAN'T she make?
A. Square B. Rectangle C. Pentagon D. Hexagon

3) Tandi makes a regular hexagon with her cube. Then she folds the regular hexagon over along one diagonal to make another 2D shape.

3a. How many sides does her new 2D shape have?
A. 3 B. 4 C. 5 D. 6

3b. What is the mathematical name of the new 2D shape?
A. Square B. Rectangle C. Trapezium D. Heptagon

1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer so as to clarify their own thinking by putting it into words, and to develop their communication skills.
3. If you have a group, try to make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the learners again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If a learner has changed from a wrong to a right answer, you could ask them to explain what made them change their mind. This will help them to understand the reasoning for themselves.

For each question, you could demonstrate the correct answer with a Collapsible Cube before going on to the next.

The correct answers are:
q1) C. 8 q2) C. Pentagon q3a) B. 4 q3b) C. Trapezium

SOLUTION

The paper stick cube can be collapsed into a number of 2D shapes, including a triangle, rectangle, regular hexagon, irregular hexagon, square, rhombus, and trapezium.

Other 3D shapes made with paper sticks behave in different ways.

Deltahedra (shapes with triangle faces) are rigid and will not change their shapes.

Some 3D shapes with differently shaped faces, such as a square-based pyramid or a triangle prism, can make a 2D shape, but some others cannot.

A pentagonal pyramid, for example, can be folded into a tetrahedron but not into a 2D shape. There are many shapes to explore, and a great deal to discover.

Learning objectives

In doing this activity students will have an opportunity to:

- Explore the properties of a cube.
- Carefully observe, identify and record a range of 2D shapes.
- Understand how one 2D shape can transform into another.
  Visualise and create different 3D shapes and explore their properties.
- Predict their transformations, and then test the predictions.

Generic competences

In doing this activity students will have an opportunity to:

- **search systematically** and identify new examples;
- **visualise** and develop the skill of interpreting and creating visual images;
- **predict** transformations and test their predictions;
- **record** their own discoveries.

Follow up

Shadows Activity https://aiminghigh.aimssec.ac.za/years-4-8-shadows-activity/
Cube Nets https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
How do you see it? https://aiminghigh.aimssec.ac.za/years-4-to-9-how-do-you-see-it/
Three Views https://aiminghigh.aimssec.ac.za/years-4-8-three-views/