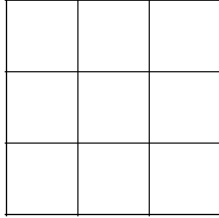


## SYMMETRY CHALLENGE



How many symmetric patterns can you make by shading whole squares in a 3 by 3 grid?

## HELP

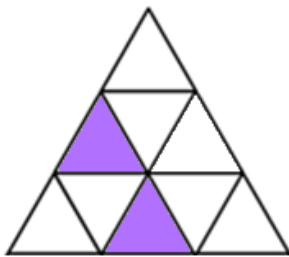
You could start with a 2 by 2 grid to help you to feel confident that you understand what to do.

## NEXT

A simple extension: do any of the patterns have rotational symmetry?

Use the chart on page 3 to fill in all possible distinct symmetric patterns.  
There are 64 solutions.

The challenge can be extended to larger square lattices, e.g. 4 by 4, and to investigating whether there are any differences between even and odd lengths of side.



Shading symmetric patterns in an isometric grid is another possibility.























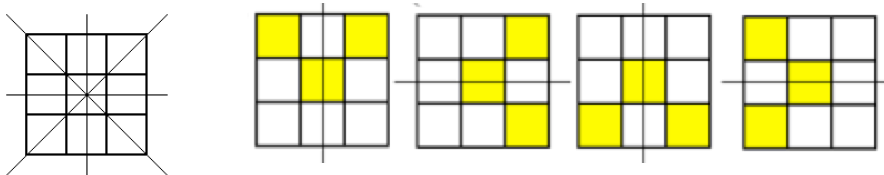


**SYMMETRIC PATTERNS IN A 3 BY 3 GRID**

<p><b>Zero or nine shaded squares</b></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 2px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> </div> <div style="border: 1px solid black; padding: 2px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> </div> </div> <p><b>One or eight shaded squares</b></p> <div style="border: 1px solid black; padding: 2px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> </div>																												<p><b>Three shaded squares</b></p> <div style="display: grid; grid-template-columns: repeat(4, 1fr); gap: 5px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> </div> <p style="text-align: right;"><b>or 6</b></p>																																					<p><b>Four shaded squares</b></p> <div style="display: grid; grid-template-columns: repeat(4, 1fr); gap: 5px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </table> </div> <p style="text-align: right;"><b>or 5</b></p>																																				
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## NOTES FOR TEACHERS

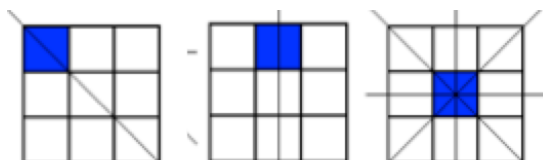
### SOLUTION



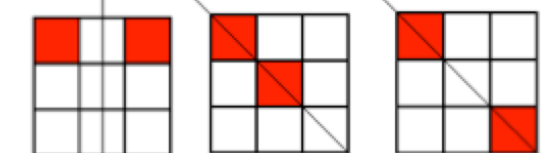
There are 4 lines of symmetry in the grid. Patterns are counted as **the same if one is an image of the other by rotation**. For example, the 4 diagrams above are counted as the same; just include ONE of these patterns, not all four. The patterns belong to 4 equivalence classes under rotation. Learners have met equivalence classes with fractions. Just as we regard  $\frac{1}{2}$  as representing **all** other fractions that have the value 0.5, here we only count one pattern from each of the equivalence classes.

This gives 64 different patterns (see below). Some patterns are repeated 4 times by images under rotation, some are repeated twice, and other patterns are not repeated.

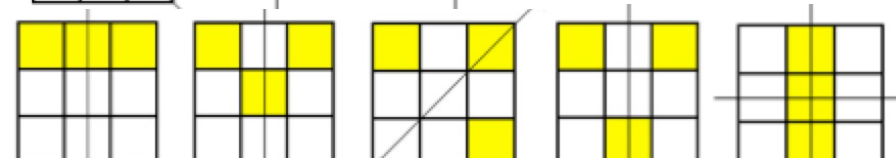
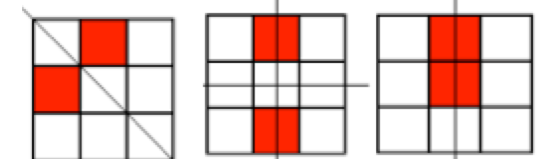
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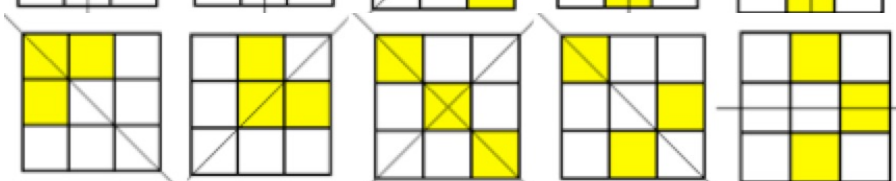
Shading just one square gives 3 symmetric patterns. The same diagrams show that, by shading the other 8 squares, there are also 3 symmetric patterns.



