AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

The theme for this INCLUSION AND HOME LEARNING GUIDE with related learning activities for all ages from 4 to 18+ is
MULTIPLICATION TABLES AND NUMBER PATTERNS

Just choose whatever seems suitable for your group of learners

The SHIFTING TIMES TABLES activity was designed for Lower Secondary

SHIFTING TIMES TABLES

Count 1 to 100 in 4s saying the 4 times table: 4, 8, 12, 16, 20, … 80, 84, 88, …

Now shift the 4 times table up 3 and count in 4s again starting from 7:

7, 11, 15, 19, 23, … 83, 87, 87, 91, …

This diagram shows a Function Machine, also called a mapping diagram. You input numbers into the black box and the machine outputs a number.

How does this mapping diagram represent shifting the 4 times table by 3?

Which tables were shifted to give the following sequences? By how much?
Explain how you know.

(a) 7, 12, 17, 22, 27, … 82, 87, 92, …

(b) 9, 11, 13, 15, 17, … 89, 91, 93, …

(c) 13, 20, 27, 34, 41, … 83, 90, 97, …

(d) 4, 7, 10, 13, 16, … 79, 82, 85, …

(e) 5, 11, 17, 23, 29, … 71, 77, 83, …

Match each of the following rules for mapping the input numbers n = 1, 2, 3 … to output numbers in the sequences above. Explain how you do this.

(1) $n \rightarrow 7n + 6$

(2) $n \rightarrow 5n + 2$

(3) $n \rightarrow 3n + 1$

(4) $n \rightarrow 6n - 1$

(5) $n \rightarrow 2n + 7$

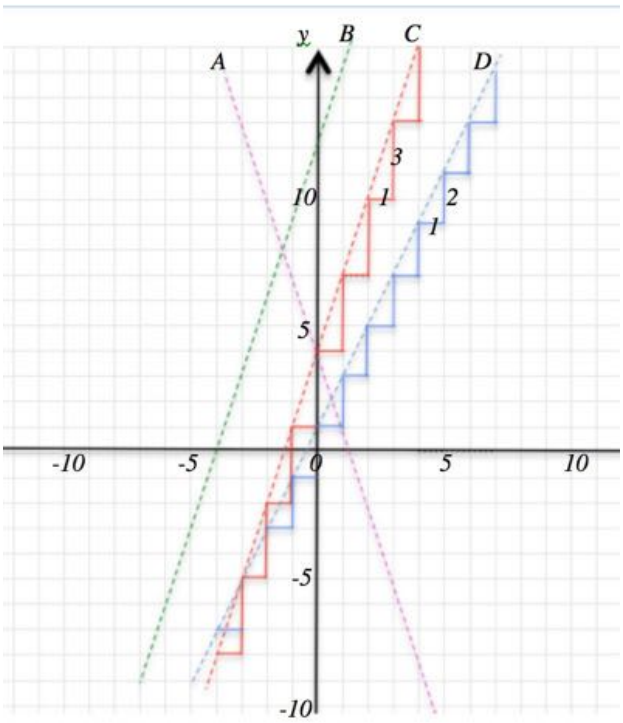
Now make up your own sequence and rule.

HELP

How well do you know your multiplication tables? It's a handicap not to know your tables but it's never too late to learn. You might like to play the game at https://www.education.com/game/radar-arrays and see how speedy you can get.

Counting 4, 8, 12, 16 ,… is called SKIP COUNTING and the numbers in the sequence are called MULTIPLES OF 4. It might help you to be more successful in maths if you practise skip counting up to 100 (or 150) in all the sequences 2s, 3s, 4s, all the way up to 10s (that's an easy one). The best way to do this is to write out the sequence. Then read it aloud to yourself a few times. Then cover it so you can't see it. Then repeat it without looking at the list you wrote down until you can do it without hesitation.

NEXT



Continue the following sequences for the next 3 terms and continue the sequences backwards to the previous 3 terms:

7, 10, 13, 16, 19, ...
15, 18, 21, 24, 27, ...
1, -2, -5, -8, -11, ...

The first 2 sequences come from a multiplication table shifted up. Which multiplication table is it? What do you notice about the third sequence? How do the sequences relate to the red steps in the diagram?

