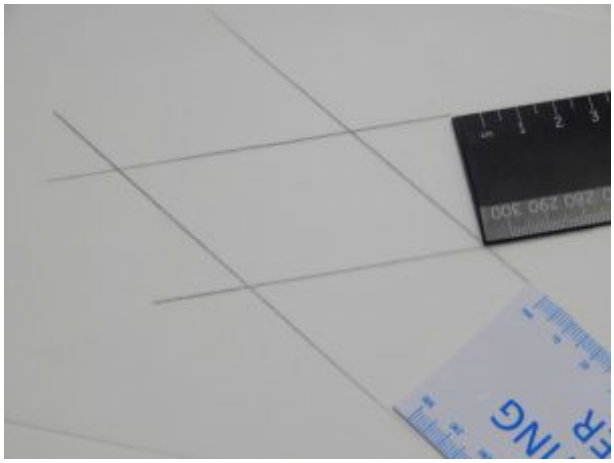
PROPERTIES OF PARALLELOGRAMS



We start from this definition:

A parallelogram is a quadrilateral with both pairs of opposite edges parallel.

In this investigation you will discover some other properties of parallelograms.

Step 1: Use two rulers with different widths to draw a parallelogram on tracing paper or baking paper. Make sure that it is not a rhombus and that the adjacent edges are not equal in length.

Step 2: Place a second piece of tracing paper over the first and copy the parallelogram.

Step 3: Slide the second tracing paper over the first to compare the lengths of the opposite edges of the parallelogram.

How do the lengths of the opposite edges compare?

What did you discover? Call this Conjecture 1:
Both pairs of opposite edges of a parallelogram are ...

Step 4: Rotate or flip the second paper to compare the sizes of the opposite angles of the parallelogram.

How do the sizes of the opposite angles compare?

What did you discover? Call this Conjecture 2: Both pairs of opposite angles of a parallelogram are ...

Step 5: What can you discover about the point of intersection of the diagonals? Draw or fold the two diagonals of the parallelogram. Place a dot on their intersection.

Fold to compare the lengths of the two segments on each diagonal.

What did you discover? Call this Conjecture 3. It could be: The point of intersection of the diagonals of a parallelogram ...

Step 6: Now fold the diagonals of one of the sheets and compare angles between the diagonals. Are the angles equal in size?

Do the diagonals bisect the angles? Justify your answer.

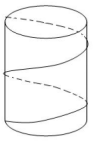
Step 8: Use the paper with the folded diagonal, rotate it and see if it the area of the triangle formed by the diagonal fits on to the triangles of the second paper.

What did you discover? Call this Conjecture 4. It could be: The diagonal of a parallelogram …

(Idea taken from: Michael Serra, "Patty Paper Geometry", Key Curriculum Press, 1994, pp 88 – 89)

HELP

This is a parallelogram made from the core of a toilet roll. It has been pulled apart very carefully, making the curved surface of the cylinder flat, so that the spiral (a helix) became two parallel edges of the parallelogram. To make it stay flat it was stuck down on the top of a desk. Try this for yourself.



If the opposite edges are parallel, which angles are equal?

If the opposite edges are parallel, which angles add up to 180?

You have the original parallelogram that you drew and a copy on tracing paper.

By matching lengths and angles in the copy to lengths and angles in the original, try to discover which lengths and angles in a parallelogram are equal.

What can you discover this way about the diagonals of a parallelogram and the angles they form?

What can you discover about the opposite edges or angles of a parallelogram?

NEXT

You started with this definition:

A parallelogram is a quadrilateral with both pairs of opposite edges parallel.

Draw one of the diagonals so that the parallelogram is split into two triangles.

Knowing that the opposite edges are parallel, what can you say about the angles?

Look for congruent triangles.

Can you prove your conjectures?

NOTES FOR TEACHERS

SOLUTION

These are only conjectures. Once convinced of these facts learners should go on to find proffs that they are in fact true.

Conjecture 1: Both pairs of opposite edges of a parallelogram are **equal.**

Conjecture 2: Both pairs of opposite angles of a parallelogram are **equal.**

Conjecture 3: The point of intersection of the diagonals of a parallelogram **bisects each diagonal**. Note that the two diagonals are not equal in length.