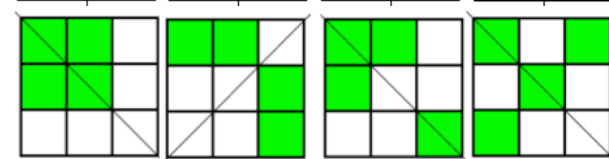
Shading 2 or 7 squares gives 6 symmetric patterns in each case.



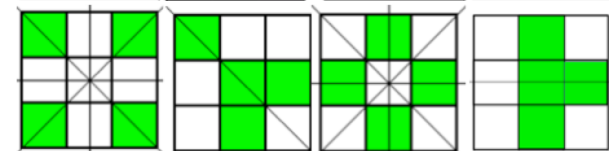
Shading 3 or 6 squares gives 10 symmetric patterns in each case.



Shading 4 or 5 squares gives 12 symmetric patterns in each case.



Shading 0 or 9 squares gives 2 symmetric patterns.



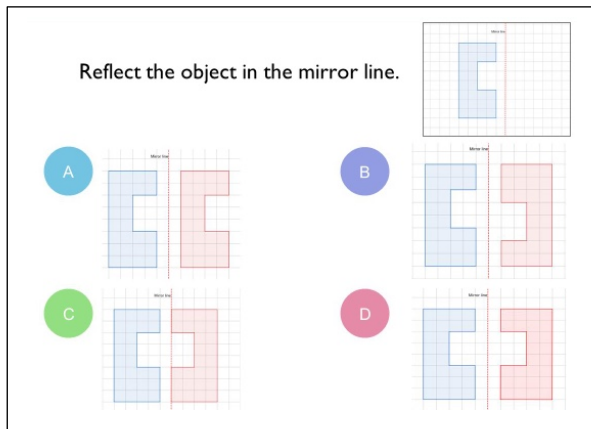
**This makes a total of 64 patterns.**

## DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes. It can be used before or after the lesson.

Show the question to the learners and say:

**“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.**



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reason for their answer because it helps them to clarify their own thinking and to develop communication skills.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers.

5. Notice if there is a change and who gave right and wrong answers.

**The correct answer is: D**

**Possible misconceptions:**

A. This is a translation

B. This is a rotation

C. The two shapes are not equidistant from the mirror line (line of symmetry).

<https://diagnosticquestions.com>

## Why do this activity?

The activity requires learners to recognise and visualise the transformations of a 2D shape. They are encouraged to work systematically in a spatial environment. The problem is accessible to most learners even if they need support in organising and presenting their ideas and ensuring the completeness of their solutions.

This Symmetry Challenge activity can be done in odd moments, over a period of weeks. By recording the new patterns as they find them, and filling in solutions on a poster, children work systematically as a team to find all possible solutions.

## Learning objectives

In doing this activity students will have an opportunity to:

- develop a familiarity with reflection and rotation symmetry;
- learn the language of transformations.

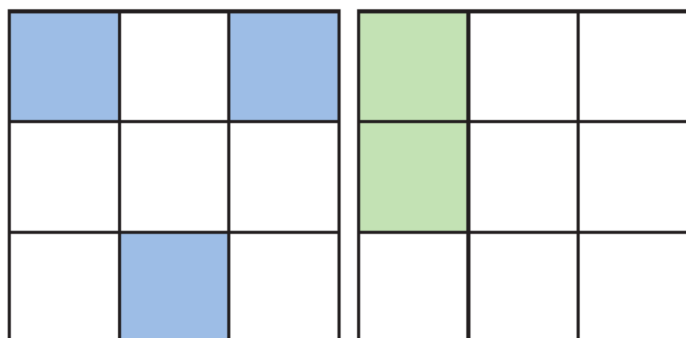
## Generic competences

In doing this activity students will have an opportunity to:

- develop visualization skills and spatial awareness;
- gain experience of working systematically to find all possible solutions to a problem;
- gain experience of sharing work in a team so as to cover all cases and achieve a set goal.