How do these sequences relate to the lines A, B and C in the diagram?
Which sequence relates to which line?

https://aiminghigh.aimssec.ac.za/steps/

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 8

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 9

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples of 10

INCLUSION AND HOME LEARNING GUIDE

THEME: MULTIPLICATION TABLES AND NUMBER PATTERNS

There is work on functions here for all ages, for a week, with related tasks.

Young children

COUNT AND CLAP: count together 1, 2, 3, 4, … and clap on every other number, on 2, 4, 6, …

Then whisper the numbers in between and say the numbers you clap on loudly, even shout them.

Graduate to counting in 2s (without the whispering). This is called SKIP COUNTING. How far can the children go with skip counting?

Then do the same for every third number 3, 6, 9, … and later for every 4^{th} number and so on.

Do this for a few minutes from time to time and, most importantly, make it fun. It could be something parents and carers do to keep children from getting bored on a journey.

Which STEPS can they skip count in?

Upper Primary

Do lots of skip counting and then introduce the word '**multiple**'.

Now count in 3s but every other number is whispered: 3, **6**, 9, **12**, 15, **18** …

When they skip count 6, 12, 18, 24 … they learn that these numbers are multiples of 6.

Multiple Patterns Talk about the pattern here that corresponds to skip counting in 6s, or multiples of 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Describe this pattern. What do you think the patterns for other multiples look like, for example multiples of 2 or 3 or 4 or 5?

On the sheet on page 3, shade in the patterns of other sequences of multiples.

Why do you think that in each pattern there are the same number of unshaded squares across and down between the shaded squares?

What can you say about the patterns for multiples of 2, 4, 6, 8 and 10?

What can you say for the patterns for multiples of 3, 6 and 9?

What can you say about the patterns for 5 and 10?

https://aiminghigh.aimssec.ac.za/multiple-patterns/

ORCHESTRA – a second activity for upper primary if you have 5 or more people in your group

For Home Learning, if you have 5 or more people in your house, and can involve everyone, do the **Orchestra Activity** as a group. It's better with 7 people but, with 5 people you will have the numbers 2, 3, 4 and 5 and take it in turns to be the conductor. 7 people make a small Orchestra. Everyone is given one of the numbers 1 to N (start with 1 to 6).

Everyone counts 1, 2, 3, 4, …. 100 (or they can choose a tune and sing the numbers together!)

People have to clap their hands above their heads when their own number is a factor of the number called.

They stand in rows facing the conductor of the orchestra.

If you have 13 people then the whole group will be lined up like this. A larger number can form lines behind these two rows. Orchestra can be enjoyed by 100s of people.

1 2 3 4 5 6

1 2 3 4 5 6

Conductor

Number 1 claps on every beat
Number 2 claps on 2, 4, 6, 8, …
Number 3 claps on 3, 6, 9, …
Number 4 claps on 4, 8, 12, …
Number 5 claps on 5, 10, 15, …
Number 6 claps on 6, 12, 18, …

Alternatives:

(1) The people can sit in a line of chairs and stand up when the number called is a multiple of their number.

(2) The class can split into groups of 7 and each group rehearse their show. Then each group must perform for the whole class. They can do actions and make noises.

(3) You can have rows of more than 6 people, 1 to 7 or 1 to 8

(4) The whole school can line up outside in ranks with the 1s behind each other, all the 2s behind each other etc. Everyone counts in unison and jumps and claps when the number called is a multiple of their number. This works well with rows of 1 to 6.

Lower Secondary

THE SHIFTING TIMES TABLE ACTIVITY from page 1.

If possible, start with **all the learners** in your class or household together for a short while. Tell the older teenagers that it would be good if they join in to inspire the younger ones.

The whole group should count in 4s aloud. Challenge them to count up to 100. Encourage the youngest children to join in the counting **when they can.** Ask them how they work out the next number each time.

Then write the second sequence 7, 11, 15, 19, 23, … 83, 87, 91, … on a board and ask the learners how they worked out the next number after 23 where there is a gap for this sequence.

Write the two sequences 4, 8, 12, 16, 20, … 80, 84, 88…

7, 11, 15, 19, 23, … 83, 87, 91… one below the other and ask the learners what they notice.

Ask what is the same and what is different about the two sequences. Your purpose should be to lead learners to suggest that the 4 times table has shifted up by 3 **without you telling them that**. Take the older teenagers into your confidence in advance and ask them not to 'give the game away'.

You want as many learners as possible to work it out for themselves. Then the group should chant the 4 times table.

For younger learners talk about multiplication tables which all the learners need to understand. You could introduce the word 'multiples'.

