AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

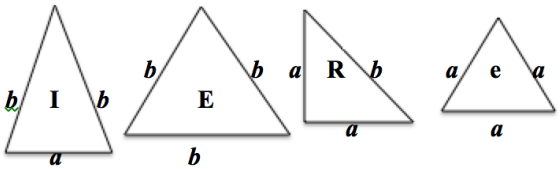
TRIANGLES is the theme for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 5 to 17+

The TRIANGLES TO TETRAHEDRA activity was designed for Secondary

Just choose whatever seems suitable for your group of learners.

TRIANGLES TO TETRAHEDRA



A tetrahedron (plural tetrahedra) is a solid with four triangular faces. Suppose you have an unlimited supply of triangles of the 4 types shown in the diagram.

How many different tetrahedra can you make with them?

Type R are right angled isosceles triangles with sides a, a and b units.
Type E are large equilateral triangles with sides of length b units.
Type e are small equilateral triangles with sides of length a units.
Type I are isosceles triangles with sides of length a, b and b units.

How can you be sure you have found them all?

HELP

To make all the tetrahedra you need:

8 large equilateral triangles,
8 small equilateral triangles,
10 isosceles triangles and
14 right angled triangles and some spares to experiment with.

If you work in a group to share the task, you should agree a system so each person looks for a different type of tetrahedron.

Cut out the triangles from scrap card using the template. First prick through the vertices to mark them. To make the tetrahedra, stick the edges together with selotape or tabs.

You could start by making:

the tetrahedron eeee with 4 e-type triangles,
the tetrahedron eIII with one e-type and 3 I-type triangles and
the tetrahedron eRRI with one e-type, two r-type and one I-type triangles.

Then look for seven more tetrahedra.

NEXT

Describe the symmetries of the tetrahedra, mentioning planes of symmetry, describing rotational symmetry and the identifying regular solids.

Template of triangles for making cardboard models.

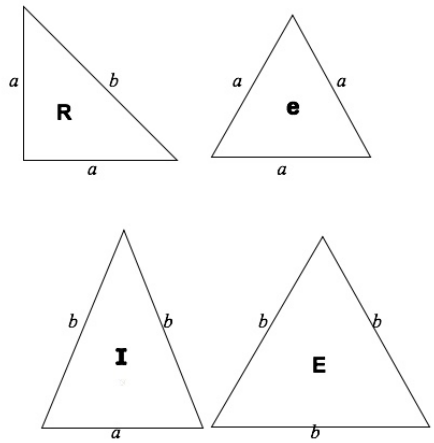
Use scrap card and prick through the vertices of the triangles to mark them on the card. Join the prick holes to mark the triangles.

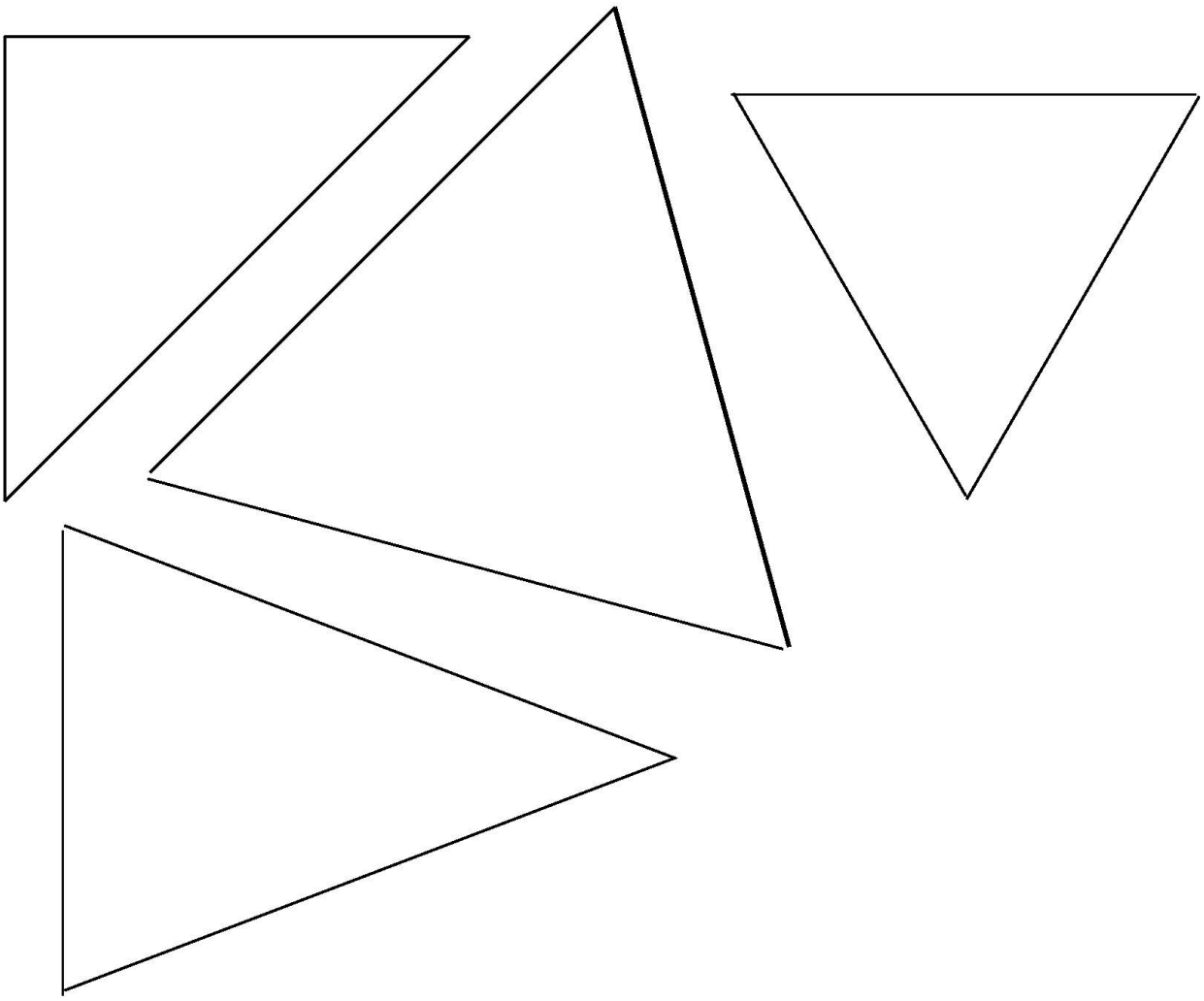
Check the lengths of the edges of the triangles are correct, then cut them out.

