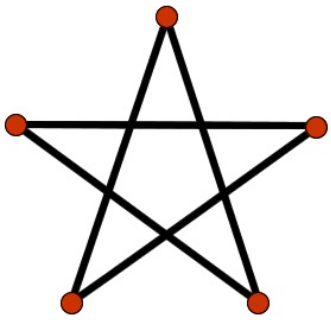
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

PATH TO THE STARS

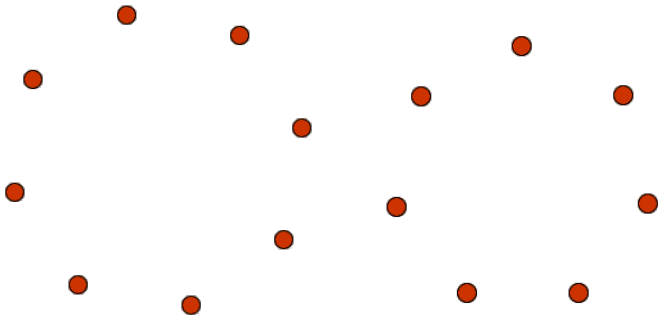


Is it possible to draw a 5-pointed star without taking your pencil off the paper?

Is it possible to draw a 6-pointed star in the same way without taking your pencil off the paper? Remember that you shouldn't join points that are right next to each other and you can't go over the same line twice. Your lines must run straight from one point of the star to another. Try it!

What about 7-pointed and 8-pointed stars?

What about a 4-pointed star? Have a go:



Do you think you'll be able to draw a 9-pointed star without taking your pencil off? Why?

What happens with more points?

HELP

Use the dotty circles on page 2 and draw some more circles of your own.

Start with circles that have 7 dots. Number the dots 0, 1, 2, 3, 4, 5 and 6.

Join every point to the point next to it until you get back to the starting point.

Join 0 to 2 continue joining every alternate point until you get back to the start.

Join 0 to 3 continue joining every third point until you get back to the starting point

Join 0 to 4 continue joining every fourth point until you get back to the starting point

Join 0 to 5 continue joining every fifth point until you get back to the starting point

Join 0 to 6 continue joining every sixth point until you get back to the starting point

What do you notice? Describe your patterns.