## Suggestions for teaching

Give each learner a sheet of 3 by 3 grids (page 2).



You could start by displaying these two shaded grids on and asking learners to say what they notice about their symmetry. Are they symmetric or not? Is there one line of symmetry or more than one? What happens to the pattern if they rotate the grid?

You might like to organise the class into 4 groups. The groups do not need to be able to work together but they could do so if that is convenient to arrange. Ask all the learners to find **symmetric patterns** by shading squares in the grids on their sheet: one group by shading just one square, the second group by shading 2 squares, the third group by shading 3 squares and the fourth group by shading 4 squares.

You might plan this with differentiation in mind and to help slower learners to get started. It is likely that the first group will find all three possibilities for shading one square quite quickly but they might have some repetitions. At this stage have a class discussion and invite learners to shade their patterns on the board. Ask the following questions:

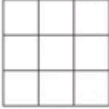
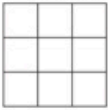
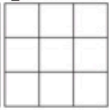
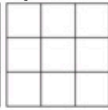


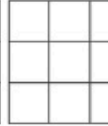
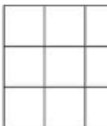


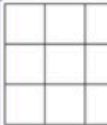
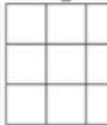

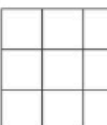
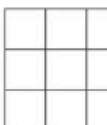
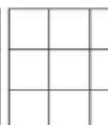
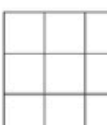

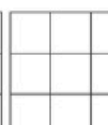
- Are all the patterns you have shaded symmetrical?
- Where are the lines of symmetry (mirror lines)?
- Which patterns are the same and which are different?
- Are there any more symmetric patterns with the same number of squares shaded?
- Ask questions so that the whole class talk about some of the issues:  
What counts as the same?
- Have they found all the possibilities? How do they know they have found them all?
- How can they find other solutions?
- What if they shaded 8 of the nine squares? Or 7 ...?

Explain to the class that the challenge is to find **all possible symmetric patterns** by shading whole squares. When any of the groups have found all the symmetric patterns with 1, 2, 3 or 4 shaded squares they can look for patterns with more shaded squares. It is important not to get too 'bogged down' with listing all the possibilities, the activity itself is a learning experience and all learners should feel happy to have found some of the solutions.

**For older learners:** Discuss the reason why it is a very important skill in the workplace, and in life, to be able to make a good plan for working systematically to make sure that you take into account all possible cases. Can the learners suggest some examples?

Then give copies of the poster template to each learner (page 3) and suggest that, as a whole class, you could together make one large poster. At the end of the lesson, if the class has not found all 64 solutions then you can put the poster on the classroom wall and invite the learners to find the missing solutions in the next few days or weeks. However, you must be sure that you don't record any solution twice by recording the solution and also one of its images. Whenever a learner finds a new solution their achievement should be celebrated.

**SYMMETRIC PATTERNS IN A 3 BY 3 GRID**

<p><b>Zero or nine shaded squares</b></p> <div style="text-align: center;">  </div> <p><b>One or eight shaded squares</b></p> <div style="display: flex; justify-content: space-around;">    </div>	<p style="text-align: center;"><b>Three shaded squares</b></p> <div style="display: flex; justify-content: space-around;">    </div>	<p style="text-align: center;"><b>Four shaded squares</b></p> <div style="display: flex; justify-content: space-around;">    </div>
<p style="text-align: center;"><b>Two shaded squares</b></p> <div style="display: flex; justify-content: space-around;">    </div> <p style="text-align: center;">or 7</p>	<div style="display: flex; justify-content: space-around;">    </div> <p style="text-align: center;">or 6</p>	<div style="display: flex; justify-content: space-around;">    </div> <p style="text-align: center;">or 5</p>

### Key questions

- Try shading 1 square only. What symmetries are possible?
- Have you found all the possibilities? How do you know that?
- If you try shading 2,3,4...squares, what symmetries are possible now?
- How will you record your findings?
- The 3 by 3 grid has four lines of symmetry and rotational symmetry of order 4. How might this help?

### Follow up

Paper Dolls <https://aiminghigh.aimssec.ac.za/years-5-8-paper-dolls/>

Tangram Pattern <https://aiminghigh.aimssec.ac.za/years-4-7-tangram-pattern/>

Mirror Mirror <https://aiminghigh.aimssec.ac.za/years-8-12-mirror-mirror/>

Reflecting Squarely <https://aiminghigh.aimssec.ac.za/grades-8-10-reflecting-squarely/>