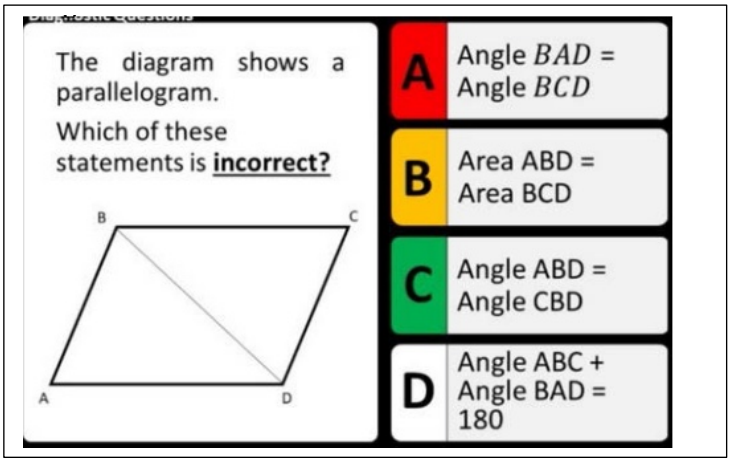
Conjecture 4: The diagonal of a parallelogram bisects it's area

DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes. It can be used before or after the lesson.

Show the question to the learners and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1.Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2.It is important for learners to explain the reason for their answer because it helps them to clarify their own thinking and to develop communication skills.

3.Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4.**Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers**

5. Notice if there is a change and who gave right and wrong answers.

The correct answer is: C

Possible misconceptions:

A. Possibly the learner did not select the intended angles or does not know that the opposite angles are equal.

B. Possibly the learner does not recognise these as congruent triangles.

D. Learners should recognise the two angles add up to 180 as they are co-interior with BC parallel to AD.

https://diagnosticquestions.com

Why do this activity?

This activity helps learners to notice the properties of parallelograms that seem apparent and to make conjectures about what appears to be true about parallelograms. It should then be used as an incentive to the learners to use and apply what they know about congruent triangles in order to prove the conjectures that they have made.

Learning objectives

In doing this activity students will have an opportunity to discover for themselves the properties of parallelograms.

Generic competences

In doing this activity students will have an opportunity to learn to use the correct vocabulary and to get into the habit of checking and asking themselves whether the answer makes sense.

Suggestions for teaching

Resources needed: Pencils. Rulers of different widths. Tracing paper or waxed baking paper.

Ideally the learners should have their own copies of the question on pages 1 and 2. Learners who find it difficult to understand the question, and to get started, should be given the HELP strip. There is a lot of reading, but this is excellent practice for learners. Think of the life skills that will give them a better chance of a good future. Maths facts in their heads alone are useless without skills like reading comprehension and the ability to use and apply what they have learnt. By reading the question for the learners you do not help them either to be able to read exam questions, or to develop life skills.

After about 10 minutes get the learners to work in pairs, to compare what they understand and what they notice, to explain their ideas to each other, and perhaps to produce a joint solution.

To help them, if they seem to need it, you can ask Key Questions, but stop yourself from telling them what to do.

For each conjecture, after most of the class have finished the steps in the work that led them to make that conjecture, you can ask some of the learners to explain the conjecture and why they believe in it.

Make sure that, by the end of the lesson, the learners understand that these are only conjectures and still need to be proved.

Key questions

- Did you answer the question?
- You know that the opposite edges are parallel, what does that tell you about the angles?
- You know that the opposite edges are parallel, so which angles add up to 180^{O}.
- What did you discover about the opposite edges of a parallelogram?
- What did you discover about the opposite angles of a parallelogram?
- What did you discover about the diagonals of a parallelogram and the angles they form?

Follow up

Properties of quadrilaterals https://aiminghigh.aimssec.ac.za/years-8-10-properties-of-quadrilaterals/
Tessellating quadrilaterals https://aiminghigh.aimssec.ac.za/years-7-12-tessellating-quadrilaterals/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6