SHIFTING TIMES TABLES

Count 1 to 100 in 4's saying the 4 times table: 4, 8, 12, 16, 20, … 80, 84, 88, …

Now shift the 4 times table up 3 and count in 4's again starting from 7:

7, 11, 15, 19, 23, … 83, 87, 87, 91, …

This diagram shows a Function Machine, also called a mapping diagram. You input numbers into the black box and the machine outputs a number.

How does this mapping diagram represent shifting the 4 times table by 3?

Which tables were shifted to give the following sequences? By how much?
Explain how you know.

(a) 7, 12, 17, 22, 27, … 82, 87, 92, …

(b) 9, 11, 13, 15, 17, … 89, 91, 93, …

(c) 13, 20, 27, 34, 41, … 83, 90, 97, …

(d) 4, 7, 10, 13, 16, … 79, 82, 85, …

(e) 5, 11, 17, 23, 29, … 71, 77, 83, …

Match each of the following rules for mapping the input numbers n = 1, 2, 3 … to output numbers in the sequences above. Explain how you do this.

(1) $n \rightarrow 7n + 6$

(2) $n \rightarrow 5n + 2$

(3) $n \rightarrow 3n + 1$

(4) $n \rightarrow 6n - 1$

(5) $n \rightarrow 2n + 7$

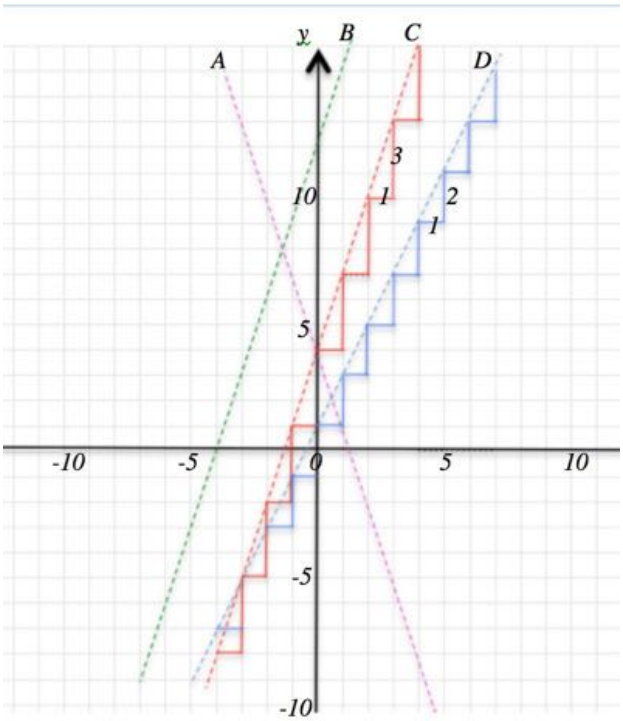
Now make up your own sequence and rule.

HELP

How well do you know your multiplication tables? It's a handicap not to know your tables but it's never too late to learn. You might like to play the game at https://www.education.com/game/radar-arrays and see how speedy you can get.

Counting 4, 8, 12, 16 ,… is called SKIP COUNTING and the numbers in the sequence are called MULTIPLES OF 4. It might help you to be more successful in maths if you practise skip counting up to 100 (or 150) in all the sequences 2s, 3s, 4s, all the way up to 10s (that's an easy one). The best way to do this is to write out the sequence. Then read it aloud to yourself a few times. Then cover it so you can't see it. Then repeat it without looking at the list you wrote down until you can do it without hesitation.

NEXT



Continue the following sequences for the next 3 terms and continue the sequences backwards to the previous 3 terms:

7, 10, 13, 16, 19, ...