NEXT

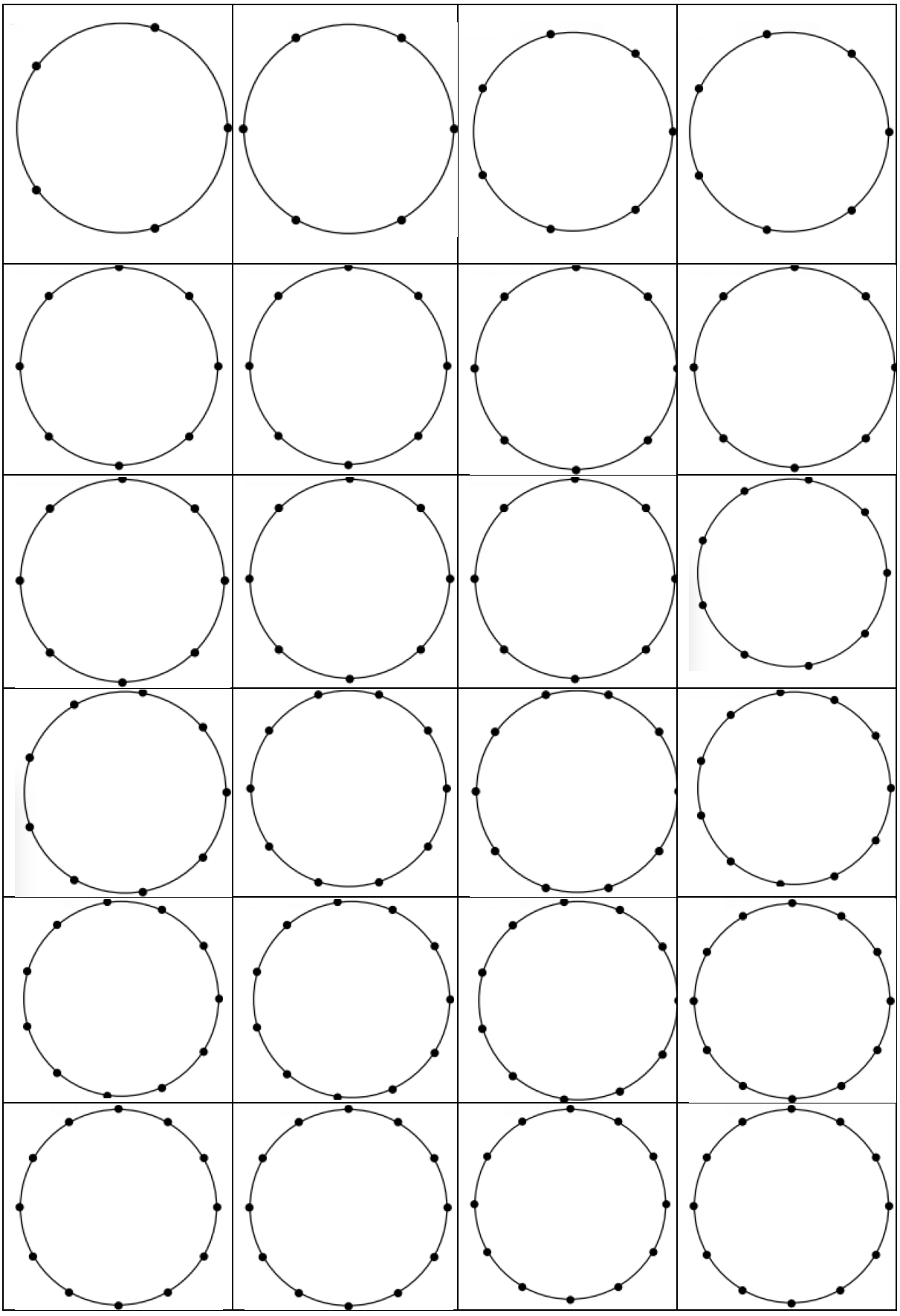
Part 1: Clocks, days of the week and seasons of the year all occur in repeating cycles. Investigate the connections between these cycles and the star patterns formed by joining dots on a circle.

Part 2: Use the sheet on page 3, shade squares containing multiples of 2, 3, 4, 5 etc on the number grids and investigate the patterns.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Part 3: Shade the grid shown here as follows:

- Put a circle around the number 2.
- Cross out all the other multiples of 2.
- Put a circle around the number 3.
- Cross out all the other multiples of 3.
- Put a circle around the number 5.
- Cross out all the other multiples of 5.
- Put a circle around the number 7.
- Cross out all the other multiples of 7.
- Now make a list of all the numbers that are NOT crossed out except the number 1.
- What do you notice about the numbers on your list?



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 8

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 9

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 10

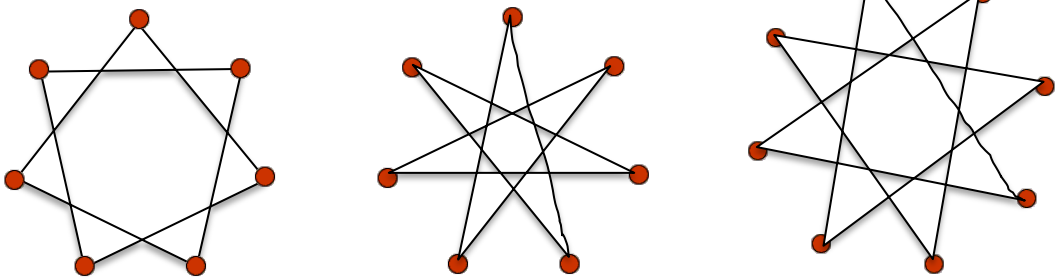
NOTES FOR TEACHERS

SOLUTION

It is possible to draw a 5-pointed star without taking your pencil off the paper by joining every alternate dot. If you count around the circle 0, 1, 2, 3, 4 then joining every alternate dot means joining multiples of 2: 0, 2, 4, 1, 3, 0 -getting back to the start.

