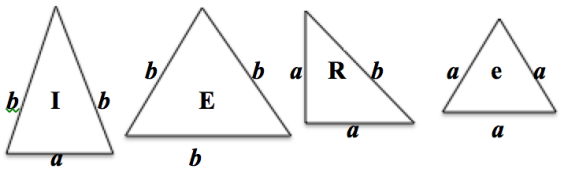
TRIANGLES TO TETRAHEDRA



A tetrahedron (plural tetrahedra) is a solid with four triangular faces. You have an unlimited supply of triangles of the 4 types shown in the diagram. How many different tetrahedra can you make with them?

Type R are right angled isosceles triangles with sides a, a and b units.

Type E are large equilateral triangles with sides of length b units.

Type e are small equilateral triangles with sides of length a units.

Type I are isosceles triangles with sides of length a, b and b units.

How can you be sure you have found them all?

HELP

To make all the tetrahedra you need 8 large equilateral triangles, 8 small equilateral triangles, 10 isosceles triangles and 14 right angled triangles and some spares to experiment with. If you work in a group to share the task, you should agree a system so each person looks for a different type of tetrahedron.

Cut out the triangles from scrap card using the template. First prick through the vertices to mark them. To make the tetrahedra, stick the edges together with selotape or tabs.

You could start by making the tetrahedra eeee, eIII and eRRI and then look for seven more.

NEXT

Describe the symmetries of the tetrahedra, mentioning planes of symmetry, describing rotational symmetry and the identifying regular solids.

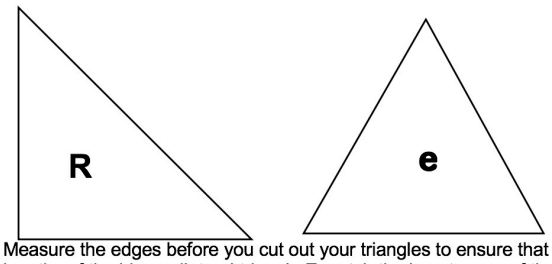
Template of smaller triangles for making cardboard models.

Make much larger demonstration triangles to scale.

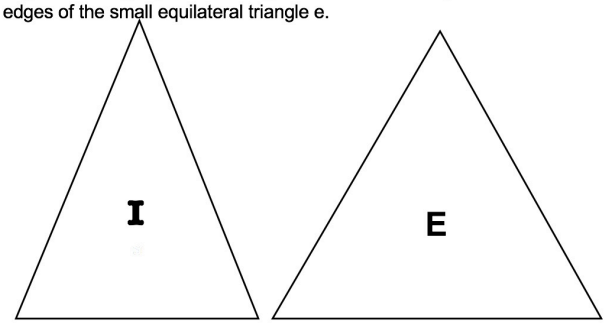
Use scrap card and prick through the vertices of the triangles to mark them on the card. Join the prick holes to mark the triangles, then cut them out.