For older learners who have met the concepts, use the language of multiples, factors, common multiples etc etc.

Show the learners the function box for the rule or mapping $n \rightarrow 4n + 3$.

Ask them what the output will be for the input n = 1. Then for n = 2, then n = 3 … What do they notice?

Give out copies of the worksheet (page 1) and ask the learners to work out what multiplication table was used in each case and how it has been shifted.

It is best to encourage your learners to read questions for themselves and decide for themselves what to do. If you have a group, then you could decide that the learners should work alone for 10 minutes and then compare their findings with a partner before sharing their findings with the whole group.

For older learners who finish the Shifting Times Tables activity quickly, tell them to make up some sequences based on shifting multiplication tables, to decide on the corresponding rules that map the counting numbers 1, 2, 3, 4, … to their sequences, and to draw a mapping diagram. These learners can test you, or other learners in the group, to find the mapping for their newly created sequences.

Ask learners to explain their answers and then to summarise what has been learned and the connections between the multiplication tables, the sequences and the rules for the functions.

Key questions

- What is the same and what is different about those 2 sequences?
- What is the next term in that sequence? And the one after that? How did you find those terms?
- What do you add on each time to get the next number in the sequence?
- What multiplication table gives you a sequence going up like that?
- By what number has the multiplication table been shifted?

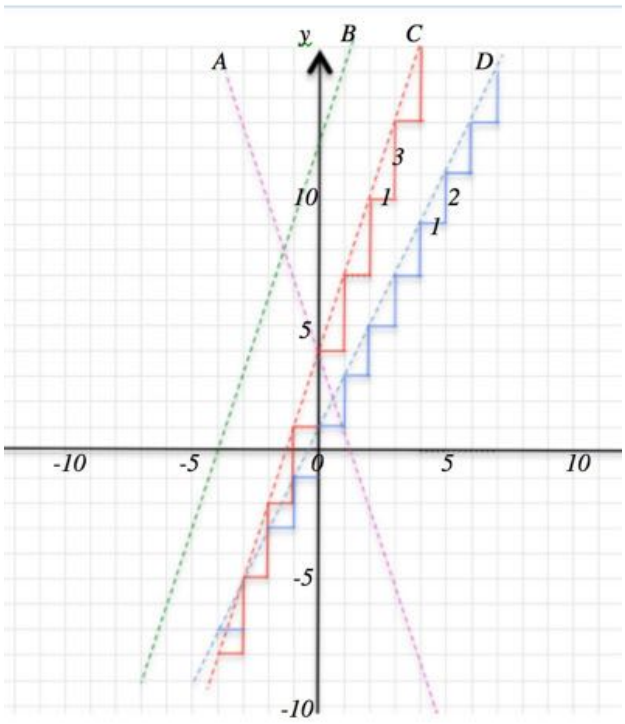
SOLUTION to the SHIFTING TIMES TABLES ACTIVITY

The sequences are given by the mapping n → a*n* + b where *a* is the multiplication table and *b* is the shift so *n* = 1 gives the first term of the sequence.

Rule	Sequence
(1) $n \rightarrow 7n+6$	(c) 13, 20, 27, 34, 41, … 83, 90, 97, …
(2) $n \rightarrow 5n+2$	(a) 7, 12, 17, 22, 27, … 82, 87, 92, …
(3) $n \rightarrow 3n+1$	(d) 4, 7, 10, 13, 16, … 79, 82, 85, …
(4) $n \rightarrow 6n - 1$	(e) 5, 11, 17, 23, 29, … 71, 77, 83, …
(5) $n \rightarrow 2n+7$	(b) 9, 11, 13, 15, 17, … 89, 91, 93, …

Years 9 and 10

Explore the ways that step counting, multiples and the gradients of lines **are connected to** the equations of straight line graphs.



Continue the following sequences for the next 3 terms and continue the sequences backwards to the previous 3 terms:

7, 10, 13, 16, 19, ...

15, 18, 21, 24, 27, ...

1, -2, -5, -8, -11, ...

The first 2 sequences come from a multiplication table shifted up.

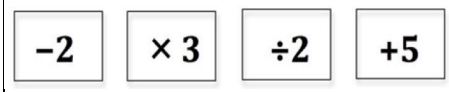
Which multiplication table is it?

What do you notice about the third sequence? How do the sequences relate to the red steps in the diagram?