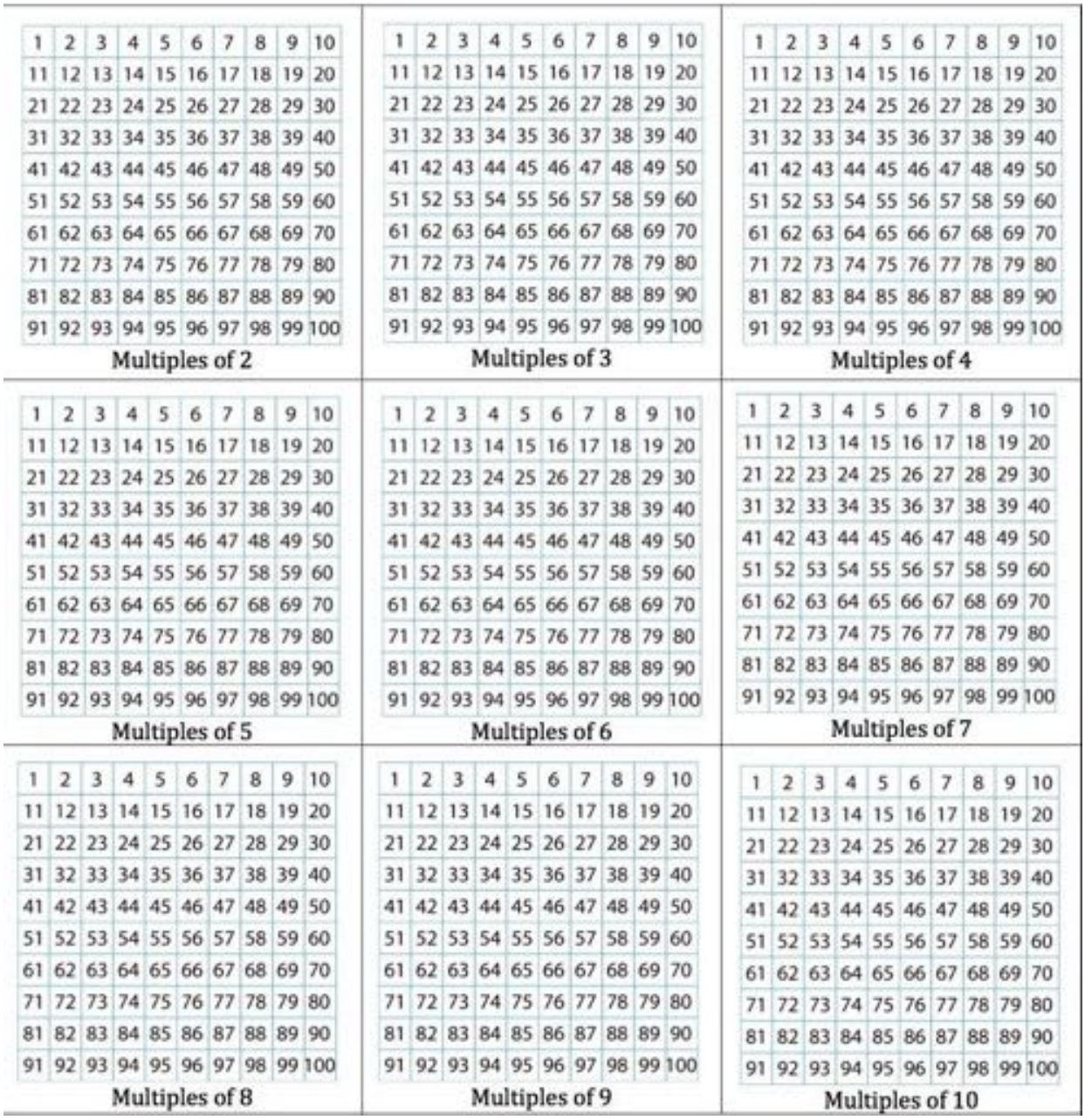
15, 18, 21, 24, 27, ...

1, -2, -5, -8, -11, ...

The first 2 sequences come from a multiplication table shifted up. Which multiplication table is it? What do you notice about the third sequence? How do the sequences relate to the red steps in the diagram?

How do these sequences relate to the lines A, B and C in the diagram? Which sequence relates to which line?

https://aiminghigh.aimssec.ac.za/steps/



Multiples of 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 8

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 9

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

See https://aiminghigh.aimssec.ac.za/multiple-patterns/

NOTES FOR TEACHERS

SOLUTION to the SHIFTING TIMES TABLES ACTIVITY

The sequences are given by the mapping n → a*n* + b where *a* is the multiplication table and *b* is the shift so *n* = 1 gives the first term of the sequence.

Rule	Sequence
(1) $n \rightarrow 7n+6$	(c) 13, 20, 27, 34, 41, … 83, 90, 97, …
(2) $n \rightarrow 5n+2$	(a) 7, 12, 17, 22, 27, … 82, 87, 92, …
(3) $n \rightarrow 3n+1$	(d) 4, 7, 10, 13, 16, … 79, 82, 85, …
(4) $n \rightarrow 6n - 1$	(e) 5, 11, 17, 23, 29, … 71, 77, 83, …
(5) $n \rightarrow 2n+7$	(b) 9, 11, 13, 15, 17, … 89, 91, 93, …

DIAGNOSTIC ASSESSMENT This could be used at the end of your sessions.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Count in steps of 6: 6, 12, 18, …
What is the 12th number in this number pattern?