To make all the tetrahedra you need

- 8 large equilateral triangles,
- 8 small equilateral triangles,
- 10 isosceles triangles and
- 14 right angled triangles.

Make as many differently shaped tetrahedra as you can using 4 of these triangles for each tetrahedron.





INCLUSION AND HOME LEARNING GUIDE

THEME: TRIANGLES

Resources: Scrap card (save packets from the kitchen). Scissors, pencils, rulers.The best sticky tape to use is the sort that can be painted but any sticky tape will do. Pins to prick through the vertices of the triangles on the template to mark them on scrap card. Allowing for spares make: 12 E-type triangles, 12 e-type triangles, 14 I-type triangles & 18-R type triangles.

Young children

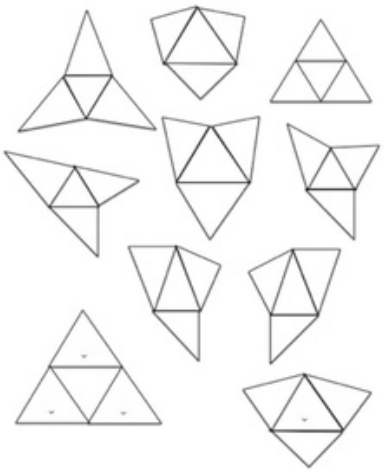
The adult could make the triangles beforehand and help the children to make the tetrahedra.

A good starting activity is to tip the triangles out of a bag and ask the children to sort the triangles into four sets.

When making the tetrahedra the children can look for triangles that will fit together by matching edges. There should be a lot of talking about the similarities and the differences between the triangles.

When the models are finished the young children can paint and decorate them.

Upper Primary



Children of ages 8 to 11 could make the triangles with help from older siblings and an adult.

They could use this picture as a guide and make the ten tetrahedra.

As with the younger children, there should be a lot of talking about the similarities and differences between the triangles.

Lower Secondary

The learners can first make the triangles. If there is a group they can share the marking and cutting out. Give them some scrap card, scissors, a pin, sticky tape and copies of the template. By pricking through the vertices of the triangles on the template they can mark out the triangles on scrap card and then cut them out.

Put one of the equilateral triangles on the table with space around it. Explain that you want to make a tetrahedron with this triangle as the base. Ask a learner to choose another triangle and to place it edge to edge with the first one to begin to make a net. Then the learners have to find two more triangles that will match edge to edge to make

the net and stick the edges together to make the first tetrahedron. Tell them that you want them to make as many **different** tetrahedra as possible using these triangles.

Share discoveries and count the number of different tetrahedra that have been found. Learners should handle the models and look at them carefully from different angles and talk about what they notice. If they have made some, but not all, of the tetrahedra, you might give a few hints, not by telling them what to do but by using key questions

Then introduce some discussion of the properties of the tetrahedra, for example symmetry.