How do these sequences relate to the lines A, B and C in the diagram?

Which sequence relates to which line?

Years 11, 12 and 13



Amy and her friends have built some functions. They all input the number 1 into their functions. Amy's output is 6, Busi's 1, Chris's 6½ and Dudu's 3½.

Amy's function $a \rightarrow$ [+5] $\rightarrow a+5$

Busi's function $b \rightarrow$ [×3] $\rightarrow$ [-2] $\rightarrow 3b-2$

Chris's function $c \rightarrow$ [÷2] $\rightarrow$ [× 3] $\rightarrow$ [+5] $\rightarrow 3c/2 +5$

Dudu's function $d \rightarrow$ [-2] $\rightarrow$ [× 3] $\rightarrow$ [÷2] $\rightarrow$ [+5] $\rightarrow 3(d-2)/2 +5=3d/2 +2$

Build some of your own functions using the operators
[subtract 2], [multiply by 3], [divide by 2] and [add 5].

Choose inputs and give the corresponding outputs to show how your functions work.

Your inputs can be numbers or variables. Make as many different functions as you can using only 2 of the 4 operations without repetition.

Use the template below to make a poster that you can fill in to show all the different functions that can made by combining 2 of these 4 operations without repetition.

In the first column put the functions that have [subtract 2] first. One example has been shown in the table. See: https://aiminghigh.aimssec.ac.za/building-functions/

HOW MANY DIFFERENT FUNCTIONS CAN YOU FIND USING 2 OF THESE 4 OPERATIONS?			
−2 first	×3 first	+2 first	+5 first
-2 ×3 $6 \rightarrow 4 \rightarrow 12$ $x \rightarrow x - 2 \rightarrow 3(x - 2)$			

Now investigate finding the input that corresponds to a given output:

https://aiminghigh.aimssec.ac.za/undoing/

Why do this activity?

This activity builds on what learners already know about multiplication tables and counting in 2s, 3s, 4s etc, and what they can easily recognise as number patterns and sequences. It takes them forward to the concept of a mapping or function.

By thinking of shifting the whole multiplication table they are naturally led to think about many pairs of numbers simultaneously, for example: (1, 7), (2, 11), (3, 15), (4, 19), (5, 23) … In this activity the learners concentrate on shifting different multiplication tables and working out the corresponding mappings.

Learning can then progress with the activity
Steps https://aiminghigh.aimssec.ac.za/steps/ moving on from shifting multiplication tables to using the pairs of numbers as coordinates of points and then joining the points to form a straight line.

The beauty of this idea is that counting in 2s leads to a line with gradient 2 (1 across, 2 up), counting in 3s leads to a line with gradient 3 (1 across, 3 up) etc. The Shifting Tables activity focuses on the rules for functions that map the natural numbers to the terms of arithmetic sequences and so the activity can be used later as an introduction to a lesson on arithmetic sequences.

Learning objectives

In doing this activity students will have an opportunity to:

- practise and learn multiplication tables and to see the link to sequences;
- meet the idea that multiplication tables and lists of multiples lead to sequences;
- work in an informal way with rules for sequences (function rules).

Generic competences

In doing this activity students will have an opportunity to think mathematically, and to make connections between what they have learnt in order to gain a deeper understanding of multiples and factors, of straight line graphs and of functions.

DIAGNOSTIC ASSESSMENT This could be used at the end of your sessions.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Count in steps of 6: 6, 12, 18, …
What is the 12th number in this number pattern?

A. 66
B. 72
C. 90
D. 78

1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and get a better understanding.
3. If you have a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Again ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and notice who gave right and wrong answers.

The correct answer is B. Learners who know their multiplication tables should get this right.

Possible misconceptions:

A. Students giving this answer may have skip counted ending up with the 11^{th} number not the 12^{th}.

C. Perhaps they did not understand the question and counted 12 more steps after 18.

D. Students giving this answer may have skip counted ending up with the 13^{th} number and not the 12^{th}.

https://diagnosticquestions.com

Follow up

Multiple Patterns https://aiminghigh.aimssec.ac.za/multiple-patterns/

Steps https://aiminghigh.aimssec.ac.za/steps/

Mind Reader https://aiminghigh.aimssec.ac.za/mind-reader

Building Functions https://aiminghigh.aimssec.ac.za/building-functions/

Undoing https://aiminghigh.aimssec.ac.za/undoing/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app or find it on Google Play.