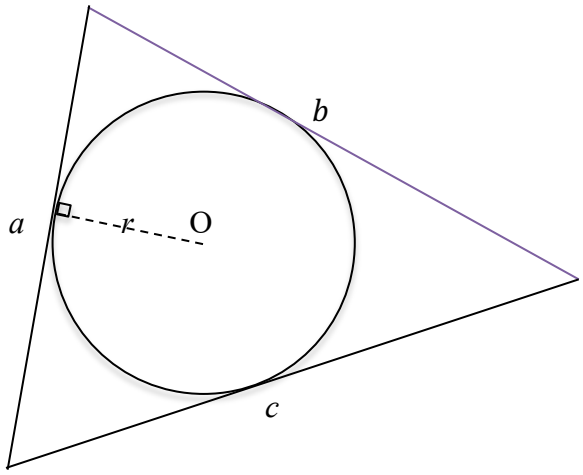
SURPRISE RATIO



A circle of radius r and centre O, is inscribed in a triangle with edges of lengths *a, b* and *c*.

Find the area and circumference of the circle.

Find the area and the perimeter of the triangle.

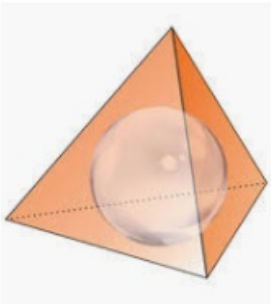
What do you notice about the ratios of the two areas and the ratios of the two perimeters?

What do you notice about the ratio of the perimeter to the area for each shape?

HELP

Draw the other two radii from the centre of the circle to the points of contact between the circle and the triangle. Imagine splitting the triangle into 3 smaller triangles each of height r and each having one of their vertices at O. Draw this into your diagram.

NEXT

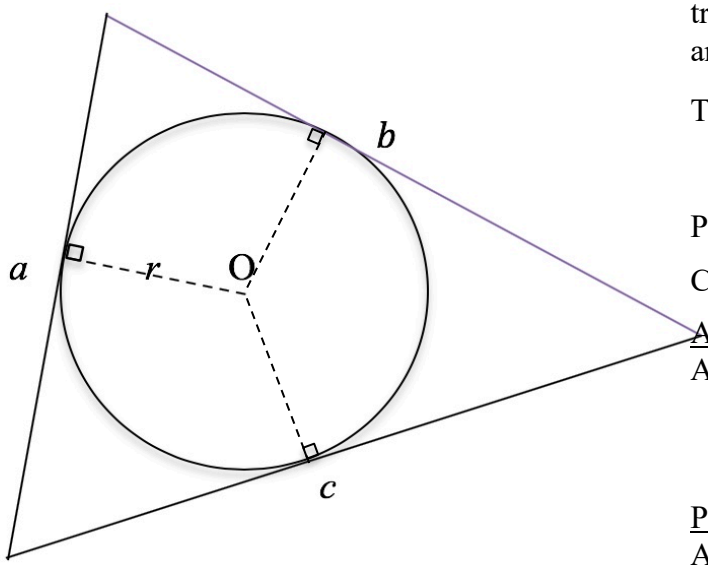


You could go up a dimension and investigate the ratios of the volumes and surface areas for a sphere inscribed in a tetrahedron, just touched by the 4 faces of the tetrahedron.

Or you might investigate similar ratios for other polygons.

NOTES FOR TEACHERS

SOLUTION



Imagine the triangle split into 3 smaller triangles, each of height *r*, with bases *a, b* and *c*.

Total area of triangle = $\frac{1}{2}(a+b+c)r$

Area of circle = πr^2

Perimeter of triangle = $(a+b+c)$

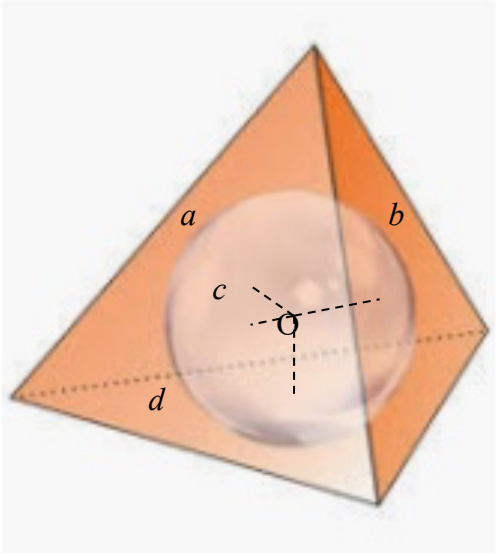
Circumference of circle = $2\pi r$

$$\frac{\text{Area of triangle}}{\text{Area of circle}} = \frac{\text{Perimeter of triangle}}{\text{Perimeter of circle}}$$