This is 5-clock arithmetic on a 5 point clock. Compare this to the 12-hour clock, the 7-day week and the 12-month year.

It is not possible to draw a **6-point star** without taking your pencil off the paper. By joining every alternate dot you get 2 triangles but you have to take your pencil off the paper. By joining a dot to the 3rd dot round the circle you get a diameter. It is impossible because 2 and 3 are factors of 6.



For a **7-point star** you can join multiples of 2 and multiples of 3 giving different stars. Multiples of 4 or 5 make the same stars joined by an anti-clockwise path.

0, 2, 4, 6, 8, 10, 12, 14 0, 3, 6, 9, 12, 15, 18, 21 0, 3, 6, 9, 12, 15, 18, 21, 24

Clock arithmetic
0, 2, 4, 6, 1, 3, 5, 0 0, 3, 6, 2, 5, 1, 4, 0 0, 3, 6, 1, 4, 7, 2, 5, 0

For **8** points join every 3rd dot to get an **8-point star**.

It is impossible to draw a **4-point star** as 2 is a factor of 4.

For **9 points you can draw 2 different stars**:

0, 2, 4, 6, 8, 10, 12, 14, 16, 18 and 0, 3, 6, 9, 12, 15, 18, 21, 24, 27

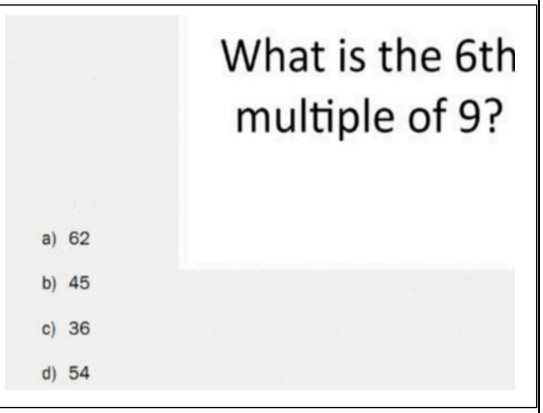
For n points you can draw stars joining multiples of any number that is not a factor of n.

DIAGNOSTIC ASSESSMENT It can be used before or after the lesson.

Show the question to the learners and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer because it helps them to clarify their own thinking and to develop communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.



4. Ask the class to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers

The correct answer is: D https://diagnosticquestions.com

Why do this activity?

This activity gives a simple introduction to ideas of factors and multiples. It builds on learners' understanding of counting in 2's or 3's or … that is on knowledge of multiplication tables. It gives learners a concrete experience on which to base their mathematical reasoning about how to draw the stars, when it is possible and when not, and which multiplication tables to use. As an option, it can be related to, and explained by, clock arithmetic.

Learning objectives

In doing this activity students will have an opportunity to develop understanding and fluency in the use of the language of factors, multiples and common multiples.

Generic competences

In doing this activity students will have an opportunity to **develop visualization** and skill to interpret or create images to represent concepts.

Suggestions for teaching

1) Start with - Clapping on Multiples.

This is a whole class activity that can be used for a short time in different lessons to build an understanding of multiples and common multiples. Everyone counts 1, 2, 3, 4, 5… clapping on the multiples of 2 up to about 30. Clap and speak loudly on the even numbers and don't clap and only speak very quietly on the odd numbers.

Now everyone counts and claps on the multiples of 3 up to about 30. Clap and speak loudly on the multiples of 3 and don't clap and whisper on all the other numbers.

Now do the same for multiples of 5.

Now split the class into 3 groups and appoint 3 leaders.

Group A Counts and claps on multiples of 2.

Group B Counts and claps on multiples of 3.

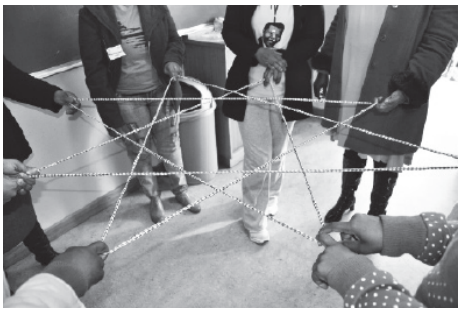
Group C Counts and claps on multiples of 5.