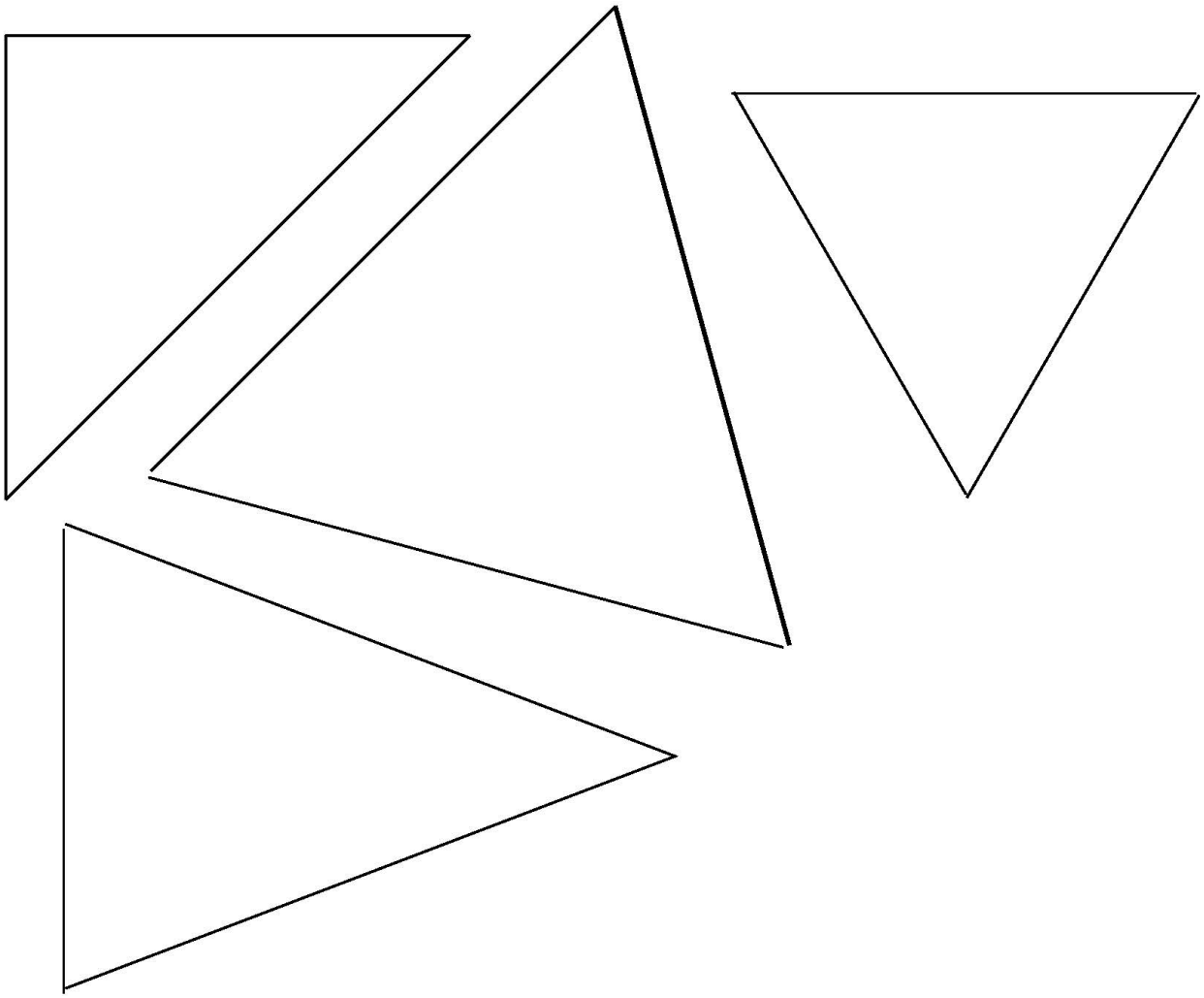
Make as many differently shaped tetrahedra as you can using 4 of these triangles.

To make all the tetrahedra you need 8 large equilateral triangles, 8 small equilateral triangles, 10 isosceles triangles and 14 right angled triangles.



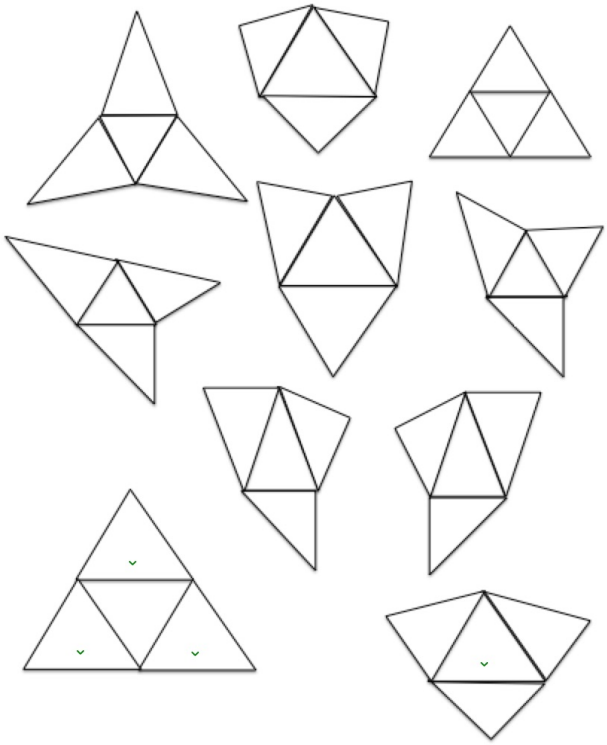
Measure the edges before you cut out your triangles to ensure that edge lengths of the big equilateral triangle E match the hypotenuse of the right-angled triangle R. The two longest edges of the isosceles triangle I match the edges of the big equilateral triangle E and the third edge of I matches the edges of the small equilateral triangle e.





NOTES FOR TEACHERS

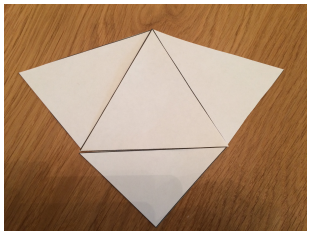
SOLUTION

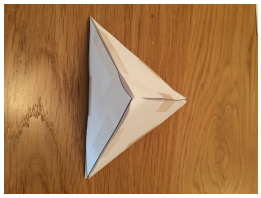


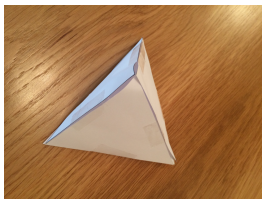
The diagram shows the nets of 10 tetrahedra made from the given triangles, two of which are mirror images.