$$= \frac{a+b+c}{2\pi r}$$

$$\frac{\text{Perimeter of circle}}{\text{Area of circle}} = \frac{\text{Perimeter of triangle}}{\text{Area of triangle}}$$

$$= \frac{2}{r}$$



A sphere of radius *r*, centre O, is inscribed in a tetrahedron which has faces of areas *a, b, c* and *d*.

Imagine the tetrahedron split into 4 smaller tetrahedra, each of height *r*, with bases *a, b, c* and *d* and vertex O.

Total surface area of tetrahedron = $(a+b+c+d)$

Surface area of sphere = $4\pi r^2$

Total volume of tetrahedron = $\frac{r}{3}(a+b+c+d)$

Volume of sphere = $\frac{4}{3}\pi r^3$

$$\frac{\text{Volume of tetrahedron}}{\text{Volume of sphere}} = \frac{\text{Surface area of tetrahedron}}{\text{Surface area of sphere}}$$

$$= \frac{a+b+c+d}{4\pi r^2}$$

$$\frac{\text{Surface area of sphere}}{\text{Volume of sphere}} = \frac{\text{Surface area of tetrahedron}}{\text{Volume of tetrahedron}}$$

$$= \frac{3}{r}$$

DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes. It can be used before or after the lesson.

Show the question to the learners and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

The diagram shows a trapezium PQRS.

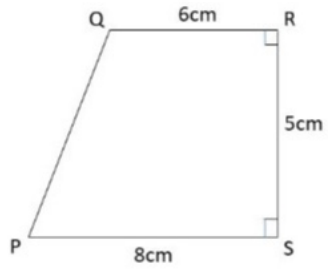


Diagram NOT accurately drawn

Calculate the area of the trapezium PQRS.

A $70cm^2$

B $35cm^2$

C $30cm^2$

D $40cm^2$

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reason for their answer because it helps them to clarify their own thinking and to develop communication skills.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

5. The correct answer is: B

Possible misconceptions:

A. The learner has misused the formula and not halved the sum of the parallel edges.

C. Could be randomly working out 6 × 5

D. Could be randomly working out 8 × 5

https://diagnosticquestions.com

Why do this activity?

This activity gives practice in working out areas of polygons and circles and in working with ratios. For older students the activity could also give practice in working with surface areas and volumes of polyhedra and spheres.

Learning objectives

In doing this activity students will have an opportunity to:

- revise methods of calculating areas and perimeters;
- revise surface areas and volumes of polyhedra and spheres (for older students);
- revise ratios.

Generic competences

In doing this activity students will have an opportunity to **think flexibly**, be creative and innovative and apply knowledge.

Suggestions for Teaching

You may want to start with the Diagnostic Quiz to check that learners can work out areas. Make sure that they can find the area of the trapezium by splitting it into a triangle and a rectangle as well as just using the formula that they may have learned by rote and understand poorly.

Students need practice in reading comprehension. They need to read questions for themselves and to work independently. Just give out the worksheet from page 1, or write the question on the board, and tell the learners to work independently.

You can give out the HELP slips to students who struggle to get started and the NEXT slips to students who finish before the rest of the class.

When most of the class have done the question get students to come to the board and explain their methods to the rest of the class.

Key questions

- Can you work out the perimeter of the triangle?
- Can you work out the perimeter of the circle?
- Can you think of another name for circumference?
- Can you work out the area of the triangle?
- Can you work out the area of the circle?
- Have you written the ratio in its lowest terms?
- Is there something you can cancel (or divide by) there?
- What is the total surface area of the tetrahedron?
- What is the surface area of the sphere?
- What is total volume of the tetrahedron?
- What is the volume of the sphere?

Follow up

The circle and the square https://aiminghigh.aimssec.ac.za/years-6-8-the-circle-and-the-square/
What's the area? https://aiminghigh.aimssec.ac.za/whats-the-area/
Can you help these farmers? https://aiminghigh.aimssec.ac.za/years-6-8-can-you-help-these-farmers/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6