Predict what you will hear. Which numbers will be loud?

Choose other multiples and repeat this activity. Each time predict what you will hear before you clap and talk about it afterwards.

2) **People Maths – Making Stars.** Start with 5 people standing in a circle.
They are numbered 0, 1, 2, 3, and 4.
The ball of string is given to 0 who holds the end of the string and passes the ball to 1.
After that the ball is passed around the circle, unrolling the string and pulling it taut.
It goes from 0 to 1 to 2 to 3 to 4 then back to 0.
Ask what shape has been made (pentagon).
Ask 'what was the rule?' (pass to next person).

Now ask the learners to start again.
0 passes the ball to 2 and it then goes to 4, then to 1 then to 3 then back to 0.
What shape is this?
Ask 'what is the rule? (pass to every alternate person in the circle).



Next choose eight learners to stand in a circle.
Number them 0, 1, 2, etc.
The ball of string is given to 0 and then to 1, then 2 and passed
around the circle, unrolling it and pulling it taut.
Ask what shape has been made (octagon).
Ask 'what was the rule?' (pass to next person).

Now ask the learners to start again. Pass the string from 0 to 2 to 4 and so on missing out one person each time until the string gets back to 0.
Ask what shape has been made (square).
Ask 'what was the rule?' (two times table, multiples of 2).
Record the numbers in order – 0, 2, 4, 6, 0 – somewhere where everyone can see them.

Start Again
Now ask the learners to pass the string from 0 to 3 to 6 and so on until the string gets back to 0.
Ask what shape has been made (eight pointed star).
Ask 'what was the rule?' (three times table, multiples of 3).
Record the numbers in order – 0, 3, 6, 1, 4, 7, 2, 5, 0 – where everyone can see them.

3) **Patterns on a circular geoboard –** Demonstrate the same patterns on a circular geoboard if one is available.

4) **Joining dots drawn on a circle –** Use the circles on page 2. Number the points and record the patterns created with the string.

   Encourage discovery. Ask the learners to explore the patterns made using 8 dots and the 4, 5, 6 and 7 times tables. Ask what is special about the number you get to when you get back to the starting point?

   Use the **One-Two-Four-More** strategy. The learners should **work alone** for a short while, **then in pairs** to discuss and try to explain what they notice about the patterns. Then the pairs can make up **foursomes** for further discussion, and finally have a **whole class discussion**. Draw out the learners' ideas and reinforce the use of the words, factors, multiples and common multiples.

   Ask the learners to investigate the different patterns they get joining points according to the different times tables using the sheet on page 2.

   Have a class discussion in which the learners share ideas and explanations of which multiples make stars and which do not and why?

Key questions

- Have you tried…
- What did you notice …
- Can you explain …
- What happens when you pass the string to the person next to you or join adjacent dots?
- Can you explain why it's impossible to join dots to make a 6-point star?
- What connection does this have to a clock, or to the days of the week?

Follow up

Primary

Strip Patterns https://aiminghigh.aimssec.ac.za/years-3-7-strip-patterns/
Multiple Patterns https://aiminghigh.aimssec.ac.za/years-5-8-multiple-patterns/

Lower Secondary

Prime Sieve https://aiminghigh.aimssec.ac.za/years-6-9-prime-sieve/
Turning Cog Wheels https://aiminghigh.aimssec.ac.za/years-6-10-turning-cogwheels/
Shifting Times Tables
https://aiminghigh.aimssec.ac.za/years-7-9-shifting-times-tables/
Factors and Multiples Game

Upper Secondary

Clock Arithmetic and Envelopes
https://aiminghigh.aimssec.ac.za/years-8-12-clock-arithmetic-and-envelopes/
Factors and Multiples Game
https://aiminghigh.aimssec.ac.za/years-6-12-factors-and-multiples-game/
Sum On
https://aiminghigh.aimssec.ac.za/years-10-12-sum-on/