To be sure to find all the tetrahedra take each of the triangles in turn, and find all the combinations of 3 triangles that can be attached around it so that the adjacent edges of the outer triangles are equal in length.

The photos below show a net with 1 large equilateral, 2 isosceles and 1 right angled triangle and 2 views of the same tetrahedron. You need to make the tetrahedra and handle them to be able to appreciate their properties.







Solutions can be labelled: eeee, eIII, eRRI, eeRR, EEEE, ERRR, EEII, EIIR and IIRR (2 versions).

To make a set of 10 tetrahedra you need 8 E-type; 8 e-type; 10 I-type and 14 R-type triangles but you need more triangles to allow for experimentation.

Why do this activity?

This practical activity gives learners experience of designing their own nets for tetrahedra, visualising the solids and working systematically to solve a problem. Templates of the triangles are provided for demonstration and a smaller version so that learners can easily cut out triangles from scrap card.

Learning Objectives

To revise nets of geometric solids. To gain experience of working systematically to solve a problem.

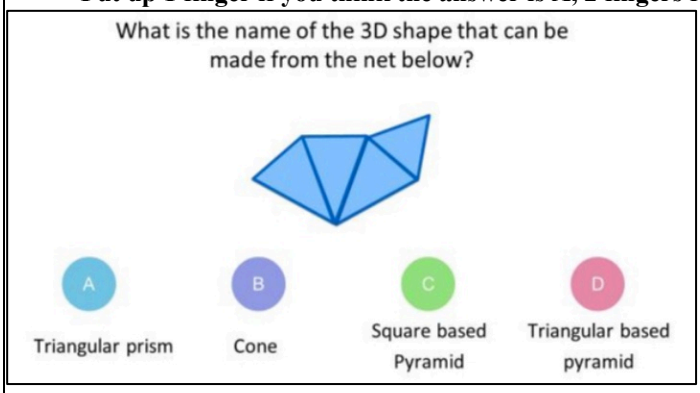
Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images of 3D objects;
- **persevere and work systematically** to investigate all possible cases.
- groups could **work together collaboratively to share the task**.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

3. It is important for learners to explain the reason for their answer, putting thoughts into words may help them to understand the ideas and to develop their communication skills.

4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

5. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**

6. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is D

Misconceptions:

A. Calling a pyramid a prism is a common misconception and vice versa.

B. A cone has a circular base and a face made from a sector of a circle.

C. A square based pyramid is made from a square and 4 triangles.

https://diagnosticquestions.com

Suggestions for Teaching

Resources: Scrap card (save packets from the kitchen). Scissors, pencils, rulers, Selotape. Prestik (or similar putty like adhesive. A supply of large triangles for demonstration made from scrap paper or card. To give some spares you need: 12 E-type triangles, 12 e-type triangles, 14 I-type triangles and 18-R type triangles. Pins to prick through the vertices of the triangles on the template to mark them on scrap card.

Stick several of each type of triangle on the board. Then take one of the equilateral triangles and stick it on the board with space around it. Explain that you want to make a tetrahedron using this triangle as the base and ask a learner to choose another triangle and to stick it on the board touching the first triangle to begin to make a net. Then the class has to find two more triangles to make the net and stick them into position.

If the learners work in groups of 4 they can share the marking out and cutting up of triangles. Give each group some scrap card, scissors, a pin, selotape and a copy of the template. By pricking through the vertices of the triangles on the template they can mark out the triangles on scrap card and then cut them out. Having cut out some cardboard triangles each group should make the net that is on the board which the class have talked about, and then make it into a solid tetrahedron. Tell the class that you want them to make as many **different** tetrahedra as possible using these triangles.

After a while the class should share what they have discovered and make a display of the different tetrahedra that have been found. The models should be passed around so that learners can handle and look at them carefully. The class could stick paper triangles on the board making the nets for the tetrahedra that the class have discovered or you could draw them on the board. You might like to give the class a few hints and time to find the rest of the solutions. You can introduce some discussion of the properties of the tetrahedra, for example symmetry.

Key questions

- Which triangles have edges to match that one, (or those two)?
- Have you checked that the edges that will go together are the same length?
- Do you have another net with exactly the same 4 triangles that will join up to make the same tetrahedron?
- Do you have another net with the same 4 triangles arranged in a different way that could be a mirror image?

Follow up

Paper stick tetrahedron https://aiminghigh.aimssec.ac.za/years-4-7-paper-stick-tetrahedron/
Cube nets https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
Cut nets https://aiminghigh.aimssec.ac.za/years-6-10-cut-nets/
Icosahedron Puzzle https://aiminghigh.aimssec.ac.za/years-7-10-icosahedron-puzzle/
Tet trouble https://aiminghigh.aimssec.ac.za/years-9-10-tet-trouble/