A. 66
B. 72
C. 90
D. 78

1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and get a better understanding.
3. If you have a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Again ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and notice who gave right and wrong answers.

The correct answer is B. Learners who know their multiplication tables should get this right.

Possible misconceptions:

A. Students giving this answer may have skip counted ending up with the 11th number not the 12th.

C. Perhaps they did not understand the question and counted 12 more steps after 18.

D. Students giving this answer may have skip counted ending up with the 13th number and not the 12th.

https://diagnosticquestions.com

Why do this activity?

This activity builds on what learners already know about multiplication tables and counting in 2s, 3s, 4s etc, and what they can easily recognise as number patterns and sequences. It takes them forward to the concept of a mapping or function.

By thinking of shifting the whole multiplication table they are naturally led to think about many pairs of numbers simultaneously, for example: (1, 7), (2, 11), (3, 15), (4, 19), (5, 23) … In this activity the learners concentrate on shifting different multiplication tables and working out the corresponding mappings.

Learning can then progress with the activity
Steps https://aiminghigh.aimssec.ac.za/steps/ moving on from shifting multiplication tables to using the pairs of numbers as coordinates of points and then joining the points to form a straight line.

The beauty of this idea is that counting in 2s leads to a line with gradient 2 (1 across, 2 up), counting in 3s leads to a line with gradient 3 (1 across, 3 up) etc. The Shifting Tables activity focuses on the rules for functions that map the natural numbers to the terms of arithmetic sequences and so the activity can be used later as an introduction to a lesson on arithmetic sequences.

Learning objectives

In doing this activity students will have an opportunity to:

- practise and learn multiplication tables and to see the link to sequences;
- meet the idea that multiplication tables and lists of multiples lead to sequences;
- work in an informal way with rules for sequences (function rules).

Generic competences

In doing this activity students will have an opportunity to think mathematically, and to make connections between what they have learnt in order to gain a deeper understanding of multiples and factors, of straight line graphs and of functions.

Suggestions for Teaching

THE SHIFTING TIMES TABLE ACTIVITY from page 1.

If possible, start with **all the learners** in your class counting in 4s aloud and challenge them to count up to 100 in unison without making a mistake.

Then write the second sequence 7, 11, 15, 19, 23, … 83, 87, 91, … on a board and ask the learners how they worked out the next number after 23 where there is a gap for this sequence.

Write the two sequences 4, 8, 12, 16, 20, … 80, 84, 88…

7, 11, 15, 19, 23, … 83, 87, 91… one below the other and ask the learners what they notice.

Ask what is the same and what is different about the two sequences. Your purpose should be to lead learners to suggest that the 4 times table has shifted up by 3 **without you telling them that**. You want as many learners as possible to work it out for themselves.

For younger learners talk about multiplication tables which all the learners need to

understand. You could introduce the word 'multiples'.

For older learners who have met the concepts, use the language of multiples, factors, common multiples etc etc.

Show the learners the function box diagram for the rule or mapping $n \rightarrow 4n + 3$. It is also called a mapping diagram.

Ask them what the output will be for the input n = 1. Then for n = 2, then n = 3 … What do they notice?

Give out copies of the worksheet (page 1) and ask the learners to work out what multiplication table was used in each case and how it has been shifted.

It is best to encourage your learners to read questions for themselves and decide for themselves what to do. If you have a group, then you could decide that the learners should work alone for 10 minutes and then compare their findings with a partner before sharing their findings with the whole group.

For learners who finish the Shifting Times Tables activity quickly, tell them to make up some sequences based on shifting multiplication tables, to decide on the corresponding rules that map the counting numbers 1, 2, 3, 4, … to their sequences, and to draw a mapping diagram. These learners can test you, or other learners in the group, to find the mapping for their newly created sequences.

Ask learners to explain their answers and then to summarise what has been learned and the connections between the multiplication tables, the sequences and the rules for the functions.

Key questions

- What is the same and what is different about those 2 sequences?
- What is the next term in that sequence? And the one after that? How did you find those terms?
- What do you add on each time to get the next number in the sequence?
- What multiplication table gives you a sequence going up like that?
- By what number has the multiplication table been shifted?

Follow up

Multiple Patterns https://aiminghigh.aimssec.ac.za/multiple-patterns/

Steps https://aiminghigh.aimssec.ac.za/steps/

Mind Reader https://aiminghigh.aimssec.ac.za/mind-reader

Building Functions https://aiminghigh.aimssec.ac.za/building-functions/

Undoing https://aiminghigh.aimssec.ac.za/undoing/