Key questions

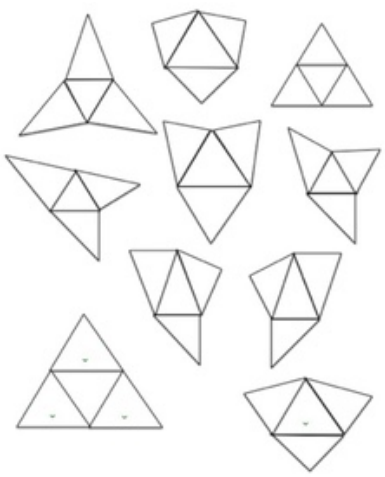
- Which triangles have edges to match that one (or those two)?
- Have you checked that those edges are the same length?
- Are there 2 nets with exactly the same 4 triangles that will make a different tetrahedron?
- Do you have another net with the same 4 triangles arranged in a different way that could be a mirror image.

Upper Secondary

You should be able to do the activity from the written instructions on page 1 and give the reasons why the ten tetrahedra are the complete set of all possibilities.

Investigate all the possibilities by taking each of the triangles in turn and finding all the combinations of 3 triangles that can be attached around it so that the adjacent edges of the outer triangles are equal in length. Make a summary of the symmetries of each of the tetrahedra.

SOLUTION



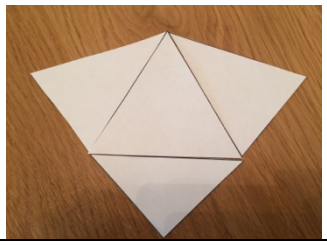
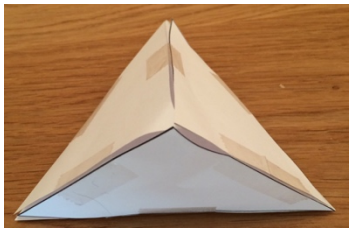
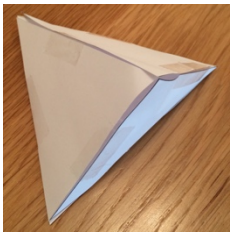
The diagram shows the nets of 10 tetrahedra made from the given triangles, two of which are mirror images.

To make a set of 10 tetrahedra you need 8 E-type; 8 e-type; 10 I-type and 14 R-type triangles. But you need more triangles to allow for experimentation.

To be sure to find all the tetrahedra take each of the triangles in turn and find all the combinations of 3 triangles that can be attached around it so that the adjacent edges of the outer triangles are equal in length.

Solutions can be labelled:
eeee; eIII; eRRI; eeRR; EEEE; ERRR; EEII; EIIR and IIRR (2 versions).

The photos below show a net with 1 large equilateral, 2 isosceles and 1 right angled triangle and 2 views of the same tetrahedron. You need to make the tetrahedra and handle them to be able to appreciate their properties.

Why do this activity?

This practical activity gives learners experience of designing their own nets for tetrahedra, visualising the solids and working systematically to solve a problem. Templates of the triangles are provided so that learners can easily cut out triangles from scrap card.

Learning objectives

In doing this activity students will have an opportunity to:

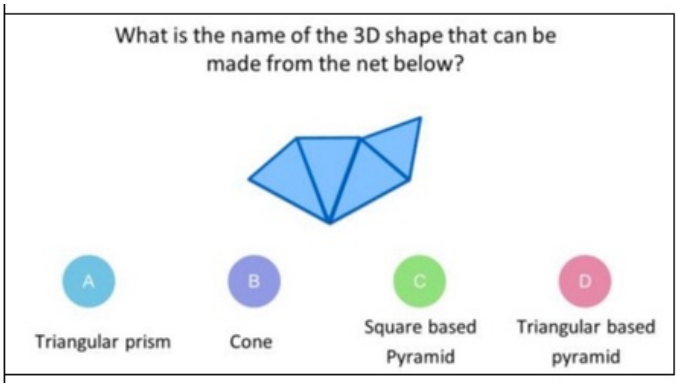
- revise nets of geometric solid;
- gain experience of working systematically to solve a problem.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize,** interpret and create visual images of 3D objects;
- **persevere and work systematically** to investigate all possible cases;
- groups could **work together collaboratively to share the task.**

DIAGNOSTIC ASSESSMENT This can be used before or after the lesson.Show this question and say: **"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".**



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and get a better understanding.
3. In a group other learners should listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change, and who gave right and wrong answers

The correct answer is: D

Misconceptions: A. Calling a pyramid a prism, and vice versa, is a common misconception.

B. A cone has a circular base and a face made from a sector of a circle.

C. A square based pyramid is made from a square and 4 triangles. https://diagnosticquestions.com

Follow up

Paper stick tetrahedron
https://aiminghigh.aimssec.ac.za/years-4-7-paper-stick-tetrahedron/
Cube nets https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
Cut nets https://aiminghigh.aimssec.ac.za/years-6-10-cut-nets/
Icosahedron Puzzle https://aiminghigh.aimssec.ac.za/years-7-10-icosahedron-puzzle/
Tet trouble https://aiminghigh.aimssec.ac.za/years-9-10-tet-trouble/

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum links:

MATHS TOYS

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Subscribe to the MATHS TOYS YouTube Channel
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13